Downside Risks and the Cross-Section of Asset Returns*

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Abstract

In an intertemporal equilibrium asset pricing model featuring disappointment aversion and changing macroeconomic uncertainty, we show that besides the market return and market volatility, three disappointment-related factors are also priced: a disappointment factor, a market downside factor, and a volatility downside factor. We find that expected returns on different asset classes reflect premiums for bearing undesirable exposures to these factors. The signs of estimated risk premiums are consistent with the theoretical predictions. Our most general, five-factor model is very successful in jointly pricing stock, option, and currency portfolios, and provides considerable improvement over nested specifications previously discussed in the literature.

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1 Introduction

Downside risk is the risk of a portfolio in case of an adverse economic scenario. Upside uncertainty is the analogue if the scenario is favorable. The asymmetric treatment of downside risk versus upside uncertainty by investors has long been accepted among practitioners and academic researchers (see, e.g., Roy, 1952; and Markowitz, 1959), and has led to the development of new concepts in asset pricing and risk management, like the value-at-risk and the expected shortfall. Also, theories of rational behavior have been developed, where investors place greater weights on adverse market conditions in their utility functions. These include the lower-partial moment framework of Bawa and Lindenberg (1977), the loss aversion of Kahneman and Tversky (1979) in their prospect theory, and the disappointment aversion of Gul (1991), which has recently been generalized by Routledge and Zin (2010). These theories suggest priced downside risks in the capital market equilibrium.

We derive and test the cross-sectional predictions of a consumption-based asset pricing model where the representative investor has generalized disappointment aversion (GDA) preferences and macroeconomic uncertainty is time-varying. In this setting, an investor with expected utility preferences requires two premiums to invest in an asset. The first one is a compensation for covariation with the market return, $Cov\left(R_i^e, r_W\right)$, which is line with the prediction of the CAPM. The second premium is a compensation for covariation with changes in market volatility, $Cov\left(R_i^e, \Delta\sigma_W^2\right)$. It has been shown by previous empirical studies that volatility risk is priced in the cross-section (see, e.g., Ang et al., 2006; and Adrian and Rosenberg, 2008).

Our main theoretical contribution is to show that the GDA investor requires three additional premiums as compensation for exposures to disappointment-related risk factors. The first is a compensation for the covariance with the disappointment factor, $Cov\left(R_i^e, I\left(\mathcal{D}\right)\right)$, where $I(\mathcal{D})$ is the indicator function that takes the value 1 if disappointment sets in and 0 otherwise. The model suggests that disappointment $(\mathcal{D})$ may set in due to two reasons: a fall in market return or a rise in market volatility. The second premium is a compensation for the covariance of asset returns with the interaction of the market return and the disappointment indicator, $Cov\left(R_i^e, r_W I(\mathcal{D})\right)$. This factor represents movements of the market return in disappointing states and we refer to it as the market downside factor. The third premium is a compensation for the covariance with the interaction of changes in market volatility and the disappointment indicator, $Cov\left(R_i^e, \Delta\sigma_W^2 I(\mathcal{D})\right)$. This factor represents changes in market volatility in disappointing states, hence we refer to it as the volatility downside factor.

In the general case, our setting thus leads to a five-factor model. Although there are five factors in the model, only two time series, the market return r_W and changes in market volatility $\Delta\sigma_W^2$ are needed to construct these factors. The disappointment factor is constructed as a function of these two series, and the two downside factors are just interactions with the disappointment indicator. We also show that if the representative investor has infinite elasticity of intertemporal substitution, then market volatility has no role in the model, and the disappointing event reduces to a fall of the market return below a reference threshold. This special case corresponds to a three-factor model with the market, the disappointment, and the market downside factors. The three-factor model is especially important when we relate our results to previous literature.

The cross-sectional implications of downside risk has already been studied in previous literature, most notably by Ang, Chen and Xing (2006) and Lettau et al. (2014). We show that our three-factor model nests the models from both of these studies, with different restrictions on the premium corresponding to the disappointment factor. We explicitly derive these restrictions and confront them with the data. Our results suggest that the restrictions imposed by the downside risk models of Ang, Chen and Xing (2006) and Lettau et al. (2014) are not supported empirically. Therefore, our three-factor model provides considerable improvement in explaining the cross-section of different asset returns, even though all three models use exactly the same information. Additionally, the five-factor model emphasizes the role of volatility in understanding downside risks. To our knowledge, little or no attention has been paid to volatility downside risk in the literature. We argue that volatility downside risk is also an important factor in explaining the cross-section of asset returns, as the five-factor model provides further improvement compared to the three-factor model.

