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International asset pricing with heterogeneous agents: Estimation and inference[☆]

Roméo Tédongap [a], Jules Tinang [b,*]

[a] Department of Finance, ESSEC Business School, 3 Avenue Bernard Hirsch, CS 50105 CERGY, 95021 Cergy-Pontoise Cedex, France
[b] University of Groningen, Department of Economics, Econometrics and Finance, Nettelbosje 2, 9747 AE Groningen, The Netherlands

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ABSTRACT

This paper empirically validates (Constantinides and Ghosh's, 2017) heterogeneous-agents consumption-based asset pricing model for predicting expected returns in international equity markets. Using the model's implications, we proxy the unobservable state variable driving income shocks with the principal component of consumption growth cumulants across agents. We confirm that both the level and changes in this cross-sectional consumption risk serve as pricing factors, emphasizing the importance of higher moments like skewness. The estimated structural parameters obtained from the Euler equations are statistically significant and plausible, while the factor risk premium estimates align with theoretical expectations. Our approach effectively explains the emerging versus developed premium, outperforming traditional methods reliant on cross-sectional variance. Our findings, robust across different model specifications and asset menus, highlight the imprecision of consumption-based factor risk premia estimates when limited to developed markets, a limitation mitigated by including emerging markets. The model demonstrates a 60% explanatory power, surpassing the global Fama–French model.

1. Introduction

This paper explores the variation in expected returns across countries' stock indices, building on the premise introduced by Bekaert and Harvey (1995). We extend the heterogeneous-agents consumption-based asset pricing models[1] (Constantinides and Duffie, 1996; Constantinides and Ghosh, 2017) to consider each agent as a country's representative. These representatives face unique consumption shocks and navigate them by investing in international stock markets.[2] While these models typically address household consumption risks within a single country, our approach applies them internationally, given similarities between household-specific risks and national challenges. Just as households encounter specific income risks, countries deal with unique productivity shocks, trade imbalances, and sector downturns, affecting their consumption and overall economic wellbeing.

Our framework focuses on country stock indices as they provide a gateway to a country's economy and major companies, offering exposure and diversification. Motivated by the globalization of financial markets and the increasing ease of cross-border investments,

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* Corresponding author.
E-mail addresses: tedongap@essec.edu (R. Tédongap), j.v.tinang@rug.nl (J. Tinang).

[1] Recent research has focused on the effects of heterogeneity in income and consumption shocks to explain financial market participation, estimate consumer preferences, and price assets (see works like Brav et al., 2002; Vissing-Jorgensen, 2002; Malloy et al., 2009; Storesletten et al., 2004).

[2] By holding a country-specific market portfolio, represented by its stock market index, there is an inherent risk linked to potential drops in that country's overall consumption growth.

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we explore whether actual country expected excess returns compensate for exposure to factors derived from a global consumption-based asset pricing model with heterogeneous agents. Using international data on country stock index returns and consumption growth rates, we examine the cross-sectional implications of the heterogeneous-agents model proposed by Constantinides and Ghosh (2017).

The model assumes that country representative agents have identical recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989) and heterogeneity in their consumption growth rates is modeled as a Poisson process with stochastic intensity. This stochastic intensity is the single-state variable of the model, which, by analogy, we refer to as *cross-sectional consumption risk* throughout the article. In this incomplete market model compatible with imperfect risk-sharing and featuring multiple stochastic discount factors (SDFs), i.e., one SDF per country representative agent, the average SDF derived by integrating out the idiosyncratic shocks experienced by agents, is driven by a trio of variables: the aggregate consumption growth rate (Δc_t), the level of the cross-sectional consumption risk (x_t), and changes in cross-sectional consumption risk (Δx_t). This common SDF serves as the primary tool in our asset pricing tests.

Theoretically, all cumulants of country consumption growth rates are linearly related to the unobserved cross-sectional consumption risk, implying perfect correlations between these cumulants. However, actual correlations vary between 0.66 and 0.79. Using Principal Component Analysis (PCA), we derive a deeper understanding of countries' consumption growth rate diversities and associated risks. The first principal component, weighing 83%, acts as a proxy for this consumption risk, encapsulating information from variance, skewness, and kurtosis. Notably, this PCA-based proxy correlates more strongly with the cumulants (0.89 with variance, −0.90 with skewness, and 0.94 with kurtosis) than pair-wise correlations, aligning better with the model's theoretical expectations than simply using the cross-sectional variance as in Sarkissian (2003) and Darrat et al. (2011). Given our proxy for cross-sectional consumption risk, we compute the model's common SDF and test its ability to price the cross-section of international assets.

We begin our tests by estimating the model's structural parameters using the generalized method of moments (GMM) based on equilibrium moment conditions. We aim to gauge how the model's nonlinear SDF prices our test assets and check if the derived preference parameters match other asset pricing studies. Our primary asset menu includes 69 stock indices from various market types (23 developed, 26 emerging, 16 frontier, and 4 standalone), primarily focusing on country-specific indices to avoid biases and reduce the diversifiable risk noise. This approach is more challenging than past research, which mainly targeted developed markets. We also explore an alternative asset pool featuring Fama–French's 25 developed portfolios sorted by size and momentum.

The GMM test supports the model at the 95% confidence level, and all parameter estimates are statistically significant with economically plausible values. The subjective discount factor range from 0.970 to 0.986, the relative risk aversion (RRA) from 1.95 to 6.50, and the elasticity of intertemporal substitution (EIS) from 1.15 to 2.86. Notably, RRA and EIS estimates exceed one, and RRA values surpass the inverse of EIS, aligning with past findings on the preference for early resolution of uncertainty, as seen in Bansal et al. (2016). Our research distinctively achieves these insights by examining a heterogeneous-agents asset pricing model on a global scale, emphasizing worldwide equities. The implied SDF time series captures both OECD and non-OECD economic cycles.

By utilizing the GMM, we build upon the growing literature on recursive preferences estimation as seen in works by Chen et al. (2013), Bansal et al. (2016), Meddahi and Tinang (2016), and Constantinides and Ghosh (2017). However, our approach identifies the model's state variable through the cross-sectional cumulants of country consumption growth rates and calculates the SDF for asset pricing explicitly. In contrast, existing studies typically rely on U.S. aggregate consumption to filter out the latent state variable for SDF computation or bypass the direct calculation of the SDF, leaning on the model's implied unconditional asset price moments for parameter estimation.

Next, we linearize the SDF as common in cross-sectional asset pricing and estimate the linear factor model using the two-pass cross-sectional regressions of Fama and MacBeth (1973). The reduced-form factor risk premiums are statistically significant, stable, and theoretically consistent. In the cross-section, a one-standard deviation drop in the beta of cross-sectional consumption risk corresponds to a 1.01% increase in risk premium, while a similar decrease in the beta for its changes results in a 0.41% rise. Notably, the latter premium is relevant only within Epstein–Zin's preference framework. Considering the average risk premium of 2.40% across the 69 MSCI indices, with quartile values of 1.50%, 2.24%, and 3.09%, our results have significant economic implications.

Our approach of employing international test assets in a global heterogeneous-agents consumption-based asset pricing model featuring a single state variable that influences cross-sectional consumption risk mirrors previous research such as Sarkissian (2003) and Darrat et al. (2011). Both papers build on Constantinides and Duffie (1996) where cross-sectional consumption risk is the cross-sectional variance of country consumption growth rates, all cross-sectional higher-order cumulants being equal to zero under the assumption of conditional normality of agents' idiosyncratic shocks. However, our study offers a distinctive perspective by considering Epstein–Zin's recursive utility and conditional non-normality of agents' idiosyncratic shocks, which provides valuable insights. Both result in the pricing of changes in cross-sectional consumption risk and integrating information from cross-sectional higher-order moments, moving beyond the traditional reliance on cross-sectional variance. We find these two novelties statistically and economically significant in our empirical analyses.

Theoretically, the cross-sectional consumption risk is a positive process. However, as our PCA-based proxy does not empirically guarantee positivity, we explore an alternative that uses model implications to back out cross-sectional consumption risk as proportional to cross-sectional variance up to a constant coefficient that depends on model structural parameters. This is particularly crucial in GMM estimation, where structural parameters, including those governing the positive cross-sectional consumption risk process, are estimated. When estimating the GMM based on the nonlinear SDF, both proxies of cross-sectional consumption risk deliver comparable pricing errors. This finding underscores the importance and ability of the nonlinear SDF to exploit interactions between aggregate consumption growth and cross-sectional variance to potentially capture the richness of information hidden in cross-sectional higher-order cumulants.

In contrast, when linearizing the SDF, the capacity to combine aggregate consumption growth and cross-sectional variance in a way that extracts information from higher-order moments vanishes. Using the PCA-based proxy of cross-sectional consumption risk incorporates the information content of the cross-sectional higher-order moments. Alternatively, we explore ad-hoc linear factor model specifications supplementing the cross-sectional variance factor with other cross-sectional cumulants. This allows us to examine empirically whether information from these cumulants, unaccounted for by cross-sectional variance, holds value in pricing international equities. Although this latter approach does not directly derive from the model, it aligns with its underlying assumptions. Our benchmark linear factor model with the PCA-based proxy of cross-sectional consumption risk performs better than comparable specifications that build on the work of Constantinides and Duffie (1996) by using cross-sectional variance and has previously been considered for pricing international equity (Darrat et al., 2011) and foreign exchange markets (Sarkissian, 2003).

To illustrate the performance and economic significance of the information content of cross-sectional higher-order moments and changes in cross-sectional consumption risk, which are the main innovations in our setting, we decompose the risk premium associated with long positions in emerging markets versus short positions in developed markets, a consistent average excess return difference of 0.98% quarterly, reinforcing the prevalent understanding that emerging markets investments bear higher risks. Our deep dive into specific risk factors revealed a consistent positive contribution to the long-short premium from cross-sectional skewness and changes in cross-sectional variance. In contrast, cross-sectional variance is not a driver of the Emerging versus Developed risk premium in all model specifications. Intriguingly, the PCA-based proxy of cross-sectional consumption risk demonstrates empirical superiority in explaining international equities, evidenced by its positive and substantial contribution to the long-short premium, contrasting with the negligible or negative contributions of cross-sectional variance. The significant performance of the PCA-based proxy relative to the traditional cross-sectional variance approach in explaining the observed premium underscores its empirical advantage and our innovative contribution to the literature.

In examining whether cross-sectional moments of country consumption growth rates are pricing factors in international stock indices, we have updated our econometric methods based on recent advancements in linear asset pricing tests. We aim to eliminate spurious factors and ensure reliable statistical inference for factor risk premia. As Kleibergen (2009) and Kleibergen and Zhan (2015) have pointed out, rank deficiencies in the initial stage of the Fama and MacBeth (1973) regressions lead to weaker identifications in the subsequent stage. This aligns with recent empirical findings, like those from Ang et al. (2020) and Gagliardini et al. (2016), which stress the need for a more expansive cross-section to achieve precise factor risk premia estimates. Results from our 23 developed market MSCI indices indicate that the risk premium tied to cross-sectional consumption risk is not statistically significant, even at the 10% level. However, when we broadened our asset menu to include a combination of developed, emerging, frontier, and standalone MSCI indices, we observed increased variability in risk exposures and expected returns. This comprehensive asset menu allows for enhanced identification and precision in estimating factor risk premia, as emphasized in the Fama–MacBeth regressions. As Giglio et al. (2021) highlighted, identifying a factor risk premium largely depends on the asset menu's scope. Our expanded menu demonstrates that a broader international asset selection offers more precise identification and estimation of risk premia associated with cross-sectional higher-order moments of consumption growth rates.

The remainder of the paper is organized as follows. Section 2 provides an overview of the theoretical framework establishing the link between asset risk premia and the cross-sectional higher-order moments of heterogeneous-agents consumption growth rates. Section 3 presents the empirical approach, together with the data and their descriptive statistics. We also describe the different econometric methodologies used in the estimation (e.g., generalized methods of moments for the structural parameters, and cross-sectional regressions for factor risk premiums), and present the estimation results. Section 4 concludes. An external appendix available from the authors' web pages includes additional analyses with the associated technical derivations, tables, and figures.

2. Theoretical background

Our framework is based on the Constantinides and Ghosh (2017) model and assumes that the economy is global. It features an infinite number of country representative consumers, each indexed by "i", and whose sum is normalized to one. The model's representation of infinitely many small consumers, in the international context, can be seen as a stylized representation of the global economic landscape. Even though we have a finite number of countries, the variations in their economic sizes, stages of development, and openness to international markets mean that their influence on global consumption dynamics varies widely. This variation can be analogously seen as the consumption disparity among infinitely many households. While the scale and specifics differ, the core idea remains, i.e., diverse entities, whether households or countries, interact in a shared economic environment, with idiosyncratic risks influencing their consumption trajectories. By extending the Constantinides and Ghosh (2017) framework to countries, we offer a novel lens through which we can view and understand international consumption dynamics and risk-sharing mechanisms.

The representative consumer in each country receives a labor income and can invest in international financial markets, which the asset menu consists of country stock indices or international portfolios. The proceeds from labor income and financial assets are then utilized to purchase the single numeraire good for consumption. Following Constantinides and Duffie (1996), the consumer cannot insure against idiosyncratic country-specific shocks through the global market, making the market incomplete. Therefore, there are no traded state-contingent goods available to hedge against the potential decline in consumption in country i.

The country representative consumers have identical recursive utility of Epstein and Zin (1989) whose one-period Stochastic Discount Factor (SDF) for agent i is given by:

$$M_{i,t+1} = \exp\left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{i,t+1} + (\theta - 1) r_{i,c,t+1}\right), \tag{1}$$

where $\Delta c_{i,t+1} = c_{i,t+1} - c_{i,t}$ with $c_{i,t} = \ln C_{i,t}$, $C_{i,t}$ is the country i representative agent's consumption level, δ is the time-preference parameter, γ is the relative risk aversion parameter, ψ is the elasticity of intertemporal substitution, $\theta = \dfrac{1-\gamma}{1-{}^{1}/_{\psi}}$, and $r_{i,c,t+1}$ is the log-return on the claim to country i representative agent's consumption.

The world aggregate consumption growth is unpredictable, homoscedastic, and evolves as follows

$$\Delta c_{t+1} = \mu_c + \sigma_c \epsilon_{t+1}, \tag{2}$$

where $\Delta c_{t+1} = c_{t+1} - c_t$ with $c_t = \ln C_t$, C_t is the world aggregate consumption level, ϵ_{t+1} is an independent and identically distributed (i.i.d.) standard normal process, and μ_c and σ_c are the mean and the volatility of aggregate consumption growth rate, respectively.

The model does not assume a complete absence of risk sharing across countries. Instead, it is compatible with imperfect risk-sharing. In this setting, agents cannot fully insure against all idiosyncratic risks, which leads to country-specific consumption growth rates that may not perfectly correlate with global consumption growth. This captures the empirical regularities often observed where countries cannot fully diversify away from country-specific risks despite their integration into global financial markets. Under the assumptions of Constantinides and Ghosh (2017), in equilibrium, agent i's consumption level $C_{i,t}$, is a share $h_{i,t}$ of aggregate consumption, i.e.,

$$C_{i,t} = h_{i,t} C_t \quad \text{where} \quad h_{i,t} = h_{i,t-1} \exp\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}} - \sigma^2 \frac{j_{i,t}}{2}\right) + \left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}} - \tilde{\sigma}^2 \frac{\tilde{j}_{i,t}}{2}\right)\right). \tag{3}$$

The processes $\eta_{i,t}$ and $\tilde{\eta}_{i,t}$ are i.i.d. standard normal and capture the idiosyncratic shock occurring in country i, and $j_{i,t}$ and $\tilde{j}_{i,t}$ are conditionally distributed Poisson processes driving the occurrence of the representative consumer i's idiosyncratic shocks at time t. Following Constantinides and Ghosh (2017), the processes $j_{i,t}$ and $\tilde{j}_{i,t}$ can be viewed as time-t random variables that characterize the agents' cross-sectional heterogeneity of consumption growth.