We use the generalized methods of moments (GMM) to explore the cross-sectional predictions of our three- and five-factor models. Our test assets are stock portfolios sorted on size and momentum, index option portfolios sorted on type, maturity, and moneyness, and currency portfolios sorted on their respective interest rates. These portfolios exhibit large heterogeneity in their average returns and thus are ideal for cross-sectional asset pricing tests. The main empirical results of the paper relate to the pricing of the disappointment-related risk factors.

All the disappointment-related factors have significant risk premiums and the signs on all the risk prices are in line with the theoretical predictions. In terms of pricing errors, when tested on all asset classes jointly, our three-factor model with a root-mean-squared pricing error (RMSPE) of 19 basis points (bps) per month provides a significant improvement over the CAPM with a RMSPE of 49bps. The corresponding pricing errors of the downside risk models of Ang, Chen and Xing (2006) (27bps) and Lettau et al. (2014) (31bps) are considerably higher than that of the three-factor model. Our five-factor model, with a RMSPE of 13bps, largely outperforms a two-factor model with market return and changes in market volatility with a RMSPE of 27bps. Moreover, the five-factor GDA model also outperforms the four-factor model of Carhart (1997) on all asset classes except for stocks where the four-factor model does only marginally better. However, the GDA model has the benefit of being motivated by dynamic consumption-based equilibrium asset pricing and behavioral decision theories, rather than being motivated by asset pricing anomalies themselves. These findings suggest the importance of disappointment-related risk in the cross-section of asset returns. Our results are robust to different data samples, to alternative measures of market volatility, and to alternative specifications of the disappointing event.

This paper contributes to the developing literature that attempts to provide empirical support for the recent generalization by Routledge and Zin (2010) of the axiomatic disappointment aversion framework of Gul (1991). In the literature, GDA preferences have appeared in consumption-based equilibrium models mainly with the goal of attempting to explain the time series behavior of the aggregate stock market, and rarely in cross-sectional asset pricing studies.[1] Parallel to our study, Delikouras (2014a) also uses GMM to test a consumption-based model with GDA preferences using annual consumption data and six Fama-French portfolios sorted on size and book-to-market. He motivates the use of annual data by potential inaccuracies and measurement errors in consumption data that are inherent to monthly and quarterly samples. In contrast, we substitute out consumption in a way similar to Campbell (1993), and rely on the market return. We also derive cross-sectional implications in the form of a factor model, and find it useful to explicitly characterize factors that are valued by disappointment averse investors. We can then avoid potential measurement problems in consumption data (of the types advocated for example by Wilcox, 1992),

[1]For instance, Bonomo et al. (2011) show that persistent shocks to consumption volatility are sufficient when coupled with GDA preferences to produce moments of asset prices and predictability patterns that are in line with the data. Schreindorfer (2014) aims at explaining properties of index option prices, equity returns, variance, and the risk-free rate using the GDA model and a heteroscedastic random walk for consumption with the multifractal process of Calvet and Fisher (2007). Delikouras (2014b) uses the GDA model to explain the credit spread puzzle.

or delayed responses of consumption to financial market news (as discussed for example by Parker and Julliard, 2005), and test the model at the monthly frequency using market return data.

To the contrary of Delikouras (2014a) who assumes constant volatility of aggregate consumption, our setting also implicitly allows for time-varying macro-economic uncertainty, a feature that is supported empirically (see for example Bansal et al., 2005). Relying on market return rather than consumption also makes our results directly comparable to recent cross-sectional studies on downside risks such as Ang, Chen and Xing (2006) and Lettau et al. (2014). Finally, because of the kink in the GDA utility function, estimation and inference problems may occur due to strong nonlinearities and discontinuity inherent to the GDA model, and due to endogeneity of the disappointment-related factors. To avoid these problems, we do not estimate the original preference parameters directly, but instead, we specify the disappointing event exogenously and estimate the cross-sectional factor risk premiums which are functions of these parameters.

The rest of the paper is organized as follows. In Section 2, we present and develop the theoretical setup from which we derive the implied cross-sectional model. In Section 3, we provide a calibration analysis of the model benchmarked to Bonomo et al. (2011). Section 4 contains the empirical analysis while Section 5 concludes. An Online Appendix available from authors' web pages contains additional material and proofs.

2 Theoretical motivation

2.1 Assumptions on investors' preferences

Following Epstein and Zin (1989) and Weil (1989), we consider an economy where the representative investor has recursive utility:

$$V_t = \begin{cases} \left[(1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t \left(V_{t+1} \right) \right]^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} & \text{if } \psi > 0 \text{ and } \psi \neq 1 \\ C_t^{1-\delta} \left[\mathcal{R}_t \left(V_{t+1} \right) \right]^{\delta} & \text{if } \psi = 1 \end{cases}, \quad (2)$$

where $0 < \delta < 1$ is the parameter of time preference and $\psi > 0$ is the elasticity of intertemporal substitution. The current period lifetime utility, V_t, is a function of the current consumption, C_t, and the certainty equivalent of next period's lifetime utility, $\mathcal{R}_t \left(V_{t+1} \right)$. Routledge and Zin (2010) introduce generalized disappointment aversion (GDA) into this framework by assuming that the certainty equivalent $\mathcal{R}$ is implicitly defined by

$$U\left(\mathcal{R} \right) = E\left[U\left(V \right) \right] - \ell E\left[\left(U\left(\kappa \mathcal{R} \right) - U\left(V \right) \right) I\left(V < \kappa \mathcal{R} \right) \right] , \quad (3)$$

with

$$U\left(X \right) = \begin{cases} \frac{X^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma > 0 \text{ and } \gamma \neq 1 \\ \ln X & \text{if } \gamma = 1 \end{cases}, \quad (4)$$

where the parameter $\gamma \geq 0$ is the coefficient of relative risk aversion. When ℓ is equal to zero, $\mathcal{R}$ reduces to expected utility (EU) preferences and V_t represents the Epstein and Zin (1989) recursive utility. GDA preferences are a two-parameter extension of the EU framework. When $\ell > 0$, outcomes lower than $\kappa \mathcal{R}$ receive an extra weight, decreasing the certainty equivalent. The larger weight given to these bad outcomes implies an aversion to losses. The parameter $\ell \geq 0$ is interpreted as the degree of disappointment aversion, while the parameter $0 < \kappa \leq 1$ is the percentage of the certainty equivalent such that outcomes below it are considered disappointing. The special case $\kappa = 1$ corresponds the original disappointment aversion preferences of Gul (1991).

The investor maximizes the lifetime utility V_t, subject to the budget constraint

$$W_{t+1} = (W_t - C_t) R_{W,t+1} \ , \tag{5}$$

where W_t is the wealth in period t and $R_{W,t+1}$ is the simple gross return on wealth which we refer to as the market return. Following Hansen et al. (2007) and Routledge and Zin (2010), consumption and portfolio choice induces an equilibrium restriction on the gross return on any asset, given by the unconditional Euler equation:

$$E\left[M_{t-1,t}\left(1+\ell I\left(\mathcal{D}_t\right)\right) R_{i,t}^e\right] = 0 \ , \tag{6}$$

where $R_{i,t}^e = R_{i,t} - R_{f,t}$ is the excess return of asset i, $R_{f,t}$ is the risk-free simple gross return, $I(\cdot)$ denotes the indicator function taking the value 1 if the condition is met and 0 otherwise, and

$$M_{t-1,t} = \delta \left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\psi}} \left(\frac{V_t}{\mathcal{R}_{t-1}\left(V_t\right)}\right)^{\frac{1}{\psi}-\gamma} \text{ and } \mathcal{D}_t = \left\{V_t < \kappa \mathcal{R}_{t-1}\left(V_t\right)\right\} \ , \tag{7}$$

where $M_{t-1,t}$ is the stochastic discount factor between periods $t-1$ and t, and $\mathcal{D}_t$ denotes the disappointing event. The logarithm of $M_{t-1,t}$ and $\mathcal{D}_t$ may be written as

$$\ln M_{t-1,t} = \ln \delta - \gamma \Delta c_t - \left(\gamma - \frac{1}{\psi}\right) \Delta z_{V,t} \text{ and } \mathcal{D}_t = \left\{\Delta c_t + \Delta z_{V,t} < \ln \kappa\right\}, \tag{8}$$

where $\Delta c_t \equiv \ln\left(\frac{C_t}{C_t - 1}\right) = \ln C_t - \ln C_{t-1}$ and $\Delta z_{V,t} \equiv \ln\left(\frac{V_t}{C_t}\right) - \ln\left(\frac{\mathcal{R}_{t-1}(V_t)}{C_{t-1}}\right)$ represent the change in the log consumption level (or consumption growth) and the change in the log welfare valuation ratio (or welfare valuation ratio growth), respectively.