In the international context, this framework could provide a fresh perspective on the well-documented puzzles related to the lack of risk sharing across countries. While many standard international macroeconomics and finance models predict higher levels of risk sharing than what is empirically observed, the current model might offer insights into the deviations from these predictions by assuming imperfect risk sharing. The nuances of this imperfect risk-sharing can help bridge the gap between theoretical predictions and the empirical realities of international consumption and investment patterns. This latter point, however, goes beyond the scope of the current article.

The Poisson distribution followed by $j_{i,t}$ is governed by a common stochastic intensity process ω_t that drives the representative consumer i's idiosyncratic income shocks:

$$\forall i, \quad \text{prob}\left(j_{i,t} = n \mid \mathcal{I}_t\right) = \exp\left(-\omega_t\right) \frac{\omega_t^n}{n!}, \quad n = 0, 1, 2, \ldots \tag{4}$$

where $\mathcal{I}_t$ denotes the information set at time t. Therefore, the process ω_t can be viewed as a global business cycle variable. It follows that the term $\eta_{i,t}\sigma\sqrt{j_{i,t}} - \sigma^2 \frac{j_{i,t}}{2}$ in Eq. (3) captures country i's idiosyncratic income shocks that are related to the global business cycle. For example, financial crises in one region can rapidly spread globally, such as the 2008 crisis in the United States, leading to decreased revenues worldwide. More recently, the COVID-19 pandemic has had considerable economic effects worldwide, leading to decreased activity and employment in many countries.

Instead, the intensity of the Poisson distribution followed by $\tilde{j}_{i,t}$ is a constant $\tilde{\omega}$ for all representative consumers:

$$\forall i, \quad \text{prob}\left(\tilde{j}_{i,t} = n \mid \mathcal{I}_t\right) = \text{prob}\left(\tilde{j}_{i,t} = n\right) = \exp\left(-\tilde{\omega}\right) \frac{\tilde{\omega}^n}{n!}, \quad n = 0, 1, 2, \ldots \tag{5}$$

It follows that the term $\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}} - \tilde{\sigma}^2 \frac{\tilde{j}_{i,t}}{2}$ in Eq. (3) captures country i's idiosyncratic income shocks that are unrelated to the global business cycle. For example, natural disasters, political instability, and public health emergencies can significantly affect a country's income.

To ease the computational exposure, Constantinides and Ghosh (2017) define the scaled state variable $x_t \equiv \left(\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right) - 1\right)\omega_t$ and assume that x_t follows an auto-regressive gamma process of order 1, ARG(1), as in Gourieroux and Jasiak (2006). Formally, we have:

$$x_{t+1} = \nu\xi + \rho x_t + \varepsilon_{x,t+1}, \tag{6}$$

where $\nu > 0$, $\xi > 0$, $\rho > 0$, and $\varepsilon_{x,t+1}$ is a martingale difference sequence. The conditional mean and variance are given by $\mathbb{E}\left[x_{t+1} \mid x_t\right] = \nu\xi + \rho x_t$ and $\text{var}\left[x_{t+1} \mid x_t\right] = \nu\xi^2 + 2\rho\xi x_t$, respectively. Likewise, the unconditional mean, variance and first-order autocovariance of x_t are given by:

$$m_x \equiv \mathbb{E}\left[x_t\right] = \frac{\nu\xi}{1-\rho}, \quad \upsilon_x \equiv \text{var}\left[x_t\right] = \frac{\nu\xi^2}{(1-\rho)^2} \quad \text{and} \quad \text{ac1}\left[x_t\right] = \frac{\rho\nu\xi^2}{(1-\rho)^2}. \tag{7}$$

As suggested by Eq. (1), there are multiple SDFs corresponding to each agent in the current incomplete market setting, aligning with the emerging view that different investors possess different SDFs for pricing assets. The difference, however, is that in the current model, all agents face the same asset menu, indicating a lack of market segmentation from the asset accessibility standpoint. Constantinides and Ghosh (2017) further show from the above consumption dynamics and preferences that, in equilibrium, each agent's valuation of any given security of the available asset menu is the same. Given this multitude of individual SDFs, they derive the average SDF by integrating out the idiosyncratic shocks experienced by agents and obtain the following:

$$M_{t+1} = \mathbb{E}\left[M_{i,t+1}|\mathcal{I}_{t+1}\right] = \exp\left(b_0 - b_1 \Delta c_{t+1} - b_2 x_{t+1} - b_3 \Delta x_{t+1}\right), \tag{8}$$

$$b_0 = \theta \log \delta + (\theta - 1)\left(q_0 - \left(1 - q_1\right) A_0\right) + \left(\exp\left(\frac{\gamma(\gamma+1)\tilde{\sigma}^2}{2}\right) - 1\right)\tilde{\omega}$$
$$b_1 = \gamma \qquad b_2 = -\frac{\exp\left(\frac{\gamma(\gamma+1)\sigma^2}{2}\right) - 1}{\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right) - 1} + \left(1 - q_1\right)(\theta - 1) A_1 \qquad b_3 = -(\theta - 1) A_1 \tag{9}$$

where q_0 and q_1 are respectively the drift and slope coefficients of the Campbell and Shiller (1988) log-linear approximation of the return to each agent's individual consumption claim, while A_0 and A_1 are respectively the drift and slope coefficients of the each agent's individual log wealth-to-consumption ratio as a linear function of the state variable x_t.[3] The average or common SDF in Eq. (8) serves as the primary tool in our asset pricing tests.

Eq. (8) shows that the equilibrium SDF depends on three risk factors: the world aggregate consumption growth, Δc_{t+1}, the cross-sectional consumption risk level x_{t+1} and the cross-sectional consumption risk changes, Δx_{t+1}. The price of aggregate consumption growth risk, γ, is positive. The price of cross-sectional consumption risk level, b_2, is negative once $\gamma > 1$ as commonly agreed in the consumption-based asset pricing literature (see, for example, Tauchen, 2011). This follows from the fact that $q_1 \approx 1$. Likewise, the risk price associated with the cross-sectional consumption risk changes $(1 - \theta) A_1$ is nonpositive in particular if $A_1 < 0$ and $\theta \leq 1$. Notice that A_1 is negative because asset markets dislike uncertainty (see, for example, Bansal et al., 2005). Also, assuming $\gamma > 1$ so that the process x_t is well-defined, the condition $\theta \leq 1$ may follow from $\psi > 1$ and $\gamma \geq \frac{1}{\psi}$, as commonly argued in the asset pricing literature with recursive utility (see for example Bansal and Yaron, 2004). The agent's preference for early resolution of uncertainty in the recursive utility model is also equivalent to $\gamma > \frac{1}{\psi}$ (see Epstein and Zin, 1989, henceforth, **EZ**). If $\gamma = \frac{1}{\psi}$, then it leads to the constant relative risk aversion (**CRRA**) model. In this case we have $\theta = 1$, i.e., changes in the cross-sectional consumption risk are not priced, and the equilibrium SDF depends only on two risk factors: world aggregate consumption growth Δc_{t+1} and the cross-sectional consumption risk level, x_{t+1}. A decrease in world consumption growth increases the SDF, whereas cross-sectional consumption risk level and changes have positive effects on the SDF.

Notice from equation $C_{i,t} = h_{i,t} C_t$ that the growth rate of $h_{i,t}$ is the difference between the country individual consumption growth rate and the world aggregate consumption growth rate, i.e., by definition, the country relative consumption growth rate. The pricing kernel (8) is not fully observable as it depends on the unobservable cross-sectional consumption risk x_t. However, as Eqs. (A.2)–(A.5) of the internal appendix show, the cross-sectional conditional cumulants of relative consumption growth across countries are linear functions of the cross-sectional consumption risk. Therefore, we can use the cumulants computed from the data to back out an empirical proxy for the cross-sectional consumption risk. In the subsequent empirical analysis, we use the principal component of the cross-sectional cumulants of country consumption growth rates to proxy the cross-sectional consumption risk. As robustness checks, we also consider the cross-sectional moments of country consumption growth rates directly as linear pricing factors in explaining the cross-sectional variation in country expected returns.

3. Empirical analysis

This section tests the consistency of the heterogeneous-agents consumption-based asset pricing model on international consumption and security data. Section 3.1 presents the data sample and summary statistics. Section 3.2 shows the estimation of the model's structural parameters. In Section 3.3, we linearize the pricing kernel and present the linear factor models' estimations.

3.1. Data and descriptive statistics

Consumption, population, and asset price data for the different countries are sourced from DataStream. Real consumption data are private final consumption expenditures (PFCE) in constant price and asset price data are MSCI stock market indexes denominated in U.S. dollar.[4] The indexes are value-weighted and adjusted for dividend reinvestment. These data are available for 69 countries:[5]

[3] These coefficients are obtained simultaneously by solving a non-linear system and a fixed point problem that determines the average wealth-consumption ratio, given in the appendix.

[4] We use the MSCI return index denominated in U.S. dollar instead of local currency. By doing so, we can easily compare foreign assets without worrying about the exchange rates.

[5] We drop some countries from the sample either because they had a negative risk premium on average (e.g., Bulgaria, Jordan, Serbia, and Ukraine), or faced a hyperinflation episode (e.g., Zimbabwe).

Table 1
List of countries and acronyms.

Country	Acronym	Country	Acronym
Panel A: Developed Markets			
Australia	**AU**	Israel	IS
Austria	**OE**	**Netherlands**	**NL**
Canada	**CN**	**Norway**	**NW**
Belgium	**BG**	New Zealand	**NZ**
France	**FR**	**Portugal**	**PT**
Germany	**BD**	**United States**	**US**
Denmark	**DK**	**United Kingdom**	**UK**
Finland	**FN**	**Switzerland**	**SW**
Hong Kong	**HK**	Singapore	SP
Italy	**IT**	**Spain**	**ES**
Japan	**JP**	**Sweden**	**SD**
Ireland	IR		
Panel B: Emerging Markets			
Argentina	AG	Malaysia	MY
Brazil	BR	Peru	PE
Chile	CL	Russian Federation	RS
China	CH	Poland	PO
Greece	**GR**	Qatar	QT
India	IN	Philippines	PH
Colombia	CB	Saudi Arabia	SI
Hungary	HN	Taiwan	TW
Indonesia	ID	Thailand	TH
Egypt	EY	**Turkey**	**TK**
Czech Republic	CZ	United Arab Emirates	UA
Mexico	MX	**South Korea**	**KO**
Czech Republic	CZ		
Kuwait	KW		
Panel C: Frontier Markets			
Bangladesh	BN	Oman	OM
Croatia	CT	Estonia	EO
Ghana	GH	Pakistan	PK
Kenya	KN	Romania	RM
Kazakhstan	KZ	Slovenia	SJ
Lithuania	LN	Tunisia	TS
Mauritius	MU	Sri Lanka	LK
Morocco	MC	Vietnam	VM
Panel D: Stand-alone Markets			
Bosnia Herzegovina	BH	Palestine	PL
Jamaica	JA	Trinidad&Tobago	TT

The table displays the list of countries included in our empirical analysis with their acronyms and classification as either developed, emerging, frontier, or stand-alone market. In bold are the 24 countries whose consumption growth rates are used to compute the factors in our main analysis.

23 developed, 26 emerging, 16 frontier, and 4 standalone markets according to the MSCI (2021) classification. The data are quarterly and cover the period 1970Q1-2021Q1. Table 1 displays the list of countries that we consider in our empirical analysis along with their acronyms and MSCI classifications. A second asset menu considered for robustness checks in our empirical analyses consists of the Fama–French 25 developed portfolios formed on size and momentum. All asset excess returns are computed by subtracting the U.S. 1-month T-bill rate from the index or portfolio returns. Portfolio returns and T-bill rate data are downloadable from the Kenneth French data library.[6]

We compute each country's real per capita consumption growth rate by subtracting its population growth rate from its aggregate real consumption growth rate.[7] To compute the cross-sectional cumulants of the country per capita real consumption growth rates used in our asset pricing tests, we consider a balanced panel of consumption data from 24 countries, including 21 developed and three emerging. These countries' names and acronyms are highlighted in boldface in Table 1. Data for these countries are available longer and deemed less noisy. The cross-sectional cumulants are weighted using the country's private final consumption expenditures based on purchasing power parity (henceforth, PPP-weighted) or equal-weighted for robustness checks. The observations span the period

[6] Available online at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[7] Table A1 of the external appendix offers descriptive statistics for consumption growth by country, while Tables A2 and A3 provide details on the excess returns for individual countries and the 25 Fama–French portfolios, respectively.

Table 2
Summary statistics: Mean, Dispersion, Skewness, Kurtosis, and $PC1_t$.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max	Ac1
A. Cross-sectional moments (equally weighted)								
aggcg	241	1.990	1.980	−15.873	1.241	3.031	5.492	0.776
vargcg	241	7.525	5.552	0.541	4.014	8.879	30.916	0.720
skewcg	241	0.033	1.141	−3.512	−0.430	0.608	3.518	0.627
kurtcg	241	4.743	2.953	1.746	2.880	5.606	15.950	0.558
B. Cross-sectional moments								
aggcg	241	2.076	1.934	−14.171	1.306	3.264	5.919	0.803
vargcg	241	4.684	4.729	0.286	2.161	5.443	43.720	0.722
skewcg	241	0.078	1.622	−4.573	−0.885	1.122	6.255	0.648
kurtcg	241	4.644	6.763	−1.503	0.568	5.934	57.055	0.617
C. Cross-sectional cumulants								
aggcg	241	2.076	1.934	−14.171	1.306	3.264	5.919	0.803
vargcg	241	4.684	4.729	0.286	2.161	5.443	43.720	0.722
skewcg	241	−5.267	53.574	−633.319	−4.675	7.212	99.823	0.541
kurtcg	241	197.950	647.786	−108.312	2.422	78.203	8,017.225	0.529
D. First principal component of cross-sectional cumulants								
PC1	241	0.000	1.000	−0.484	−0.346	−0.062	11.759	0.609
ΔPC1	240	0.003	0.879	−8.249	−0.083	0.071	6.256	−0.025

The table presents the summary statistics (mean, standard deviation, minimum, maximum, 25% and 75% percentiles, and first-order autocorrelation) of the cross-sectional moments of the real per capita growth rate (in percentage) of the quarterly private final consumption expenditure and the proxy of cross-sectional consumption risk. Panel A presents the equally weighted cross-sectional moments and Panel B presents the similar moments where the weights are the proportion of the country consumption based on purchasing power parity (PPP) to world consumption. Panel C presents the weighted cross-sectional cumulants where the weight is the share of country consumption based on purchasing power parity (PPP) to world consumption. Panel D presents the proxy of cross-sectional consumption risk obtained as the first principal component of the cross-sectional cumulants of country consumption growth. We used a balanced panel data of 24 developed and emerging countries over the period from 1961Q1 to 2021Q1.

1961Q1-2021Q1. World aggregate real per capita consumption growth rate is measured by the cross-sectional mean of country aggregate real per capita consumption growth rates.

The time-series descriptive statistics of the cross-sectional moments of countries' consumption growth rates are presented in the first two panels of Table 2. The equal-weighted cross-sectional mean has a sample average and standard deviation of 1.99% and 1.98%, respectively, while similar statistics for the PPP-weighted cross-sectional mean are 2.08% and 1.93%, respectively. The equal-weighted and PPP-weighted cross-sectional variance sample averages are 7.53 and 4.68 percent-squared, and their standard deviations are 5.55 and 4.73, respectively. The cross-sectional skewness is positive on average and close to zero. However, its time-series evolution as displayed in Panel A of Fig. 1 clearly evidences periods of negatively-skewed cross-sectional distribution of countries' consumption growth rates that often coincide with economic recessions, and episodes of positively-skewed cross-sectional distribution. Finally, cross-sectional excess kurtosis is, on average, positive (i.e., the kurtosis is larger than 3) and fluctuates through time. As a result, the country's consumption growth distribution often exhibits a thinner tail than the normal distribution.

The last two panels of Table 2 present the time-series descriptive statistics of the cross-sectional cumulants of countries' consumption growth rates, and the level ($PC1_t$) and changes ($\Delta PC1_t$) of their (standardized) first principal component. The first principal component captures about 83% of the variability in the second to fourth cross-sectional cumulants of consumption growth. Its correlations with the second, third, and fourth-order cumulants are 0.89, −0.90, and 0.94, respectively. Interestingly, these correlations are significantly higher than the pair-wise correlations between the cross-sectional cumulants, therefore aligning more closely with the model's theoretical predictions.

Panel A of Fig. 1 plots the time-series of the standardized cross-sectional moments, and the shaded bars represent the OECD and non-OECD country recession periods. Data on these recession indicators are available from the Federal Reserve Economic Data (FRED) services. This figure highlights some interesting patterns. The aggregate consumption growth factor displays troughs around recessions, especially the first and second oil crises in 1973–1974 and 1979–1980, the 1990 oil price shock, the 2008 financial crisis, and recently during the Covid-19 pandemic. Second, the cross-sectional variance factor displays some peaks around the same recession periods except for the 2008 financial crisis. However, we observe a peak in the cross-sectional variance factor following the 1997 Asian financial crisis while the cross-sectional mean factor is barely negatively affected.

Finally, cross-sectional skewness differs from aggregate consumption growth, slightly peaking during the oil crises and troughing in later recessions. Similarly, the kurtosis factor's path differs from the variance factor; it barely responded to the 1990 oil price shock yet peaked during the 2001–2002 period, marked by the September 11 attacks, while the variance factor remained low. These cross-sectional moments do not consistently synchronize and can reflect diverse global economic events. The principal component captures these different facets of cross-sectional consumption risk, and assets more susceptible to these risks offer higher expected

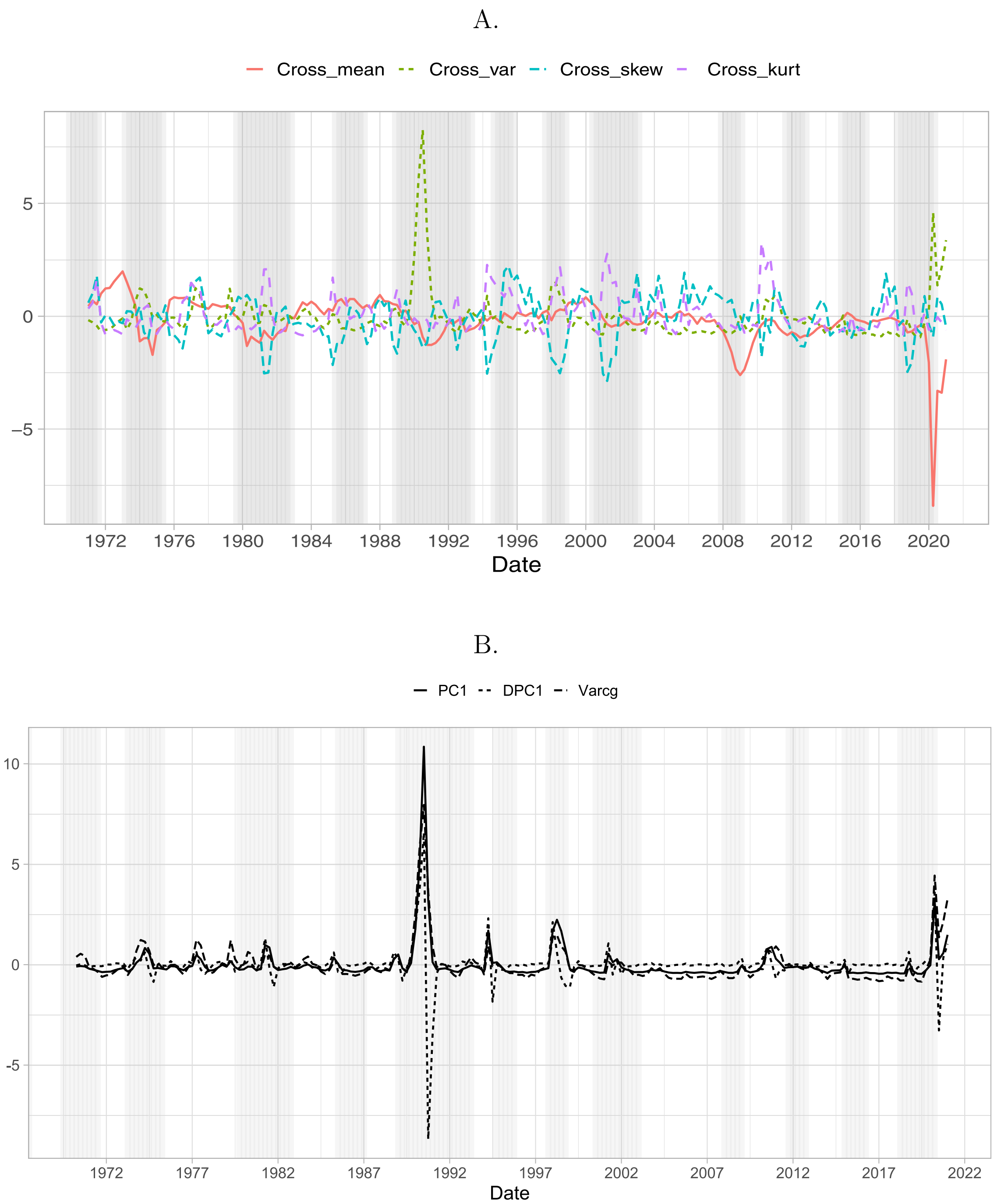

Fig. 1. Cross-sectional moments of consumption and first principal component.
Panel A shows the time series evolution of the cross-sectional (standardized) moments (mean, variance, skewness and kurtosis) of international consumption growth. Panel B shows the evolution of the standardized first principal component of the consumption growth cross-sectional cumulants and its changes. The cross-sectional moments and cumulants are weighted by country consumption expenditures based on purchase power parity measured in dollars. Panel (A) uses an balanced panel of 24 developed and emerging economies. The variables on the figure have been standardized. The gray bars represent the OECD and Non-OECD countries recession periods.

returns. The moments of countries' consumption growth rates effectively represent the global business cycle and can elucidate international asset pricing, our article's core focus. Panel B of Fig. 1 illustrates the evolution of the consumption growth's first principal component and cross-sectional variance, both standardized. Their close movement is verified by their strong correlation in Table 3.

Table 3
Correlation matrix: Cross-sectional cumulants and $\mathrm{PC1}_t$.

	PC1	aggcg	vargcg	skewcg
aggcg	−0.23			
vargcg	0.89	−0.28		
skewcg	−0.90	0.22	−0.66	
kurtcg	0.94	−0.13	0.78	−0.79

This table presents the correlations between the Cross-sectional weighted cumulants and the principal component $\mathrm{PC1}_t$. We used a balanced panel data of 24 developed and emerging countries over the period from 1961Q1 to 2021Q1.

3.2. Estimation of the structural parameters

This subsection describes the estimation procedure for the model structural parameters and presents the results. We employ the Generalized Method of Moments (GMM) to estimate the asset pricing model described in Section 2, rooted in the conventional Euler equations. These equations posit that the expected product of the stochastic discount factor (SDF) and the gross return of an asset should equate to one. Notably, our model characterizes the SDF as being exponential affine in the factors. Within this structure, the drift of this affine function is contingent on several parameters, yet not all of these exert a substantial influence on the factor risk prices. Some parameters have only a marginal impact. In light of this observation, it stands to reason that parameter identification might be enhanced by judiciously restricting certain parameters that primarily influence the drift, e.g., $\tilde{\omega}$. Specifically, setting $\tilde{\omega} = 0$ could foster a more parsimonious and identifiable model, enhancing the validity and interpretability of our subsequent findings. Under this restriction, the full model has nine parameters which are collected in the vector $\Theta = \left(\delta, \gamma, \psi, \mu_c, \sigma_c, \nu, \xi, \rho, \sigma\right)$.

The vector of moment conditions denoted by $e_t(\Theta)$ combines the model's pricing errors (i.e., the Euler equation errors) with the differences between sample and theoretical moments of the factors. We also add the unconditional mean and variance of the aggregate consumption growth rate. Thus, our model is overidentified, enabling us to test overidentifying restrictions.

Our GMM uses a diagonal weighting matrix with a unitary weight on the Euler moments and a more considerable weight on the remaining moments related to the dynamics of the state variables.[8]

3.2.1. Estimation of the structural parameters using PCA

The common SDF derived in Eqs. (8) depends on the unobserved cross-sectional consumption risk x_t. However, as shown in Eqs. (A.2)–(A.5) of the internal appendix, the cross-sectional cumulants of country consumption growth rates are linear functions of the rescaled cross-sectional consumption risk. Therefore, we apply a principal component analysis to these cumulants and use the first principal component as a proxy of the unobserved state variable. Our observable proxy for cross-sectional consumption risk x_t is computed as follows:

$$\hat{x}_t = m_x + \sqrt{\upsilon_x} \cdot \mathrm{PC1}_t \tag{10}$$

where $\mathrm{PC1}_t$ is the standardized first principal component of the cross-sectional cumulants of country consumption growth rates, m_x and υ_x denote the theoretical unconditional mean and variance, respectively, of the unobserved cross-sectional consumption risk x_t as given in Eq. (7). This approach forces the empirical proxy to match the theoretical mean and variance of x_t. However, the positivity of cross-sectional consumption risk process is not empirically guaranteed. We subsequently propose an alternative estimation strategy to overcome this issue.

For the GMM estimation, the empirical proxy $\hat{x}_t$ is substituted in the expression of the SDF in Eq. (8) to get an observable SDF $\hat{M}_t$ given the model parameters Θ. We then form the moment conditions, putting together the model pricing errors with the centered unconditional moments of the aggregate consumption growth and cross-sectional consumption risk. Formally, we have:

$$\mathbb{E}\left[e_t(\Theta)\right] = 0, \quad \text{with} \quad e_t(\Theta) = \begin{pmatrix} 1 - \hat{M}_t \left[R'_{s,t} \;\; R'_{f,t}\right]' \\ \mu_c - \widehat{\mathrm{aggcg}}_t \\ \sigma_c^2 - \left(\widehat{\mathrm{aggcg}}_t^2 - \overline{\widehat{\mathrm{aggcg}}}^2\right) \\ \rho - \mathrm{PC1}_t \cdot \mathrm{PC1}_{t-1} \end{pmatrix}, \tag{11}$$

where R_s denotes the vector of test assets used to estimate Θ, R_f is the return of the risk-free asset as measured by the U.S. T-bill, and $\widehat{\mathrm{aggcg}}_t$ denotes our estimate of the global aggregate consumption growth in the data. In total, there are $K+3$ moment conditions, where K is the number of risky securities considered in our asset menu plus the risk free asset. We consider three different estimation scenarios. First, the asset menu is composed by developed countries' MSCI indexes with the longest available MSCI index series; there are 18 of them, the same considered by Darrat et al. (2011), and the number of moment conditions with this scenario is 22. Second, the asset menu contains the MSCI indexes of all 69 countries; therefore, the number of moment conditions is 73. Third, the asset menu is made by the Fama–French 25 developed market portfolios sorted on size and momentum, which yields 29 moment conditions.

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[8] The goal is to prioritize the matching of these other moments, before evaluating the asset pricing ability of the model (for similar approaches, see also Parker and Julliard, 2005, Yogo, 2006, and Tédongap, 2015, among others).

Table 4
GMM: Principal component as proxy for cross-sectional consumption risk.

A. Estimation with the Developed market MSCI indices.					
Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.986	6.496	1.152	1.684e−02	1.853e−02
95% CI .	(0.986, 0.987)	(6.371, 6.620)	(1.063, 1.240)	(1.683e−02, 1.684e−02)	(1.830e−02, 1.876e−02)
Par.	ν	ξ	ρ	σ	MAE
Est.	1.855e−03	6.590e−02	0.609	0.345	0.49
95% CI	(1.829e−03, 1.882e−03)	(6.585e−02, 6.595e−02)	(0.608, 0.609)	(0.345, 0.346)	

B. Estimation with all market MSCI indices.					
Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.970	1.948	2.859	1.684e−2	1.849e−2
95% CI.	(0.969, 0.970)	(1.932, 1.965)	(2.636, 3.083)	(1.683e−2, 1.684e−2)	(1.834e−2, 1.864e−2)
Par.	ν	ξ	ρ	σ	MAE
Est.	1.178e−3	6.815e−2	0.609	0.383	0.99
95% CI	(1.105e−3, 1.252e−3)	(6.807e−2, 6.824e−2)	(0.608, 0.609)	(0.382, 0.383)	

C. Estimation with Developed market 25 Fama–French portfolios.					
Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.980	4.858	1.745	1.208e−2	1.958e−2
95% CI.	(0.979, 0.981)	(4.786, 4.931)	(1.714, 1.776)	(1.205e−2, 1.209e−2)	(1.917e−2, 2.000e−2)
Par.	ν	ξ	ρ	σ	MAE
Est.	9.508e−05	0.139	0.490	0.375	0.65
95% CI	(9.426e−05, 9.590e−05)	(0.138, 0.140)	(0.488, 0.491)	(0.347, 0.403)	

The table shows the estimation of the model structural parameters obtained by GMM where the moments are formed by the asset pricing equilibrium condition that discounted expected returns should equal to one. The SDF is computed by substituting the proxy of cross-sectional consumption risk obtained as the first principal component of cross-sectional moments of country consumption growth. Panel A shows the estimation with the Developed market MSCI indices. Panel B shows the structural parameters estimation with all countries market MSCI indices. Panel C shows the structural parameters estimation with Fama–French 25 developed portfolios sorted by size and momentum. The mean absolute error (MAE) are expressed in percentage. The J-stats are respectively 0.17 in Panel A, 2.13 in Panel B and 0.25 in Panel C, and for all the cases, the model is not rejected at the 5% level of significance. The critical value of J-stat is obtained by 95% quantile of the bootstrap distribution of the objective function because we did not use the optimal weighting matrix. The confidence intervals are computed using block-bootstrap by resampling of 7-quarters blocks. We made 1000 replications.

Table 4 displays the GMM estimation results based on the moment conditions of Eq. (11) for three different asset menus: the developed MSCI indexes (Panel A), all MSCI indexes (Panel B), and the Fama–French 25 developed portfolios sorted by size and momentum (Panel C). In each panel of the table, structural parameter estimates are shown with their 95% confidence intervals. These confidence bounds rely on standard errors that we obtain from a block-bootstrap by resampling blocks of a seven-quarter length to compute the distribution of the parameter estimates. The empirical distribution of the J-stat is a byproduct of the same procedure.[9] The first striking observation is that all model structural parameter estimates are statistically significant at the conventional level of 5% because zero belongs to none of the 95% confidence intervals in the table.

Due to the sufficiently large weight[10] put on non-pricing moments of Eq. (11), estimates of the unconditional mean (μ_c) and standard deviation (σ_c) of the aggregate consumption growth and the autocorrelation (ρ) of cross-sectional consumption risk match their sample counterparts perfectly. Estimates of the preference parameters are well within the range of the values found in the literature. The pure discount factor δ is estimated between 0.970 and 0.986. The risk aversion coefficient is estimated at 6.50 with the developed markets asset menu, 1.95 with all market indexes, and 4.86 with the Fama–French 25 developed market portfolios. The EIS coefficient is above 1 for all asset menus (1.15 with developed markets MSCI indexes, 2.86 with all markets MSCI indexes, and 1.75 with Fama–French 25 developed market portfolios). In all cases, the risk aversion is greater than the inverse of the elasticity of inter-temporal substitution, leaning toward investors' preference for an early rather than late resolution of uncertainty on international markets. This result corroborates existing estimation findings by Chen et al. (2013), Bansal et al. (2016), Meddahi and Tinang (2016) and Constantinides and Ghosh (2017), and is deemed helpful in resolving many asset pricing puzzles.

Using the model parameter estimates, we back out the model-implied stochastic discount factor. The left panels of Fig. 2 present the evolution of the model-implied SDFs computed with the parameter estimates from the three asset menus under consideration.