In some special cases such as $\ell = 0$ and $\gamma = 1/\psi$, the moment condition (6) is readily testable by GMM using actual data on aggregate consumption growth and asset returns. Earlier results for this test of the standard model are presented in Hansen and Singleton (1982, 1983). In the general case, the moment condition (6) is not directly testable by GMM since the continuation value is not observable from the data. Following the long-run risks asset pricing literature pioneered by Bansal and Yaron (2004), an assumed endowment dynamics can be exploited, together with the utility recursion (2) and the certainty equivalent definition (3), to express welfare valuation ratios in terms of economic state variables such as aggregate volatility, which may be estimated from the data.

2.2 Cross-sectional implications of GDA preferences

In order to obtain the cross-sectional implications that form the basis of our empirical investigation, we make two substitutions. First we substitute out consumption growth following Epstein and Zin (1989), Hansen et al. (2007) and Routledge and Zin (2010) who show that in equilibrium, the market return, is related to consumption growth and the welfare valuation ratio growth through

$$r_{W,t} = -\ln\delta + \Delta c_t + \left(1 - \frac{1}{\psi}\right)\Delta z_{V,t} \ . \tag{9}$$

Second, assuming that aggregate consumption growth is heteroscedastic and unpredictable as in Bollerslev et al. (2009), Tauchen (2011) and Bonomo et al. (2011), and consistent with the empirical evidence presented in Beeler and Campbell (2012) among many others, we can solve for the welfare valuation ratio endogenously and express the welfare valuation ratio growth, $\Delta z_{V,t}$, in terms of changes in the volatility of the market return, which we refer to as market volatility.

Making these substitutions, and after some algebraic manipulation, the Euler equation in (6) may be written as a cross-sectional linear factor model[2]

$$E\left[R_{i,t}^{e}\right] = p_W \sigma_{iW} + p_X \sigma_{iX} + p_{\mathcal{D}} \sigma_{i\mathcal{D}} + p_{W\mathcal{D}} \sigma_{iW\mathcal{D}} + p_{X\mathcal{D}} \sigma_{iX\mathcal{D}} \ , \tag{10}$$

where

$$\begin{aligned} \sigma_{iW} &\equiv Cov\left(R_{i,t}^{e}, r_{W,t}\right) \\ \sigma_{iX} &\equiv Cov\left(R_{i,t}^{e}, \Delta\sigma_{W,t}^{2}\right) \\ \sigma_{i\mathcal{D}} &\equiv Cov\left(R_{i,t}^{e}, I\left(\mathcal{D}_t\right)\right) \\ \sigma_{iW\mathcal{D}} &\equiv Cov\left(R_{i,t}^{e}, r_{W,t} I\left(\mathcal{D}_t\right)\right) \\ \sigma_{iX\mathcal{D}} &\equiv Cov\left(R_{i,t}^{e}, \Delta\sigma_{W,t}^{2} I\left(\mathcal{D}_t\right)\right) \ . \end{aligned} \tag{11}$$

The covariance risk prices $p_W > 0$, $p_X < 0$, $p_{\mathcal{D}} < 0$, $p_{W\mathcal{D}} > 0$, and $p_{X\mathcal{D}} < 0$ are functions of the preference parameters δ, γ, ψ, ℓ and κ, as well as functions of the parameters governing the endowment dynamics. Let us have a detailed look at the signs of the covariance risk prices. First, as $p_W > 0$, investors require a premium for a security that has positive covariance with the market return. This is in line with the CAPM theory of Sharpe (1964) and Lintner (1965). The second risk price is $p_X < 0$, thus, investors are willing to pay a premium for a security that has positive covariance with $\Delta\sigma_{W,t}^{2}$. This is consistent with the existing empirical literature (see, e.g., Ang, Hodrick, Xing and Zhang, 2006; Adrian and Rosenberg,

[2]The details of the derivation are outlined in the Online Appendix.