9 The bootstrap procedure allows us to better capture the finite-sample distribution of the parameter estimates and the J-stat. This approach is also used by Bansal et al. (2016) and Constantinides and Ghosh (2017) among others. Overall, the model is not rejected based on the bootstrap distribution of the J-statistic and we refer the reader to the corresponding figures reported in the caption of Table 4.

10 Parker and Julliard (2005) and Tédongap (2015) use a similar approach in their cross-sectional asset pricing model estimations via GMM. Yogo (2006) shows the importance of including the non-pricing moments. In our analysis, the weight put on non-pricing moments is 100, corresponding to weighting the mean squared errors generated by these moments 10,000 times compared to the asset pricing moment conditions.

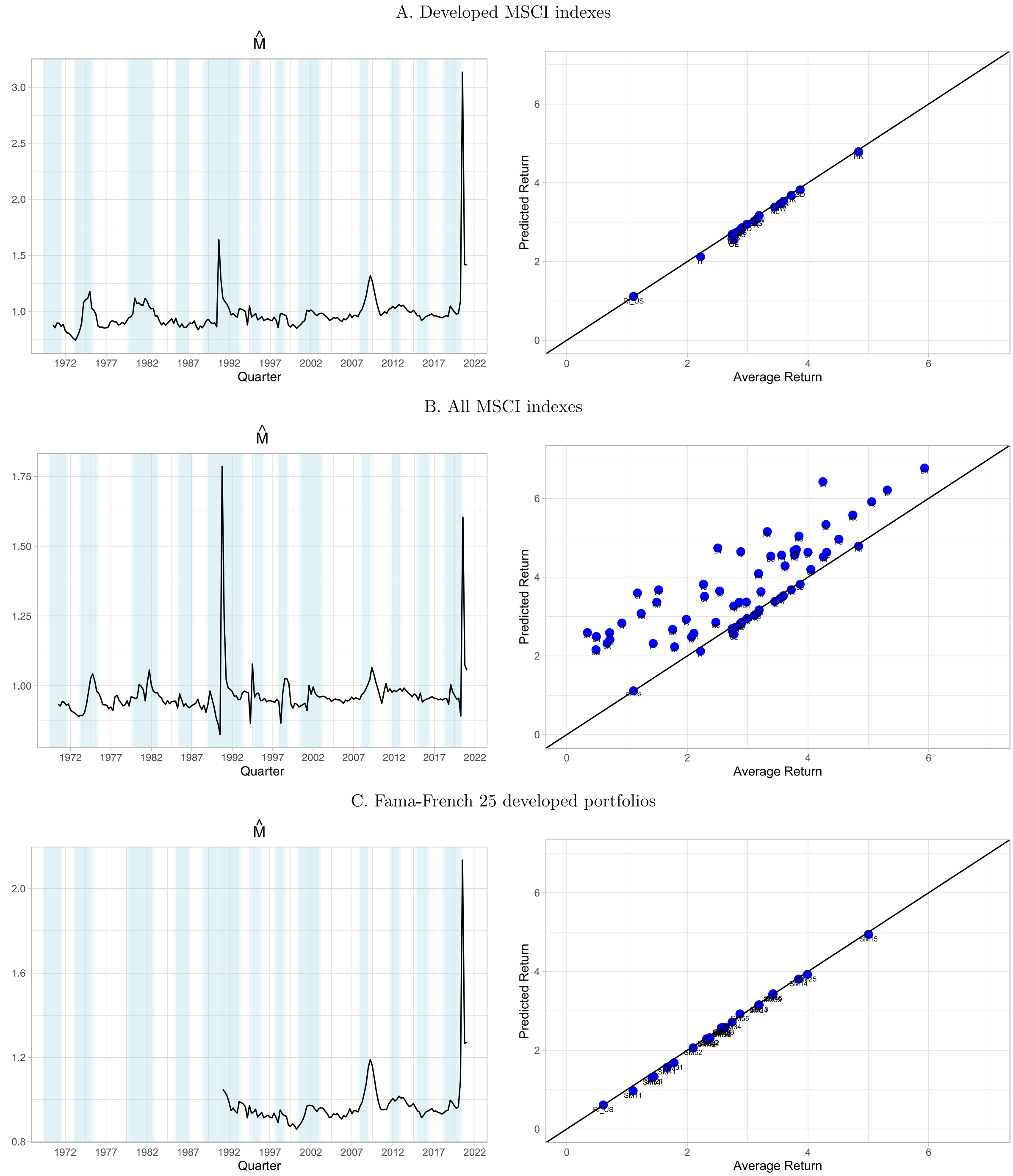

Fig. 2. Implied stochastic discount factor, realized and predicted expected returns.
This figure represents the evolution of the model-implied stochastic discount factor (see Eq. (8)) in the left panels, and the model's predicted average returns against the realized average returns. The sample period is from 1970Q1 to 2021Q1 in Panels A & B, and from 1991Q1 to 2021Q1 in Panel C. Rf_US represents the average T-bill rate in US considered as the risk free rate. The country consumption risk is proxied by the first principal component of the cross-sectional cumulants of country consumption growth rates. OECD and NON OECD recessions are represented by the shaded bars.

The ability of these SDFs to capture the business cycles is obvious and corroborates the observations from Panel B of Fig. 1. We see that the model-implied SDFs peak during recessions; in particular, they spike during the 1970s oil shock, the 2008–2009 financial crisis, and the COVID-19 pandemic in 2020.

Table 5
GMM: Cross-sectional variance as proxy for cross-sectional consumption risk.

A. Estimation with the Developed market MSCI indices.

Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.979	1.019	2.098	1.684e−2	1.849e−2
95% CI .	(0.979, 0.979)	(1.014, 1.024)	(2.083, 2.113)	(1.683e−2, 1.684e−2)	(1.848e−2, 1.850e−2)
Par.	ν	ξ	ρ	σ	MAE
Est.	6.710e−3	2.569e−5	0.958	0.063	0.49
95% CI	(6.690e−3, 6.731e−3)	(2.534e−5, 2.605e−5)	(0.957, 0.959)	(0.062, 0.065)	

B. Estimation with all market MSCI indices.

Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.972	1.464	2.325	1.684e−2	1.849e−2
95% CI .	(0.972, 0.973)	(1.456, 1.471)	(2.309, 2.340)	(1.683e−2, 1.684e−2)	(1.844e−2, 1.854e−2)
Par.	ν	ξ	ρ	σ	MAE
Est.	3.876e−2	3.067e−5	0.992	0.063	0.99
95% CI	(3.874e−2, 3.878e−2)	(3.018e−5, 3.116e−5)	(0.991, 0.993)	(0.061, 0.064)	

C. Estimation with Fama–French 25 developed portfolios.

Par.	δ	γ	ψ	μ_c	σ_c
Est.	0.981	2.056	1.821	1.208e−2	1.951e−2
95% CI .	(0.980, 0.981)	(2.052, 2.061)	(1.816, 1.826)	(1.207e−2, 1.208e−2)	(1.951e−2, 1.952e−2)
Par.	ν	ξ	ρ	σ	MAE
Est.	1.010e−2	1.844e−2	0.519	0.054	0.66
95% CI	(1.009e−2, 1.011e−2)	(1.816e−2, 1.873e−2)	(0.517, 0.520)	(0.053, 0.055)	

The table shows the estimation of the model structural parameters obtained by GMM where the moments are formed by the asset pricing equilibrium condition that discounted expected returns should equal to one. The SDF is computed by substituting the rescaled cross-sectional variance of country consumption growth as proxy of cross-sectional consumption risk. Panel A shows the estimation with the Developed market MSCI indices. Panel B shows the structural parameters estimation with all market MSCI indices. Panel C shows the structural parameters estimation with Fama–French 25 developed portfolios sorted by size and momentum. The J-stat are respectively 0.19 for Panel A, 2.18 for Panel B and 0.26 for Panel Cand for all the cases, the model is not rejected at the 5% level of significance. The critical value of J-stat is obtained by 95% quantile of the bootstrap distribution of the objective function because we did not use the optimal weighting matrix. The confidence intervals are computed using block-bootstrap by resampling of 7-quarters blocks. We made 1000 replications.

For each scenario, we also report in Table 4 the model's mean absolute pricing error (MAE). Given the structural parameter estimates, we can compute the x_t series from Eq. (10) and plug it into Eq. (8) to obtain the SDF series $\hat{M}_t$. The predicted (or model-implied) Euler equation error, the predicted asset expected returns, and the associated MAE are computed as follows:

$$\bar{e}_i = \overline{\hat{M} R_i} - 1, \quad \bar{R}_i^{\text{pred}} = \frac{1}{\bar{\hat{M}}} \left[\bar{e}_i + 1 - \widehat{\text{cov}} \left(\hat{M}, R_i \right) \right], \quad \text{and} \quad \text{MAE} = \frac{1}{N} \sum_{i=1}^{N} \left| \bar{e}_i \right|. \tag{12}$$

where $\overline{\hat{M} R_i}$ is the sample average of the discounted asset i's gross return series, $\bar{\hat{M}}$ is the sample average of the model-implied SDF series, and $\widehat{\text{cov}} \left(\hat{M}, R_i \right)$ is the sample covariance between the model-implied SDF and the asset i's gross return series. The table shows that the MAE is 0.49% with the developed market MSCI indexes, 0.99% with all market indexes, and 0.65% with the Fama–French 25 developed market portfolios. A more meaningful comparison of MAE across different model specifications or estimation alternatives can prove relevant as in the next section.

Although the MAE statistics give a summary measure of the overall fit of each scenario or model specification, it is also helpful to have a visual impression of the relative empirical performance of each specification we investigate as advocated by Lettau and Ludvigson (2001); thus, these are reported in the right panels of Fig. 2. For a given scenario, each panel plots the predicted expected return of each asset, obtained using the corresponding structural parameter estimates, against the realized average return. If the model fits perfectly, then all the assets should lie along the 45-degree line that is also plotted. Deviations from this 45-degree line represent pricing errors, and assets situated on the line are considered as fairly priced according to our model. Assets above the line are considered as over-priced, whereas assets below the line are under-priced. Fig. 2 illustrates well the impressive asset pricing performance of the heterogeneous-agents consumption-based asset pricing model for the developed country indices and the Fama–French 25 developed portfolios, as all assets lie on the 45-degree line. For the broader asset menu, we observe that developed market indices remain fairly priced, whereas the emerging and frontier are all over-valued.

3.2.2. Estimation of the structural parameters using the cross-sectional moments

In the previous GMM specification, we use the principal component of the cross-sectional cumulants of consumption growth rates to summarize their information content, which is fully embedded within cross-sectional consumption risk. Our model, however, assumes a positive process for cross-sectional consumption risk, which contrasts with the PCA-based proxy. To rectify this incongruity, we present an alternative approach that ensures the positivity of the proxy for cross-sectional consumption risk. This is particularly crucial in the GMM estimation, where structural parameters, including those governing the positive cross-sectional consumption risk process, are estimated. In the following GMM specification, the cross-sectional variance only is used to back out cross-sectional consumption risk. To harness information from the cross-sectional skewness and kurtosis, we introduce associated moment conditions that compel the GMM model estimation to align with their observed dynamics, encompassing parameters such as autoregressive coefficients, means, and variances.

We recall that $x_t \equiv \left(\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right)-1\right)\omega_t$ and we use the formula of the conditional cross-sectional variance given in Eq. (A.3) of the internal appendix, assuming that all idiosyncratic income shocks are related to the global business cycle, i.e., $\tilde{\omega}=0$. In this case, we can directly back out another empirical proxy of x_t as follows:

$$\hat{x}_t = \frac{\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right)-1}{\sigma^2+\frac{\sigma^4}{4}}\widehat{\text{vargcg}}_t \tag{13}$$

where $\widehat{\text{vargcg}}_t$ is an estimate of the cross-sectional variance in the data.

Eq. (13) guarantees the positivity of $\hat{x}_t$, consistent with the theoretical assumption, provided that $\gamma > 1$. From Eq. (8), we can now compute an estimate $\hat{M}_t$ of the common SDF that prices the test assets at hand, and form the following moment conditions:

$$\mathbb{E}\left[e_t(\Theta)\right]=0, \quad \text{with} \quad e_t(\Theta)=\begin{pmatrix} 1-\hat{M}_t\left[R'_{s,t}\ \ R'_{f,t}\right]' \\ \mu_c-\widehat{\text{aggcg}}_t \\ \sigma_c^2-\left[\widehat{\text{aggcg}}_t^2-\overline{\widehat{\text{aggcg}}}^2\right] \\ m_x-\hat{x}_t \\ v_x-\left(\hat{x}_t^2-\bar{\hat{x}}^2\right) \\ \rho v_x-\left(\hat{x}_t\hat{x}_{t-1}-\bar{\hat{x}}^2\right) \\ \mathbb{E}\left[\text{skewcg}_t\right]-\widehat{\text{skewcg}}_t \\ \text{var}\left[\text{skewcg}_t\right]-\left(\widehat{\text{skewcg}}_t^2-\overline{\widehat{\text{skewcg}}}^2\right) \\ \text{ac1}\left[\text{skewcg}_t\right]-\left(\widehat{\text{skewcg}}_t\cdot\widehat{\text{skewcg}}_{t-1}-\overline{\widehat{\text{skewcg}}}^2\right) \\ \mathbb{E}\left[\text{kurtcg}_t\right]-\widehat{\text{kurtcg}}_t \\ \text{var}\left[\text{kurtcg}_t\right]-\left(\widehat{\text{kurtcg}}_t^2-\overline{\widehat{\text{kurtcg}}}^2\right) \\ \text{ac1}\left[\text{kurtcg}_t\right]-\left(\widehat{\text{kurtcg}}_t\cdot\widehat{\text{kurtcg}}_{t-1}-\overline{\widehat{\text{kurtcg}}}^2\right) \end{pmatrix}, \tag{14}$$

where $\mathbb{E}\left[\text{skewcg}_t\right]$, $\text{var}\left[\text{skewcg}_t\right]$, and $\text{ac1}\left[\text{skewcg}_t\right]$ are the theoretical unconditional moments of the cross-sectional skewness, and $\mathbb{E}\left[\text{kurtcg}_t\right]$, $\text{var}\left[\text{kurtcg}_t\right]$, and $\text{ac1}\left[\text{kurtcg}_t\right]$ are the cross-sectional kurtosis equivalent, as defined in Appendix A. In total, there are $K+11$ moment conditions.

Table 5 provides the GMM estimation results based on the moment conditions of Eq. (14) for the same three different asset menus we consider in the previous section. Table 5 is structured identically to Table 4. Combined information including from Table A4 of the external appendix show that results from this alternative GMM strategy are robust as estimated Euler equation errors remain statistically close to zero. All structural parameter estimates are still statistically significant at the 95% confidence level. The subjective discount factor estimates range between 0.972 and 0.982, the RRA coefficient estimates range between 1.01 and 2.74, suggesting that country representative agents are more risk-tolerant than usually assumed in the asset pricing literature, and the EIS estimates range between 1.61 and 2.33, depending on the asset menu. It also exceeds the inverse of the risk aversion coefficient, still corresponding to a preference for early resolution of uncertainty and, therefore, corroborating previous findings in the literature.

The cross-sectional consumption risk is more persistent with an autocorrelation coefficient ρ that is between 0.96 and 0.99 when the asset menu comprises MSCI indexes. This estimate deviates from the sample value as the current GMM specification loses its target due to extra overidentifying restrictions compared to the previous specification. However, the persistence of cross-sectional consumption risk (0.52) with the present GMM specification is comparable in magnitude to the previous one (0.49) when the test assets are the Fama–French 25 developed portfolios.

Looking at the MAE, we see that the model's fit with the current GMM specification does not change compared to the previous one.[11] When the asset menu comprises the developed MSCI indexes, both GMM specifications deliver an MAE of 0.49%. The MAE values for both are equal to 0.99% when the test assets are all countries' MSCI indexes. Likewise, MAE values are 0.66% and 0.65% for the current and the previous GMM specification, respectively, when testing both on the Fama–French 25 developed portfolios. While this observation is reassuring, it questions the information content of cross-sectional higher-order cumulants and warrants some explanations.

Using only the cross-sectional variance as a proxy may seem more restrictive, focusing merely on one dimension. Our SDF's nonlinear (exponential affine) form uniquely captures the interactions between aggregate consumption growth and the cross-sectional consumption risk. Using the cross-sectional variance as a proxy within this nonlinear SDF allows for complex, nonlinear combinations of aggregate consumption growth with the cross-sectional variance of consumption growth rates, which can potentially encapsulate the relevant information in cross-sectional higher-order cumulants, namely skewness and kurtosis. When estimating the GMM based on the nonlinear SDF, both proxies of cross-sectional consumption risk deliver comparable pricing errors. This finding underscores the importance and ability of the nonlinear SDF to exploit the nonlinear interactions between aggregate consumption growth and cross-sectional variance to potentially capture the richness of information hidden in cross-sectional higher-order cumulants such as skewness and kurtosis.

Overall, the model performs well in predicting the average expected returns of country stock market indices. Thus, the original idea of Constantinides and Ghosh (2017) regarding the cross-sectional distribution of household consumption as a driver of asset prices in the U.S. also applies internationally. That is, the cross-sectional distribution of country consumption growth rates can explain the variation of expected returns across international financial markets.

3.3. Linear factor models

In this subsection, we explore commonly used linear factor pricing methods, where the SDF can be approximated by a linear combination of the factors. This approach is the most popular in cross-sectional asset pricing and ties asset risk premia directly to their comovement with the factors. In our main specification, factors are the world consumption growth, Δc_t, and the cross-sectional consumption risk level and changes, x_t and Δx_t, respectively. We recall that, since conditional cross-sectional moments of the country relative consumption growth rates in date t solely depend on x_t (through its perfect linear dependence with ω_t) as shown in the internal Appendix A, we can use these moments as factors to capture the information embedded in x_t.

The return on a given asset i in excess of the return on the risk-free asset should satisfy the standard Euler asset pricing equation given by:

$$\mathbb{E}\left[M_t R^e_{i,t}\right] = 0, \tag{15}$$

where the non-linear SDF specified in Eq. (8) can be approximated by first-order log-linear approximation as follows:

$$-\frac{M_t}{\mathbb{E}\left[M_t\right]} \approx 1 + m_t - \mathbb{E}\left[m_t\right] = b_0 + b' f_t, \tag{16}$$

where $b_0 = -1 - b_1\mu_c - b_2 m_x$ and the risk prices are given by $b' = \left(b_1, b_2, b_3\right)$ with b_1, b_2, and b_3 expressed in Eq. (9), and the factors are given by $f_t' = \left(\Delta c_t, x_t, \Delta x_t\right)$. Similar to our analysis in Section 3.2, we consider two baseline model specifications. In the first, information about cross-sectional consumption cumulants is aggregate, i.e., we have a three-factor linear model where x_t is proxied by the first principal component of the conditional cumulants of country consumption growth rates. In the second, information about cross-sectional consumption cumulants is disaggregate, i.e., we have a seven-factor model where instead of x_t and Δx_t, we use directly the three cross-sectional moments (variance, skewness, and kurtosis) and their changes.

The standard Euler Eq. (15) enables to obtain the following beta formulation of expected returns:

$$\mathbb{E}\left[R^e_{i,t}\right] = \mathrm{cov}\left[-\frac{M_t}{\mathbb{E}\left[M_t\right]} R^e_{i,t}\right] = b'\mathrm{cov}(f_t, R^e_{i,t}) = \lambda'\beta_i, \tag{17}$$

where $\lambda = \Sigma_{ff} b$ and $\beta_i = \Sigma_{ff}^{-1}\sigma_{fi}$, and where $\Sigma_{ff} = \mathbb{E}\left[\left(f_t - \mu_f\right)\left(f_t - \mu_f\right)'\right]$, $\mu_f = \mathbb{E}\left[f_t\right]$, and $\sigma_{fi} = \mathbb{E}\left[\left(f_t - \mu_f\right) R^e_{i,t}\right]$. The components of the vectors λ and β_i are the factor risk premia and the amounts of factor risks embedded in asset i, respectively.

We use the Fama and MacBeth (1973), henceforth **FM**, regressions to estimate the factor loadings for each asset and the risk premium attached to each factor. In short, in a first step, we do a time-series regression of the excess returns on the factors to obtain an estimate of β_i for each asset:

$$R^e_{i,t} = a_i + \beta_i' f_t + \epsilon_{i,t}, \quad i = 1, \ldots, N \quad \text{and} \quad t = 1, \ldots, T. \tag{18}$$

In a second step, for each time period t, we do a cross-sectional regression of excess return on the beta to get a time-series estimate of the vector λ of factor risk premia as well as time-series estimates of the pricing errors α_i. From the beta formulation in Eq. (17), the excess return on asset i for each time period can be expressed as follows:

$$R^e_{i,t} = \lambda_t'\beta_i + \alpha_{i,t}, \quad i = 1, 2, \ldots, N \text{ for each } t. \tag{19}$$

[11] A visual comparison of the model's fit across the two GMM specifications can be done by looking at Figure B1 of the external appendix. It is configured as Fig. 2. Both figures are indistinguishable visually.

If the expected returns are fully spanned by the betas, then the average pricing error $\alpha_i \equiv \mathbb{E}\left[\alpha_{i,t}\right]$ should be equal to zero. Otherwise, $\alpha_i \neq 0$ and the model is considered as misspecified.

Following Cochrane (2005), λ and α_i can be estimated as the time-series average of the cross-sectional regressions estimates. These estimates and their sampling errors are given by:

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\lambda_t,\ \hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{it},\ \sigma^2\left(\hat{\lambda}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\lambda}_t - \hat{\lambda}\right)^2,\ \sigma^2\left(\hat{\alpha}_i\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)^2. \tag{20}$$

The joint significance of the pricing errors can be tested using the chi-squared asymptotic distribution as follows:

$$\alpha'\hat{\Omega}^{-1}\alpha \sim \chi^2(N-K) \quad \text{where} \quad \hat{\Omega} = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)'. \tag{21}$$

Using Eq. (21) for testing the joint significance of the alphas does not account for the fact that factor risk exposures (betas) have been estimated in the first stage; it also assumes that the pricing errors are uncorrelated through time (no serial correlation). To correct these two limitations, we use the Shanken (1992) correction for the first and the Newey and West (1987) heteroskedasticity and autocorrelation-adjusted variance–covariance matrix for the second.

We finally provide two additional goodness-of-fit measures commonly used in the literature (e.g., Campbell and Vuolteenaho, 2004; Yogo, 2006; Darrat et al., 2011), namely, the mean absolute pricing error (MAE), and the pseudo R-squared ($\bar{R}^2$), defined by:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{\alpha}_i\right| \ \text{and}\ \bar{R}^2 = 1 - \left(\frac{\hat{\alpha}'\hat{\alpha}}{\left(\bar{R}^e - \frac{1}{N}\sum_{i=1}^{N}\bar{R}_i^e\right)'\left(\bar{R}^e - \frac{1}{N}\sum_{i=1}^{N}\bar{R}_i^e\right)}\right), \tag{22}$$

where $\hat{\alpha}_i$ is the average pricing error of asset i, $\hat{\alpha}$ is the vector of the $\hat{\alpha}_i$s, $\bar{R}_i^e$ is the average excess return of asset i, and $\bar{R}^e$ is the vector of the $\bar{R}_i^e$s.

3.3.1. *Estimation of the factor risk premia using the benchmark dataset*

The second-step FM regression results reported in Table 6 are based on our benchmark asset menu consisting of 69 developed, emerging, frontier, and standalone market MSCI indices.[12] Column (1) corresponds to the representative-agent consumption-based CAPM model (**CCAPM**), where the unique pricing factor is the world consumption growth. Column (2) corresponds to the model with heterogeneous agents and CRRA preferences (**HCRRA**), where the two pricing factors are the world consumption growth and the level of cross-sectional consumption risk. Finally, column (3) matches the specification with heterogeneous agents and Epstein–Zin preferences (**HEZ**). In this latter case, there are three pricing factors: world consumption growth, and the level and changes of cross-sectional consumption risk. The proxy for cross-sectional consumption risk is the first principal component of the cross-sectional cumulants of country consumption growth rates.

We first comment on the global performance of the different models before looking closely at the estimated coefficients. Model performance measures are reported in the bottom panel of Table 6. Column (1) shows that the CCAPM model explains 47.05% of the cross-sectional variability of expected returns on country MSCI indexes, with an MAE of 0.80%. However, it cannot fully capture the average expected excess returns as the pricing error of a typical asset appears to be significantly different from zero (the average absolute t-stat of the alphas is 2.15). On the other hand, the HCRRA model in column (2) shows an improvement compared to the CCAPM model. Indeed, it explains 58.74% of the variability of the countries' MSCI indexes expected excess returns, with an MAE of 0.71%, and pricing errors that on average are not statistically different from zero at the standard level of significance. Thus, accounting for agents' heterogeneity is important in explaining the expected returns of international equity indices. Finally, the HEZ model in column (3) displays further improvement over the HCRRA. The model captures 60.31% of the variability of the countries' MSCI indexes expected returns and performs pretty well in mean absolute pricing error at 0.69%. Likewise, the average absolute t-stat of the model's alphas is 1.54, thus the typical alpha is not statistically different from zero at a 10% significance level, even though the joint nullity of the model pricing errors is rejected by the corresponding test.

Turning to the coefficients estimates, we see that the factor risk premia are strongly identified as the rank tests in the bottom panel of Table 6 reject the null hypothesis of their weak identification.[13] Moreover, these reduced-form factor risk premiums are statistically significant, stable across different model specifications, and possess the expected theoretical sign. Focusing on the HEZ model in column (3), both the level and changes in cross-sectional consumption risk carry negative risk premia estimates under the PCA-based proxy, amounting to −0.64 and −0.28, with t-statistics of −1.96 and −2.19, respectively. To put these estimates in perspective, a one-standard deviation reduction in the cross-sectional consumption risk's beta (equivalent to a change in $\beta_{i,x}$ by −1.576 in the cross-section) aligns with a 1.01% increment in the risk premium, derived as −0.64×−1.576. Likewise, a one-standard deviation

[12] Similar results without the intercept in the second-step regressions are presented in Tables A5, A6 and A7 of the external appendix. We also refer the reader to it for discussions about weakly identified factors in Sections A and B, including illustrations in Figure B2 and Table A8. Darrat et al. (2011) examine a linear factor model featuring cross-sectional variance of country consumption growth rates but do not address the weak-identification issue.

[13] The results in Tables A9, A10 and A11 of the external appendix show that our tests are very powerful in detecting a non-zero risk premium, both in the single factor and the multi-factor cases. Assuming a data-generating process similar to the one we have in our observed data, the probability of not rejecting the null hypothesis of a zero risk premium for a factor when it is false, and therefore committing a type II error, is less than 1%. Our simulations' results support that the prices of risk are well-estimated and our tests do not lack power provided we consider the actual variation in empirical betas of the assets.

Table 6
Fama-MacBeth regressions results: All countries MSCI indexes.

	Consumption-based Models							CAPM and 3 factors Models		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)	(9)
	CCAPM	HCRRA	HEZ	DLP	HDLP	HCRRA-MOM	HEZ-MOM		CAPM	FF-3
λ_0	1.60	2.04	2.18	1.52	1.16	1.48	1.67	λ_0	0.85	1.65
	(2.84)	2.41)	(2.61)	(2.70)	(1.81)	(2.39)	(2.21)		(0.59)	(1.96)
	[2.83]	[2.32]	[2.25]	[2.62]	[1.68]	[2.34]	[0.97]		[0.58]	[1.70]
λ_{aggcg}	0.35	−0.26	−0.53	0.42	0.52	0.36	−0.10	λ_{CAPM}	1.16	−2.29
	(0.98)	(−0.55)	(−1.09)	(1.77)	(2.37)	(1.35)	(0.41)		(−0.79)	(−1.58)
	[0.97]	[−0.53]	[−0.95]	[1.73]	[2.25]	[1.32]	[−0.20]		[0.76]	[−1.41]
λ_x		−0.18	0.03							
		(−0.56)	(0.10)							
		[−0.54]	[0.09]							
$\lambda_{\triangle x}$			−0.41							
			(−3.38)							
			[−3.01]							
λ_{vargcg}				0.34	−0.32	−0.16	−0.29	λ_{HML}		1.43
				(0.27)	(−0.23)	(−0.18)	(−0.30)			(0.69)
				[0.27]	[−0.22]	[−0.18]	[−0.14]			[0.60]
λ_{skewcg}						0.16	−0.22	λ_{SMB}		0.05
						(0.39)	(−0.70)			(0.06)
						[0.38]	[0.32]			[0.05]
λ_{kurtcg}						0.45	0.53			-
						(0.41)	(0.52)			
						[0.40]	[0.25]			
$\lambda_{\triangle vargcg}$					−1.00		−0.68			
					(−1.20)		(−0.87)			
					[−1.12]		[−0.38]			
$\lambda_{\triangle skewcg}$							0.55			
							(1.97)			
							[1.18]			
$\lambda_{\triangle kurtcg}$							2.23			
							(2.63)			
							[1.15]			
R^2	8.63%	12.42%	35.27%	11.78%	22.92%	10.51%	45.13%		8.41%	12.18%
MAE	0.52	0.50	0.42	0.51	0.50	0.51	0.40		0.52	0.57
Rank test	4.34	6.54	5.77	3.08	1.81	4.19	0.31		495.15	1.02
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(1.00)		(0.00)	(0.44)
Alpha test	414.23	96.47	60.22	243.76	223.77	273.65	28.57		68.71	42.63
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)		(0.00)	(0.00)
Avrge.|t-stat (α)|	1.46	1.51	1.15	1.54	1.56	1.67	0.65		1.30	1.07

The table shows the Fama–MacBeth regressions results when the asset menu comprises the 69 developed, emerging, frontier, and standalone market MSCI indices. Consumption-based factors are computed from a balanced panel of 24 developed and emerging country consumption data. The data frequency is quarterly for the consumption-based model specifications, and the sample period runs from 1961Q1 to 2021Q1. The data frequency is monthly for the global CAPM and the global Fama–French three-factor model, with different starting months, 1961M1 and 1990M1, respectively, and the same end month, 2021M3. Our estimation sample is constrained by the shortest time series. The global CAPM factor is the MSCI World index excess return over the three-month U.S. Tbill rate. For each factor risk premium estimate, the Newey–West (Shanken) t-statistics are shown in parentheses (square brackets). R^2 is the r-squared from the cross-sectional regression of average excess returns on the betas. MAE is the mean absolute pricing error. The rank test statistic is computed as in Kleibergen and Zhan (2020) and the Alpha test statistic is computed from equation (21). Avrge.$|t\text{-stat}(\alpha)|$ is the average absolute Shanken-adjusted Newey–West t-statistics of the pricing errors.

decrease in the beta for changes in cross-sectional consumption risk, resulting in a change of $\beta_{i,\Delta x}$ by −1.464 in the cross-section, signifies a 0.41% rise in the risk premium (calculated as −0.28×−1.464), everything else being equal. Note that this last contribution to the risk premium is pertinent only within the Epstein–Zin's preference framework. Given that the average risk premium across the 69 MSCI indices is 2.40%, and the quartile values are 1.50%, 2.24%, and 3.09%, respectively, these findings hold substantial economic weight. In summary, these results confirm the empirical validity of the heterogeneous-agents consumption-based asset pricing model with Epstein–Zin preferences as changes, in addition to the level, of cross-sectional consumption risk appear to be a significant cross-sectional pricing factor for international equity indices.