2008). The third risk price is $p_{\mathcal{D}} < 0$, showing that disappointment-averse investors are willing to pay a premium for a security that has positive covariance with the $I(\mathcal{D}_t)$ factor. Such an asset tends to move upward when the disappointing event occurs. Letting π denote the disappointment probability, note that $Cov\left(R^e_{i,t}, I(\mathcal{D}_t)\right) = \pi\left(E\left[R^e_{i,t} \mid \mathcal{D}_t\right] - E\left[R^e_{i,t}\right]\right)$. That is, assets with $\sigma_{i\mathcal{D}} < 0$ are undesirable because they have lower expected payoffs than usual when disappointment sets in. The fourth factor in (10) is $r_{W,t} I(\mathcal{D}_t)$. It represents changes in market return when the economy is in the disappointing state. We refer to it as the market downside factor throughout the paper. The associated risk price is positive. Investors require a premium for a security that has positive covariance with $r_{W,t} I(\mathcal{D}_t)$, since such an asset tends to have a negative return when there is a low market return in a disappointing state. The fifth and final factor is $\Delta\sigma^2_{W,t} I(\mathcal{D}_t)$, representing changes in market volatility when the economy is in the disappointing state. We subsequently refer to it as the volatility downside factor. The associated risk price is negative. Investors are willing to pay a premium for a security that has positive covariance with the volatility downside factor. Such an asset tends to have positive returns when market volatility increases in a disappointing state.

We also show that the disappointing event may be written as

$$\mathcal{D}_t = \left\{ r_{W,t} - a\left(\sigma_W / \sigma_X\right) \Delta\sigma^2_{W,t} < b \right\}, \tag{12}$$

where $\sigma_W = Std\left[r_{W,t}\right]$ and $\sigma_X = Std\left[\Delta\sigma^2_{W,t}\right]$ are the respective standard deviations of market return and changes in market volatility. Similar to the covariance risk prices, the coefficients $a > 0$ and b are also functions of the preference parameters and the parameters governing the endowment dynamics. The term $\left(\sigma_W / \sigma_X\right) \Delta\sigma^2_{W,t}$ may be viewed as the return on a volatility index with same standard deviation as the market return. In this case, disappointment occurs if the return of holding a long position in the market index combined with a times a short position in the volatility index falls below a constant threshold b. In particular, if the coefficient a is equal to one, the long position in the market index is exactly balanced by the short position in the volatility index in determining disappointment. As a decreases from one towards zero, disappointment is more likely to occur due to a fall in the market index rather than an increase in the volatility index. The opposite happens as a increases from one towards infinity.

Equation (10) corresponds to a linear multifactor representation of expected excess returns in the cross-section. The unrestricted model is a five-factor model which we refer to as GDA5 throughout the rest of the paper. It states that in addition to the market return and changes in market volatility, three additional factors command a risk premium: the disappointment factor $I(\mathcal{D}_t)$, the market downside factor $r_{W,t}I(\mathcal{D}_t)$, and the volatility downside factor $\Delta\sigma^2_{W,t}I(\mathcal{D}_t)$.

There are two special cases worth considering. First, if the elasticity of intertemporal substitution is infinite ($\psi = \infty$), then changes in market volatility disappear from the model, i.e., $a = 0$ and $p_X = p_{X\mathcal{D}} = 0$. In this case the cross-sectional model (10) reduces to a three-factor model with the market, the disappointment, and the market downside factors, and the disappointing event has the simple form $\mathcal{D}_t = \{r_{W,t} < \ln(\kappa/\delta)\}$. We refer to this restricted model as GDA3 throughout the paper. Second, if the representative investor is not disappointment averse ($\ell = 0$), then $p_{\mathcal{D}} = p_{W\mathcal{D}} = p_{X\mathcal{D}} = 0$, i.e., all disappointment-related related factors disappear from the model. In this case (10) reduces to a two-factor model where only market risk and volatility risk are priced.