We further delve into the economic significance of our two novel factors by dissecting the risk premium associated with taking a long position in emerging markets and a short position in developed markets, say the EMD premium, breaking down their contributions from different factors and the portion unexplained by the model. The results are presented in Panels A, B, and C of Table 7. In the data, the difference in average excess returns between emerging and developed countries (i.e., the EMD premium), amounting to 0.98% quarterly, aligns with the well-established notion in the literature that investment in emerging countries entails higher risk when compared to investments in developed countries. This is a consensus understanding supported by extensive research. When we examine emerging markets' exposure relative to developed markets concerning a specific risk factor, it is reasonable to anticipate a positive long-short premium. This premium should be positive, calculated as the factor lambda

Table 7
Decomposition of the risk premium.

Data	Emerging	Developed	Difference	Explained
R^{ex}	3.03	2.04	0.98	0.98
Model	β_{Emerging}	$\beta_{\text{Developed}}$	$\beta_{\text{Difference}}$	$\lambda_{\text{factor}}\beta_{\text{Difference}}$
Panel A: CCAPM				
aggcg	1.51	1.28	0.23	0.35
Panel B: HCRRA				
aggcg	1.13	1.12	0.01	0.01
x	−1.70	−0.87	−0.83	0.53
Panel C: HEZ				
aggcg	1.09	1.11	−0.02	−0.03
x	−1.18	−0.72	−0.46	0.29
Δx	−1.37	−0.42	−0.94	0.26
Panel D: DLP				
aggcg	1.53	1.27	0.26	0.43
vargcg	0.00	−0.02	0.02	−0.06
Panel E: HDLP				
aggcg	1.32	1.23	0.09	0.14
vargcg	0.22	0.06	0.16	−0.46
Δvargcg	−0.74	−0.26	−0.48	0.80
Panel F: HCRRA-MOM				
aggcg	1.77	1.34	0.43	0.59
vargcg	0.25	0.08	0.17	−0.46
skewcg	1.60	0.82	0.78	0.51
kurtcg	−0.19	−0.08	−0.11	0.00
Panel G: HEZ-MOM				
aggcg	1.73	1.35	0.38	0.51
vargcg	0.56	0.17	0.39	−1.00
skewcg	1.63	0.82	0.81	0.50
kurtcg	−0.37	−0.19	−0.18	0.01
Δvargcg	−0.79	−0.28	−0.51	0.72
Δskewcg	−0.06	0.10	−0.17	−0.03
Δkurtcg	0.36	0.26	0.10	−0.01
Panel H: HEZ-MOM (Parsimonious)				
aggcg	1.37	1.25	0.11	0.15
vargcg	0.38	0.12	0.26	−0.61
skewcg	1.63	0.85	0.78	0.49
Δvargcg	−0.70	−0.24	−0.46	0.60

This table shows the decomposition of the risk premium with respect to consumption factors. It reports the risk exposures (βs) and their implied risk premiums. R^{ex} is the average excess return reported for emerging countries, developed countries, and the difference between the two. βs are obtained from the time series regressions of return on consumption factors. The risk premium is computed as the product of risk exposure and risk price. These betas and risk prices correspond to models estimated in Table 6

multiplied by the spread between the betas of emerging and developed markets. This expectation aligns with the inherent risk differential between emerging and developed markets, regardless of the specific factor being considered.

Within the observed EMD premium of 0.98%, the CCAPM's aggregate consumption factor contributes 0.35%. However, when considering the HCRRA and HEZ models, this factor's contribution is minimal at 0.01% and −0.03%, respectively. Instead, the PCA-based proxy for cross-sectional consumption risk becomes predominant in these models. Specifically, the HCRRA and HEZ models account for 0.54% and 0.52% of the EMD premium, which surpasses the 0.35% attributed to the CCAPM. Consistent with theoretical expectations and the extensive empirical evidence confirming that emerging markets investments bear higher risk, betas of emerging and developed stocks on the level and changes of cross-sectional consumption risk are negative, with the negative trend being significantly pronounced for emerging market stocks. For the level, the betas are −1.18 for emerging stocks and −0.72 for developed stocks. For changes, the values are −1.37 and −0.42, respectively. Both the level and changes in our PCA-based proxy for cross-sectional consumption risk exhibit positive and substantial contributions to the EMD premium.

Table 6's columns (4) through (7) present the outcomes of the second-step FM regressions, replacing the cross-sectional consumption risk with actual cross-sectional moments of countries' consumption growth rates. An international investor typically seeks to diversify their portfolio across various markets to mitigate risk and enhance potential returns. However, the nature of the relationship between international stock markets and the cross-sectional moments of countries' consumption growth rates can significantly impact the attractiveness of these investments.

An international stock market negatively correlated with the cross-sectional variance of countries' consumption growth rates tends to underperform during times of significant variability in these rates, such as the 1990 oil price shock or the 2020 COVID-19 pandemic, as shown in Panel A of Fig. 1. Such performance can be disadvantageous during global economic uncertainties when investors seek stability in portfolios. Consequently, global investors may shy away from markets that struggle during heightened cross-sectional variance and expect a higher premium for investing in them. This perspective highlights the link between a negative risk premium and cross-sectional variance.

In column (4) of Table 6, the cross-sectional consumption risk is swapped with the cross-sectional variance of country consumption growth rates, resulting in a two-factor model, the Darrat et al. (2011) model, here referred to as **DLP**, akin to column (2). Similarly, column (5), resembling column (3), introduces the **HDLP** model, a theoretically-driven extension of the DLP model, with the addition of changes in cross-sectional variance. Both factor risk premiums are negative and highly statistically significant, as indicated in columns (4) and (5). Comparing the adjusted R^2 values between the HCRRA and the DLP model provides an immediate measure of our PCA-based proxy's performance compared to cross-sectional variance. The two-factor HCRRA model achieves an R^2 of 58.74%, surpassing the DLP model's R^2 of 51.35%. Similarly, the three-factor HEZ model, with an R^2 of 60.31%, outperforms the three-factor HDLP model, which yields an R^2 of 54.00%.

Let us revisit the breakdown of the 0.98% quarterly EDM premium. In the DLP model, 0.36% is attributed to it, with 0.43% coming from aggregate consumption growth and −0.06% from the cross-sectional variance of country consumption growth rates. As shown in Panel D of Table 7, once exposure to aggregate consumption growth is factored in, the exposure of emerging and developed markets to the cross-sectional variance factor becomes negligible. Consequently, the contribution of the cross-sectional variance factor to the EMD premium is also negligible.

Moving on to the HDLP model, it explains 0.48% of the EMD premium, as depicted in Panel E of Table 7. Interestingly, this model's predicted premium is primarily driven by changes in cross-sectional variance, a factor rooted in recursive preferences akin to Epstein–Zin.

The PCA-based proxy for cross-sectional consumption risk (HEZ specification) outperforms the cross-sectional variance factor (HDLP specification). Our results highlight the substantial positive contribution of the PCA-based proxy, amounting to 0.29%, in explaining the EMD premium. In contrast, the contribution from the cross-sectional variance factor is negative at −0.46%. Overall, our findings support Epstein–Zin preferences over CRRA preferences and suggest that, while the cross-sectional variance factor is significant, our PCA-based proxy for cross-sectional consumption risk, which incorporates information about cross-sectional skewness and kurtosis factors, is more effective in explaining variations in expected excess returns across international assets.

Recognizing that cross-sectional cumulant factors in empirical data exhibit imperfect correlations compared to the model's assumptions, we explore ad-hoc specifications. These specifications supplement the cross-sectional variance factor with cross-sectional higher-order moments, allowing us to examine empirically whether information from these moments, unaccounted for by cross-sectional variance, holds value in pricing international equities. Although this approach does not directly derive from the model, it aligns with its underlying assumptions. In various empirical analyses, the stock markets of different countries show diverse sensitivities to specific cross-sectional moments of consumption growth rates. This suggests that beyond the typical factors like aggregate consumption growth and cross-sectional variance, actual cross-sectional higher-order moments could also play a pivotal role in pricing international stock indices.

Column (6) of Table 6 departs from column (2) like column (4) but replaces the cross-sectional consumption risk with cross-sectional variance, skewness, and kurtosis, forming a four-factor model known as the **HCRRA-MOM**. Lastly, column (7), paralleling column (3), presents the seven-factor **HEZ-MOM** model, an intuitively guided extension of the HCRRA-MOM based on Epstein–Zin's recursive utility in our theoretical framework, which includes changes in the cross-sectional variance, skewness, and kurtosis of country consumption growth rates as extra factors.

Negative cross-sectional skewness of countries' consumption growth rates indicates many countries experiencing below-average growth. A stock market declining in such scenarios shows its sensitivity to widespread economic downturns. Events like the 1997 Asian Financial Crisis and the early 2000s Dot-com bubble caused significant negative skewness in global consumption growth rates, as seen in Panel A of Fig. 1. Markets declining during these periods posed investment risks, with investors facing losses both from economic challenges and underperforming stock markets. Hence, investors might anticipate higher premiums for entering such vulnerable markets, tying a positive risk premium to cross-sectional skewness. Our empirical findings, particularly in the HCRRA-MOM and HEZ-MOM models, support this skewness pricing.

High cross-sectional kurtosis of consumption growth rates signifies extreme values in the distribution. Markets underperforming in such conditions fail to offer protection during economic tail events, crucial for risk management. For example, the early 2010s European debt crisis introduced significant consumption growth rate variations, resulting in high kurtosis as visualized in Panel A of Fig. 1. Markets negatively correlated with this high kurtosis proved unreliable during financial distress. Nevertheless, our findings lack empirical support for international asset pricing based on the cross-sectional kurtosis of country consumption growth rates.

Overall, Table 6 showcases estimates of consumption-based factor risk premiums, most of which are statistically significant, except for the cross-sectional kurtosis in the HCRRA-MOM and HEZ-MOM models, as well as the changes in the cross-sectional skewness and kurtosis within the HEZ-MOM model. This leads us to propose the **Parsimonious** four-factor model, as delineated in column (8) of Table 6. This model omits factors from the HEZ-MOM model that are not statistically significant, also mitigating concerns of having excessive factors in the original HEZ-MOM setup.

The Parsimonious and HEZ-MOM models demonstrate similar efficacy, with their risk price estimates and explanatory prowess closely aligned. Further, the signs of the factor risk premium estimates from both models align with theoretical predictions. When we juxtapose the three-factor HEZ model with the four-factor parsimonious HEZ-MOM model, they exhibit comparable performance metrics: their R^2 values are 60.31% and 61.98%, respectively, with an identical MAE of 0.69%. They also have average absolute t-statistics of 1.54 and 1.01 for alphas, respectively, indicating that typical asset pricing errors are statistically insignificant at the 10% level. However, there is a notable difference in the rank test; it confirms the accurate identification of factor risk premiums in the HEZ model, a characteristic absent in the parsimonious HEZ-MOM model.

We further juxtapose the efficacy of our heterogeneous-agents consumption-based model specifications with the global Capital Asset Pricing Model (**CAPM**) and the global Fama–French three-factor (**FF-3**) model. In Table 6, these models align with column (9) and (10), respectively. Firstly, the rank test implies that the factor risk premiums in both CAPM and the FF-3 model are accurately identified. Secondly, in comparison to all previous consumption-based model specifications, these models display subpar performance, accounting for merely 26.92% and 19.32% of the variability in the expected returns across global financial markets, respectively.[14] In a similar vein to the CCAPM, but contrary to the HEZ and HDLP models, neither the CAPM nor FF-3 models fully capture the average expected excess returns. This shortfall is evidenced by the significant deviation of a typical asset's pricing error from zero, as corroborated by the average absolute t-stat of the alphas, which stand at 2.42 for the CAPM and 1.89 for the FF-3 model.

Examining the remaining data in Table 7, which details the contribution of various factors to the 0.98% quarterly EMD premium across different model specifications, we observe varied predictions from the HCRRA-MOM, HEZ-MOM, and the streamlined HEZ-MOM models. These models estimate the EMD premium at 0.65%, 0.70%, and 0.63%, respectively.

In all these scenarios, factors such as cross-sectional kurtosis and changes in cross-sectional skewness and kurtosis have a minimal impact, aligning with their previously established insignificance. However, the novel consumption factors we endorse, derived from higher-order cross-sectional moments (like cross-sectional skewness) or Epstein–Zin preferences (such as changes in cross-sectional variance), make substantial positive contributions to the EMD premium.

Overall, our comprehensive exploration of scenarios in Table 7 consistently highlights the pivotal role of cross-sectional skewness and changes in cross-sectional variance in elucidating the risk premium associated with investing in emerging markets versus developed markets. They consistently offer a positive contribution to the EMD premium. Conversely, the cross-sectional variance factor consistently contributes negatively, underscoring its limited role in accounting for the EMD premium across the various model specifications.

3.3.2. Estimation of the factor risk premia using alternative datasets

In this section, we evaluate the effectiveness of our proposed linear factor model specifications, applying them to alternative international asset menus. We first focus on the 23 developed market MSCI indices, with results presented in Table 8. This table offers results similar to those in Table 6, allowing for a direct comparison.

Upon analysis, it is apparent that the R^2 values for the CCAPM, the heterogeneous-agents model specifications with the PCA-based proxy for cross-sectional consumption risk (HCCRA and HEZ), as well as their counterparts utilizing cross-sectional variance as the proxy (DLP and HDLP), are significantly lower when estimated solely on developed market indices. This contrasts the results obtained in Table 6, where the models are applied across all categories of country MSCI indices.

More specifically, the R^2 values for the HEZ model are reduced by approximately two-fifths, about three-fifths for the HDLP model, and nearly four-fifths for the CCAPM, HCRRA, and DLP models.[15] These reductions are noteworthy, even though factor risk premia show strong identification, as indicated by rank test p-values of 0.02 or less. Additionally, the average absolute t-statistics for the alphas are 1.56 or less, implying that the typical asset pricing errors are not statistically significant at the 10% level. The HCCRA-MOM and HEZ-MOM models in Table 8 are no exception to these observations of poor performance when compared to their versions in Table 6.

We explore the subpar performance of models on developed market MSCI indices by examining asset exposures to various factors. Table 9 presents the statistics of factor loadings from first-step FM regressions, comparing all asset menus. Notably, the standard deviation and range of asset betas are consistently greater for the all-inclusive asset menu than the one restricted to only developed countries. For instance, in the HEZ model, betas on country consumption risk level and changes show standard deviations of 1.58 and 1.46 for the broader menu and 0.72 and 0.85 for developed countries. Similarly, for the HEZ-MOM model, betas on cross-sectional variance, skewness, and changes have standard deviations of 0.52, 0.99, and 0.56 with the broader menu but reduce to 0.29, 0.49, and 0.32 for developed countries. This heightened risk exposure diversity in the broader asset menu aligns with findings from Ang et al. (2020) and Gagliardini et al. (2016), emphasizing the importance of a comprehensive cross-section for accurate risk premia estimates and model efficacy.

The noticeable decline in model performance, evidenced by lower R^2 values and overall statistically insignificant risk premia estimates of consumption-based factors when estimated solely on developed country MSCI indices, underscores the pivotal role of developing countries, such as emerging, frontier, and standalone markets. As seen in Table 9, developed countries, despite their financial sophistication, exhibit a constrained diversity in risk exposures. This is attributed to the synchronized behavior of their

14 The consumption-based model specifications and the global CAPM have been evaluated over a longer sample period than the global Fama–French three-factor model, the data for which only commences from 1990Q1.

15 The relative discrepancies in performance metrics, specifically the R^2 values, when comparing our results to those obtained by Darrat et al. (2011) are noteworthy. Our calculated R^2 of 9% for the CCAPM and 12% for the DLP specification fall short of the 11% and 27% respectively achieved in their study, despite their focus on a similar asset menu consisting of developed markets MSCI indices up to 2007Q4. However, our analysis extends to a more recent timeframe, up to 2021Q1, encompassing a broader array of market conditions and economic events. In addition, our approach to constructing consumption factors diverges from that of Darrat et al. (2011). While they opt for GDP-weighted cross-sectional moments, we employ PPP-weighted cross-sectional moments.

Table 8
Fama–MacBeth regressions results: Developed countries MSCI indexes.

	Consumption-based Models								CAPM and 3 factors Models	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)	(9)
	CCAPM	HCRRA	HEZ	DLP	HDLP	HCRRA-MOM	HEZ-MOM		CAPM	FF-3
	3.13	2.70	0.25	1.62	1.90	0.91	0.63		1.80	1.92
λ_0	(4.55)	(3.64)	(0.32)	(2.53)	(2.88)	(1.29)	(0.86)	λ_0	(2.33)	(3.18)
	[4.13]	[1.54]	[0.09]	[1.71]	[1.65]	[0.52]	[0.26]		[1.18]	[1.46]
	1.70	1.99	1.99	1.38	1.00	1.37	1.11		13.79	12.98
λ_{aggcg}	(2.54)	(3.57)	(3.68)	(2.37)	(1.47)	(2.99)	(2.59)	λ_{CAPM}	(3.66)	(3.38)
	[2.36]	[1.58]	[1.08]	[1.64]	[0.86]	[1.28]	[0.84]		[1.89]	[1.58]
		−2.44	−3.55							
λ_x		(−2.47)	(−3.69)							
		[−1.05]	[−1.03]							
			−2.71							
$\lambda_{\triangle x}$			(−4.03)							
			[−1.13]							
				0.84	1.39	−3.67	−3.27			−1.40
λ_{vargcg}				(0.70)	(1.05)	(−2.60)	(−3.65)	λ_{HML}		(−0.86)
				[0.48]	[0.62]	[−1.07]	[−1.16]			[−0.40]
						3.87	3.38			2.08
λ_{skewcg}						(3.14)	(3.89)	λ_{SMB}		(1.69)
						[1.27]	[1.18]			[0.83]
						−2.07	−2.86			
λ_{kurtcg}						(−0.70)	(−0.94)			-
						[−0.28]	[−0.28]			
					−2.38		−1.68			
$\lambda_{\triangle vargcg}$					(−2.04)		(−1.83)			
					[−1.20]		[−0.55]			
							2.19			
$\lambda_{\triangle skewcg}$							(4.19)			
							[1.44]			
							3.32			
$\lambda_{\triangle kurtcg}$							(0.96)			
							[0.29]			
R^2	45.30%	53.37%	78.35%	65.99%	69.13%	85.77%	88.03%		76.01%	75.83%
MAE	0.45	0.38	0.32	0.35	0.35	0.30	0.23		0.30	0.29
Rank test	10.30	1.27	1.25	3.34	1.54	1.19	0.09		219.40	2.85
p-value	(0.00)	(0.21)	(0.22)	(0.00)	(0.08)	(0.28)	(1.00)		(0.00)	(0.00)
Alpha test	237.52	14.68	7.86	120.63	83.20	25.38	9.04		62.77	36.52
p-value	(0.00)	(0.88)	(0.99)	(0.00)	(0.00)	(0.23)	(0.96)		(0.00)	(0.003)
Avrge.$\lvert t$-stat $(\alpha)\rvert$	1.38	0.59	0.46	1.03	1.03	0.61	0.48		0.50	0.61

The table shows the Fama–MacBeth regressions results when the asset menu comprises the 23 developed market MSCI indices. Consumption-based factors are computed from a balanced panel of 24 developed and emerging country consumption data. The data frequency is quarterly for the consumption-based model specifications, and the sample period runs from 1961Q1 to 2021Q1. The data frequency is monthly for the global CAPM and the global Fama–French three-factor model, with different starting months, 1961M1 and 1990M1, respectively, and the same end month, 2021M3. Our estimation sample is constrained by the shortest time series. The global CAPM factor is the MSCI World index excess return over the three-month U.S. Tbill rate. For each factor risk premium estimate, the Newey–West (Shanken) t-statistics are shown in parentheses (square brackets). R^2 is the r-squared from the cross-sectional regression of average excess returns on the betas. MAE is the mean absolute pricing error. The rank test statistic is computed as in Kleibergen and Zhan (2020) and the Alpha test statistic is computed from Eq. (21). Avrge.$\lvert t$-stat$(\alpha)\rvert$ is the average absolute Shanken-adjusted Newey–West t-statistics of the pricing errors.

financial markets, influenced by similar policy responses to economic shocks and international trade openness. Such uniformity obscures the distinction of consumption-based factors in their MSCI indices, leading to imprecise risk premia estimates for factors. However, our research accentuates the importance of developing countries. These countries, deemed investable by the MSCI for their robust and globally accessible stock markets, are instrumental in shaping global asset pricing dynamics. Our findings imply that these markets react to global risk factors even when the foundational factors are derived predominantly from developed countries' consumption data.

Table 10 showcases estimation results for Fama–French 25 developed portfolios sorted by size and momentum. The single-factor CCAPM has a cross-sectional R^2 of 45.30%, closely matching its 47.05% in the benchmark asset menu. Both HCCRA and HEZ models indicate weak risk premia identification, evident from their rank test p-values of 0.21 and 0.22 and further supported by the low standard deviations of betas in Table 9. We interpret risk premia estimates carefully, but the consistent sign with theory and the benchmark is notable. The HEZ model explains 78.35% of cross-sectional variation with an insignificant mean absolute pricing error of 0.32%.

In columns (4) to (7) of Table 10, robustness tests reveal inconsistencies in the DLP and HDLP models regarding the risk premium of cross-sectional variance of country consumption growth rates. The significance of this factor becomes evident only when combined with skewness and kurtosis, as in HCRRA and HEZ models. These factors are jointly priced in line with theoretical expectations.

Table 9
Summary statistics of the βs.

	Developed					All Countries					25 Fama–French PF				
Statistic	N	Mean	St. Dev.	Min	Max	N	Mean	St. Dev.	Min	Max	N	Mean	St. Dev.	Min	Max
Panel A: CCAPM															
β_{aggcg}	23	1.28	0.42	0.60	1.91	69	1.34	0.60	−0.06	2.96	25	−0.64	0.34	−1.49	−0.28
$tstat(\beta_{\text{aggcg}})$	23	2.40	0.70	1.16	3.54	69	2.41	1.22	−0.22	7.13	25	−1.35	0.70	−2.88	−0.38
Panel B: HCRRA															
β_{aggcg}	23	0.56	0.36	−0.23	1.38	69	1.09	0.66	−0.69	2.69	25	−0.73	0.36	−1.64	−0.33
$tstat(\beta_{\text{aggcg}})$	23	1.10	0.65	−0.29	2.66	69	1.59	1.04	−1.32	5.90	25	−1.31	0.62	−2.72	−0.44
β_x	23	−0.85	0.73	−2.88	0.18	69	−1.16	1.53	−6.03	2.74	25	−0.32	0.14	−0.58	−0.07
$tstat(\beta_x)$	23	−0.97	0.76	−2.67	0.21	69	−1.08	1.13	−3.84	1.76	25	−0.44	0.18	−0.82	−0.11
Panel C: HEZ															
β_{aggcg}	23	0.55	0.36	−0.23	1.35	69	1.06	0.66	−0.75	2.69	25	−0.78	0.36	−1.69	−0.38
$tstat(\beta_{\text{aggcg}})$	23	1.11	0.66	−0.30	2.68	69	1.56	1.07	−1.41	5.85	25	−1.49	0.67	−2.98	−0.54
β_x	23	−0.70	0.72	−2.42	0.82	69	−0.76	1.58	−5.44	3.78	25	0.20	0.21	−0.20	0.57
$tstat(\beta_x)$	23	−0.81	0.73	−2.36	0.54	69	−0.68	1.14	−3.23	4.01	25	0.24	0.25	−0.22	0.75
$\beta_{\Delta x}$	23	−0.45	0.85	−2.81	1.22	69	−1.01	1.46	−4.60	3.24	25	−1.49	0.33	−2.19	−1.11
$tstat(\beta_{\Delta x})$	23	−0.39	0.76	−2.61	1.13	69	−0.65	0.94	−2.78	2.22	25	−1.57	0.45	−2.42	−0.95
Panel D: DLP															
β_{aggcg}	23	1.27	0.42	0.52	2.12	69	1.50	0.81	−0.54	3.85	25	−0.03	0.43	−1.00	0.66
$tstat(\beta_{\text{aggcg}})$	23	2.22	0.76	1.22	4.01	69	1.93	1.04	−0.51	5.13	25	−0.01	0.51	−1.05	0.83
β_{vargcg}	23	−0.02	0.13	−0.17	0.43	69	0.10	0.38	−0.77	1.46	25	0.53	0.17	0.33	0.94
$tstat(\beta_{\text{vargcg}})$	23	−0.27	0.64	−1.33	0.99	69	0.20	1.07	−3.21	3.63	25	1.56	0.48	0.94	2.41
Panel E: HDLP															
β_{aggcg}	23	1.23	0.44	0.49	2.12	69	1.35	0.81	−0.71	3.87	25	0.02	0.43	−0.92	0.71
$tstat(\beta_{\text{aggcg}})$	23	2.14	0.79	0.87	3.98	69	1.74	1.08	−0.64	5.22	25	0.06	0.53	−1.01	0.92
β_{vargcg}	23	0.06	0.20	−0.14	0.85	69	0.27	0.44	−0.68	1.50	25	0.44	0.18	0.25	0.86
$tstat(\beta_{\text{vargcg}})$	23	0.10	0.56	−0.77	1.76	69	0.64	1.12	−1.71	5.61	25	1.16	0.47	0.52	1.99
$\beta_{\Delta\text{vargcg}}$	23	−0.26	0.32	−1.23	0.15	69	−0.57	0.52	−1.70	0.42	25	0.21	0.07	0.08	0.40
$tstat(\beta_{\Delta\text{vargcg}})$	23	−0.89	1.63	−7.90	0.39	69	−2.02	2.69	−14.37	1.90	25	0.68	0.23	0.28	1.14
Panel F: HCRRA-MOM															
β_{aggcg}	23	1.34	0.43	0.56	2.14	69	1.57	0.82	0.09	4.15	25	0.31	0.40	−0.58	1.00
$tstat(\beta_{\text{aggcg}})$	23	2.35	0.79	1.04	4.10	69	1.91	1.12	0.23	5.10	25	0.45	0.49	−0.62	1.22
β_{vargcg}	23	0.08	0.18	−0.13	0.58	69	0.22	0.33	−0.55	1.72	25	0.79	0.20	0.56	1.27
$tstat(\beta_{\text{vargcg}})$	23	0.23	0.63	−0.68	1.64	69	0.57	1.11	−2.80	5.03	25	2.60	0.54	1.86	3.89
β_{skewcg}	23	0.82	0.47	0.02	1.90	69	1.24	0.92	−0.81	3.53	25	0.83	0.26	0.51	1.39
$tstat(\beta_{\text{skewcg}})$	23	1.62	0.78	0.05	3.64	69	1.67	1.05	−0.68	4.05	25	2.49	0.55	1.70	3.54
β_{kurtcg}	23	−0.08	0.15	−0.51	0.25	69	−0.08	0.26	−0.74	0.70	25	−0.19	0.05	−0.30	−0.10
$tstat(\beta_{\text{kurtcg}})$	23	−0.53	0.84	−2.37	1.35	69	−0.28	1.19	−2.37	4.38	25	−1.72	0.54	−2.72	−0.90
Panel G: HEZ-MOM															
β_{aggcg}	23	1.35	0.42	0.55	2.15	69	1.57	0.95	−0.34	4.40	25	0.25	0.40	−0.65	0.93
$tstat(\beta_{\text{aggcg}})$	23	2.37	0.80	1.16	4.18	69	1.98	1.34	−0.84	6.30	25	0.41	0.53	−0.69	1.28
β_{vargcg}	23	0.17	0.29	−0.10	1.06	69	0.47	0.52	−0.45	1.82	25	0.52	0.18	0.31	0.97
$tstat(\beta_{\text{vargcg}})$	23	0.62	0.95	−0.45	4.32	69	1.14	1.54	−1.46	10.64	25	1.54	0.56	0.74	2.72
β_{skewcg}	23	0.82	0.49	0.06	1.87	69	1.30	0.99	−0.93	3.84	25	0.37	0.21	0.01	0.76
$tstat(\beta_{\text{skewcg}})$	23	1.46	0.87	0.12	3.79	69	1.56	1.06	−1.21	4.11	25	0.99	0.53	0.03	2.03
β_{kurtcg}	23	−0.19	0.16	−0.68	0.04	69	−0.24	0.33	−0.95	0.96	25	−0.12	0.03	−0.19	−0.06
$tstat(\beta_{\text{kurtcg}})$	23	−1.26	1.09	−5.07	0.22	69	−0.88	1.26	−5.07	4.30	25	−1.03	0.28	−1.65	−0.66
$\beta_{\Delta\text{vargcg}}$	23	−0.28	0.32	−1.22	0.05	69	−0.59	0.56	−2.14	0.38	25	0.45	0.10	0.25	0.64
$tstat(\beta_{\Delta\text{vargcg}})$	23	−1.00	1.69	−7.98	0.13	69	−2.11	2.81	−18.05	2.23	25	1.54	0.33	0.98	2.21
$\beta_{\Delta\text{skewcg}}$	23	0.10	0.48	−0.79	0.86	69	−0.07	1.05	−2.90	2.90	25	1.16	0.27	0.79	1.73
$tstat(\beta_{\Delta\text{skewcg}})$	23	0.17	0.84	−1.41	1.40	69	−0.11	1.08	−2.71	1.86	25	1.74	0.57	0.81	2.80
$\beta_{\Delta\text{kurtcg}}$	23	0.26	0.11	0.12	0.54	69	0.31	0.32	−0.51	1.26	25	−1.03	0.28	−1.65	−0.66
$tstat(\beta_{\Delta\text{kurtcg}})$	23	1.77	0.54	0.90	2.83	69	1.40	1.13	−1.80	3.58	25	−0.88	0.30	−1.65	−0.47
Panel H: HEZ-MOM (Parsimonious)															
β_{aggcg}						69	1.36	0.79	−0.56	3.92					
$tstat(\beta_{\text{aggcg}})$						69	1.81	1.10	−0.57	5.53					
β_{vargcg}						69	0.35	0.41	−0.50	1.65					
$tstat(\beta_{\text{vargcg}})$						69	0.93	1.22	−1.70	6.73					
β_{skewcg}						69	1.21	0.99	−1.36	3.92					

(*continued on next page*)

Table 9 (*continued*).

$tstat(\beta_{\text{skewcg}})$	69	1.62	1.15	−2.20	4.41
$\beta_{\Delta\text{vargcg}}$	69	−0.53	0.50	−1.64	0.44
$tstat(\beta_{\Delta\text{vargcg}})$	69	−2.00	3.00	−19.67	1.84

This table shows the summary statistics of risk exposures to consumption factors (βs). βs are obtained from the time series regressions of return on consumption factors. These betas correspond to models and risk prices estimated in Tables 6, 8, and 10.

Table 10
Fama–MacBeth regressions results: Fama–French 25 developed portfolios.

	Consumption-based Models								CAPM and 3 factors Models	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)	(9)
	CCAPM	HCRRA	HEZ	DLP	HDLP	HCRRA-MOM	HEZ-MOM		CAPM	FF-3
λ_0	3.13 (4.55) [4.13]	2.70 (3.64) [1.54]	0.25 (0.32) [0.09]	1.62 (2.53) [1.71]	1.90 (2.88) [1.65]	0.91 (1.29) [0.52]	0.63 (0.86) [0.26]	λ_0	1.80 (2.33) [1.18]	1.92 (3.18) [1.46]
λ_{aggcg}	1.70 (2.54) [2.36]	1.99 (3.57) [1.58]	1.99 (3.68) [1.08]	1.38 (2.37) [1.64]	1.00 (1.47) [0.86]	1.37 (2.99) [1.28]	1.11 (2.59) [0.84]	λ_{CAPM}	13.79 (3.66) [1.89]	12.98 (3.38) [1.58]
λ_x		−2.44 (−2.47) [−1.05]	−3.55 (−3.69) [−1.03]							
$\lambda_{\triangle x}$			−2.71 (−4.03) [−1.13]							
λ_{vargcg}				0.84 (0.70) [0.48]	1.39 (1.05) [0.62]	−3.67 (−2.60) [−1.07]	−3.27 (−3.65) [−1.16]	λ_{HML}		−1.40 (−0.86) [−0.40]
λ_{skewcg}						3.87 (3.14) [1.27]	3.38 (3.89) [1.18]	λ_{SMB}		2.08 (1.69) [0.83]
λ_{kurtcg}						−2.07 (−0.70) [−0.28]	−2.86 (−0.94) [−0.28]			-
$\lambda_{\triangle\text{vargcg}}$					−2.38 (−2.04) [−1.20]		−1.68 (−1.83) [−0.55]			
$\lambda_{\triangle\text{skewcg}}$							2.19 (4.19) [1.44]			
$\lambda_{\triangle\text{kurtcg}}$							3.32 (0.96) [0.29]			
R^2	45.30%	53.37%	78.35%	65.99%	69.13%	85.77%	88.03%		76.01%	75.83%
MAE	0.45	0.38	0.32	0.35	0.35	0.30	0.23		0.30	0.29
Rank test	10.30	1.27	1.25	3.34	1.54	1.19	0.09		219.40	2.85
p-value	(0.00)	(0.21)	(0.22)	(0.00)	(0.08)	(0.28)	(1.00)		(0.00)	(0.00)
Alpha test	237.52	14.68	7.86	120.63	83.20	25.38	9.04		62.77	36.52
p-value	(0.00)	(0.88)	(0.99)	(0.00)	(0.00)	(0.23)	(0.96)		(0.00)	(0.003)
Avrge.|t-stat (α)|	1.38	0.59	0.46	1.03	1.03	0.61	0.48		0.50	0.61

The table shows the Fama–MacBeth regressions results when the asset menu comprises the Fama–French 25 developed portfolios. Consumption-based factors are computed from a balanced panel of 24 developed and emerging country consumption data. The data frequency is quarterly for the consumption-based model specifications, and the sample period runs from 1961Q1 to 2021Q1. The data frequency is monthly for the global CAPM and the global Fama–French three-factor model, with different starting months, 1961M1 and 1990M1, respectively, and the same end month, 2021M3. Our estimation sample is constrained by the shortest time series. The global CAPM factor is the MSCI World index excess return over the three-month U.S. Tbill rate. The Newey–West (Shanken) t-statistics are shown in parentheses (square brackets). R^2 is the r-squared from the cross-sectional regression of average excess returns on the betas. MAE is the mean absolute pricing error. The rank test statistic is computed as in Kleibergen and Zhan (2020) and the Alpha test statistic is computed from Eq. (21). Avrge.|t-stat(α)| is the average absolute Shanken-adjusted Newey–West t-statistics of the pricing errors.

The alternative asset menu displays higher R^2 values and lower mean absolute errors than the benchmark. Although Newey–West t-statistics confirm significance, Shanken t-statistics are lower than the benchmark. Our findings validate the heterogeneous-agents consumption-based model's implications for international portfolios.

The high cross-sectional R^2 from the Fama–French 25 developed portfolios may raise potential concerns. We recognize that, as Lewellen et al. (2010) pointed out, portfolios with a strong factor structure can naturally yield high R^2 values since factor loadings can align with expected returns, provided there is a weak correlation with common factors. While accepting this critique, our inclusion of these portfolios serves three-fold: they offer a conventional comparison framework, emphasize the robustness and significance of our model, and contrast with our primary 69-country MSCI indices to demonstrate our model's adaptability across diverse datasets. While valuing the contextual insights of the Fama–French portfolios, our primary focus remains on the broader MSCI indices, underscoring our model's empirical prowess.

4. Conclusion

Our article delves into international asset pricing using a framework of heterogeneous agents, aiming to bridge the gap between country-specific consumption shocks and global stock market investment strategies. Our primary goal was to understand the variability of expected returns across different national stock indices and assess whether these returns adequately account for the risks in the global consumption-based asset pricing model with heterogeneous agents.

Our research has revealed several key findings. Firstly, our model received robust support through GMM testing, providing significant parameter estimates that are both statistically and economically meaningful. By applying the heterogeneous-agents asset pricing model on an international scale, we gained more profound insights into the significance of global equities.

Our study has significant economic implications. The risk premiums we identified, particularly those related to cross-sectional consumption risk levels and changes, have substantial economic importance, especially considering the average risk premium across the MSCI indices.

We differentiate our work from previous studies by emphasizing the relevance of Epstein–Zin's recursive utility and the conditional non-normality of agents' idiosyncratic shocks. These aspects enhance our understanding by incorporating the pricing of changes in cross-sectional consumption risk and leveraging insights from cross-sectional higher-order moments, moving beyond the traditional focus on cross-sectional variance. Factors like cross-sectional skewness, previously overlooked, gain prominence, underscoring their importance in the model's explanatory power.

Furthermore, our detailed analysis of emerging versus developed markets reinforces that investments in emerging markets come with elevated risks. Our innovative approach, particularly the PCA-based proxy for cross-sectional consumption risk, provides a deeper understanding of observed premiums, making a novel contribution to academic discourse.

From a practical standpoint, investors and policymakers should consider the heterogeneity of consumption shocks when evaluating international investment opportunities and crafting investment strategies. As factors like cross-sectional skewness emerge as significant in our study, financial professionals should expand their risk assessment tools beyond traditional metrics. For countries seeking to attract international investments, understanding the drivers of risk premiums in their stock indices can be pivotal in shaping policies to stabilize consumption growth and mitigate extreme consumption shocks.

The limited identification of consumption-based factors in developed countries presents opportunities for further research. Future studies could delve into the intricacies of these markets, potentially examining disaggregated stocks or mutual funds. Additionally, exploring how global economic events, such as recessions, impact the parameters of the heterogeneous-agents consumption-based asset pricing model could be a fruitful area of investigation. Given our model's robust incorporation of Epstein–Zin preferences, studying how these preferences evolve over time and under different economic conditions holds significant academic value.

CRediT authorship contribution statement

Roméo Tédongap: Conceptualization, Supervision, Validation, Writing – review & editing. **Jules Tinang:** Methodology, Data curation, Empirical investigation, Writing – original draft.

Appendix A. Derivation of the cross-sectional moments of consumption growth

The derivation of the cross-sectional moments of the consumption growth distribution closely follows Constantinides and Ghosh (2017). This derivation uses the following identity:

$$e^{-\omega}\sum_{k=0}^{\infty} e^{kn}\omega^n/n! = e^{-\omega}\sum_{k=0}^{\infty}\left(e^k\omega\right)^n/n! = e^{-\omega}e^{e^k\omega}. \tag{A.1}$$

Differentiating one, two, three, and four times with respect to k and setting $k=0$, we obtain:

$$e^{-\omega}\sum_{k=0}^{\infty} n\omega^n/n! = \omega \quad\text{and}\quad e^{-\omega}\sum_{k=0}^{\infty} n^2\omega^n/n! = \omega^2+\omega,$$
$$e^{-\omega}\sum_{k=0}^{\infty} n^3\omega^n/n! = \omega^3+3\omega^2+\omega \quad\text{and}\quad e^{-\omega}\sum_{k=0}^{\infty} n^4\omega^n/n! = \omega^4+6\omega^3+7\omega^2+\omega.$$

The country relative consumption growth following Eq. (3) is given by:

$$\begin{aligned}\mathrm{cg}_{i,t} &= \ln\left(\frac{C_{i,t}/C_t}{C_{i,t-1}/C_{t-1}}\right)\\ &= \ln\left(h_{i,t}\right)-\ln\left(h_{i,t-1}\right)\\ &= \left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right).\end{aligned}$$

Thus, the conditional cross-sectional first to fourth cumulants obtain as follows:

$$\begin{aligned}
\text{meancg}_t &= \mu_1\left[\text{cg}_{i,t}|\omega_t\right] \\
&= \mathbb{E}\left[\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)|\omega_t\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)|j_{i,t},\tilde{j}_{i,t}\right]|\omega_t\right] \\
&= -\frac{\sigma^2}{2}\omega_t-\frac{\tilde{\sigma}^2}{2}\tilde{\omega}
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\text{vargcg}_t &= \mu_2\left[\text{cg}_{i,t}|\omega_t\right] \\
&= \mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^2|\omega_t\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^2|j_{i,t},\tilde{j}_{i,t}\right]|\omega_t\right] \\
&= \left(\sigma^2+\frac{\sigma^4}{4}\right)\omega_t+\left(\tilde{\sigma}^2+\frac{\tilde{\sigma}^4}{4}\right)\tilde{\omega}
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
\text{skewcg}_t &= \mu_3\left[\text{cg}_{i,t}|\omega_t\right] \\
&= \mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^3|\omega_t\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^3|j_{i,t},\tilde{j}_{i,t}\right]|\omega_t\right] \\
&= -\left(\frac{3}{2}\sigma^4+\frac{1}{8}\sigma^6\right)\omega_t-\left(\frac{3}{2}\tilde{\sigma}^4+\frac{1}{8}\tilde{\sigma}^6\right)\tilde{\omega}
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\text{kurtcg}_t &= \mu_4\left[\text{cg}_{i,t}|\omega_t\right]-3\left(\mu_2\left[\text{cg}_{i,t}|\omega_t\right]\right)^2 \\
&= \mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^4|\omega_t\right]-3\text{vargcg}_t^2 \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\left(\eta_{i,t}\sigma\sqrt{j_{i,t}}-\sigma^2\frac{j_{i,t}}{2}\right)+\left(\tilde{\eta}_{i,t}\tilde{\sigma}\sqrt{\tilde{j}_{i,t}}-\tilde{\sigma}^2\frac{\tilde{j}_{i,t}}{2}\right)\right)^4|j_{i,t},\tilde{j}_{i,t}\right]|\omega_t\right]-3\text{vargcg}_t^2 \\
&= \left(3\sigma^4+\frac{3}{2}\sigma^6+\frac{1}{16}\sigma^8\right)\omega_t+\left(3\tilde{\sigma}^4+\frac{3}{2}\tilde{\sigma}^6+\frac{1}{16}\tilde{\sigma}^8\right)\tilde{\omega}
\end{aligned} \tag{A.5}$$

We can now compute the unconditional first and second moments of cross-sectional consumption growth cumulants, i.e., their mean, variance and first-order autocovariance as functions of the model parameters.

The unconditional moments of the cross-sectional variance of consumption growth is given by:

$$\begin{aligned}
\mathbb{E}\left[\text{vargcg}_t\right] &= \left(\sigma^2+\frac{\sigma^4}{4}\right)\mathbb{E}\left[\omega_t\right]+\left(\tilde{\sigma}^2+\frac{\tilde{\sigma}^4}{4}\right)\tilde{\omega} \\
\text{var}\left[\text{vargcg}_t\right] &= \left(\sigma^2+\frac{\sigma^4}{4}\right)^2\text{var}\left[\omega_t\right] \\
\text{ac1}\left[\text{vargcg}_t\right] &= \left(\sigma^2+\frac{\sigma^4}{4}\right)^2\text{ac1}\left[\omega_t\right],
\end{aligned} \tag{A.6}$$

the unconditional moments of the cross-sectional skewness of consumption growth is given by:

$$\begin{aligned}
\mathbb{E}\left[\text{skewcg}_t\right] &= -\left(\frac{3}{2}\sigma^4+\frac{1}{8}\sigma^6\right)\mathbb{E}\left[\omega_t\right]-\left(\frac{3}{2}\tilde{\sigma}^4+\frac{1}{8}\tilde{\sigma}^6\right)\tilde{\omega} \\
\text{var}\left[\text{skewcg}_t\right] &= \left(\frac{3}{2}\sigma^4+\frac{1}{8}\sigma^6\right)^2\text{var}\left[\omega_t\right] \\
\text{ac1}\left[\text{skewcg}_t\right] &= \left(\frac{3}{2}\sigma^4+\frac{1}{8}\sigma^6\right)^2\text{ac1}\left[\omega_t\right],
\end{aligned} \tag{A.7}$$

the unconditional moments of the cross-sectional kurtosis of consumption growth is given by:

$$\begin{aligned}
\mathbb{E}\left[\mathrm{kurtcg}_t\right] &= \left(3\sigma^4 + \frac{3}{2}\sigma^6 + \frac{1}{16}\sigma^8\right)\mathbb{E}\left[\omega_t\right] + \left(3\tilde{\sigma}^4 + \frac{3}{2}\tilde{\sigma}^6 + \frac{1}{16}\tilde{\sigma}^8\right)\tilde{\omega} \\
\mathrm{var}\left[\mathrm{kurtcg}_t\right] &= \left(3\sigma^4 + \frac{3}{2}\sigma^6 + \frac{1}{16}\sigma^8\right)^2 \mathrm{var}\left[\omega_t\right] \\
\mathrm{ac1}\left[\mathrm{kurtcg}_t\right] &= \left(3\sigma^4 + \frac{3}{2}\sigma^6 + \frac{1}{16}\sigma^8\right)^2 \mathrm{ac1}\left[\omega_t\right],
\end{aligned} \tag{A.8}$$

where

$$\begin{aligned}
\mathbb{E}\left[\omega_t\right] &= \frac{\mathbb{E}\left[x_t\right]}{\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right)-1} \\
\mathrm{var}\left[\omega_t\right] &= \frac{\mathrm{var}\left[x_t\right]}{\left(\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right)-1\right)^2} \\
\mathrm{ac1}\left[\omega_t\right] &= \frac{\mathrm{ac1}\left[x_t\right]}{\left(\exp\left(\frac{\gamma(\gamma-1)\sigma^2}{2}\right)-1\right)^2},
\end{aligned} \tag{A.9}$$

and where $\mathbb{E}\left[x_t\right]$, $\mathrm{var}\left[x_t\right]$, and $\mathrm{ac1}\left[x_t\right]$ are given in Eq. (7).

Appendix B. Derivation of the wealth-to-consumption ratio

Constantinides and Ghosh (2017) show that the equilibrium consumption dynamics and preferences correspond to an autarchy equilibrium where each country representative's valuation of any security other than her individual consumption's claim, is the same. The common SDF for all agents is obtained by integrating out the country's consumption growth idiosyncratic shocks. To compute the common SDF, it is assumed that the wealth-to-consumption ratio is common to all agents and is an affine function of the state variable:

$$z_{i,c,t} = z_{c,t} = A_0 + A_1 x_t. \tag{B.1}$$

Using the Campbell and Shiller (1988) approximation, the log-return on the investor's wealth portfolio can be expressed as follows:

$$r_{i,c,t+1} = q_0 + q_1 z_{c,t+1} - z_{c,t} + \Delta c_{i,t+1}, \tag{B.2}$$

where the log-linearization coefficients q_0 and q_1 are endogenous. These coefficients are obtained simultaneously with the wealth-to-consumption ratio coefficients A_0 and A_1 in Eq. (B.1), by solving a non-linear equation system and a fixed point problem that determines the average wealth-to-consumption ratio $\bar{z}_c$. All things together lead to the following system:

$$\begin{cases}
\theta A_0 &= \theta \log\delta + \theta q_0 + \theta q_1 A_0 + \left(\exp\left(\frac{\gamma(\gamma-1)\tilde{\sigma}^2}{2}\right)-1\right)\tilde{\omega} \\
&\quad + (1-\gamma)\mu_c + \frac{(1-\gamma)^2}{2}\sigma_c^2 + \left(1+\theta q_1 A_1\right)\nu\xi\left(1+\frac{1}{2}\left(1+\theta q_1 A_1\right)\xi\right) \\
\theta A_1 &= \rho\left(1+\theta q_1 A_1\right)\left(1+\left(1+\theta q_1 A_1\right)\xi\right) \\
q_1 &= \frac{\exp\left(\bar{z}_c\right)}{1+\exp\left(\bar{z}_c\right)} \\
q_0 &= \log\left(1+\exp\left(\bar{z}_c\right)\right) - q_1 \bar{z}_c \\
\bar{z}_c &= A_0 + A_1\left(\frac{\nu\xi}{1-\rho}\right).
\end{cases} \tag{B.3}$$

Appendix C. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jempfin.2023.101459.

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