Equation (10) may ultimately be expressed as a multivariate beta pricing model:

$$E\left[R_{i,t}^{e}\right] = \lambda_F^{\top}\beta_{iF} \tag{13}$$

where β_{iF} is the vector containing the multivariate regression coefficients of asset excess returns onto the factors, and λ_F is the vector of factor risk premiums, respectively given by

$$\beta_{iF} = \Sigma_F^{-1}\sigma_{iF} \quad \text{and} \quad \lambda_F = \Sigma_F p_F. \tag{14}$$

The vector σ_{iF} contains the covariances of the asset excess returns with the priced factors, the vector p_F contains the associated factor risk prices, and Σ_F is the factor covariance matrix. If the covariance between the market return and changes in market volatility is negative, $Cov\left(r_{W,t}, \Delta\sigma_{W,t}^2\right) < 0$, consistent with the leverage effect as postulated by Black (1976) and documented by Christie (1982) and others, then the signs of the elements of λ_F are the same as of the corresponding elements of p_F. The model in (13) is the basis of our empirical analysis.

3 Calibration assessment

In this section, we analyze the factor risk premia, λ_f with $f \in \{W, X, \mathcal{D}, W\mathcal{D}, X\mathcal{D}\}$, generated by a GDA endowment economy, reasonably calibrated to match the risk-free rate and the aggregate stock market behavior. In setting up the calibration, we closely follow Bonomo et al. (2011). They study an asset pricing model with generalized disappointment aversion preferences and long-run volatility risk and show that it produces first and second moments of price-dividend ratios and asset returns as well as return predictability patterns in line with the data. Using the same endowment dynamics, we focus on the cross-sectional implications by studying the model-implied factor risk premia.

We assume that the consumption growth and the equity dividend growth are conditionally normal, unpredictable and their conditional variances fluctuate according to a two-state Markov chain:

$$\begin{aligned} \Delta c_t &= \mu + \sqrt{\omega_c\left(s_{t-1}\right)}\varepsilon_{c,t} \\ \Delta d_t &= \mu + \nu_d\sqrt{\omega_c\left(s_{t-1}\right)}\varepsilon_{d,t}\ , \end{aligned} \tag{15}$$

where Δc_t is the aggregate consumption growth, Δd_t is the equity dividend growth, s_{t-1} indicates the state of the world, and $\varepsilon_{c,t}$ and $\varepsilon_{d,t}$ follow a bivariate IID standard normal process with mean zero and correlation ρ. The two states of the economy naturally correspond to a low (L) and a high (H) volatility state.

The endowment dynamics is calibrated at the monthly frequency to match the sample mean, volatility, and first-order autocorrelation of the real annual US consumption growth and stock market dividend growth from 1930 to 2012. These moments remain stable if the data are updated until more recently. Panel A of Table 1 shows the parameters of the calibrated endowment process. The state transition probabilities are $p_{LL} = 0.9989$ and $p_{HH} = 0.9961$, and the corresponding long-run probabilities are 78.9% and 21.1% for the low and high volatility states, respectively. We set the preference parameters similar to the benchmark calibration of Bonomo et al. (2011). The values are presented in Panel C of Table 1. For the GDA3 model, we simply set $\psi = \infty$, everything else being equal.

The first set of results in Panel B shows that our calibration matches well the first and second moments of consumption and dividend growth in the data. The model-implied annualized (time-averaged) mean, volatility and first-order autocorrelation of consumption growth are respectively 1.80%, 2.07%, and 0.25, and are consistent with the observed annual values of 1.84%, 2.20%, and 0.48, respectively. The mean, volatility and first-order autocorrelation of dividend growth are respectively 1.80%, 13.29% and 0.25, and the observed annual values are 1.05%, 13.02% and 0.11, respectively.

Given these endowment dynamics, we solve for welfare valuation ratios in closed form, which we combine with consumption growth to derive the endogenous market return and market variance processes. We refer the reader to Bonomo et al. (2011) for formal derivations. The second set of results in Panel B shows that the model generates moments of asset prices that are consistent with empirical evidence. The level of risk free rate, 0.46% for GDA3 and 0.76% for GDA5, is close to the actual value of 0.57%. The equity premium, 8.06% for GDA3 and 6.61% for GDA5, are slightly larger than the actual value of 5.50%, but remain comparable to other sample values estimated in the literature, for example 7.25% in Bonomo et al. (2011). The equity volatility generated by the model, 17.65% for GDA3 and 16.84% for GDA5, is also comparable to the actual value of 20.25%.

The main purpose of this calibration is to study the model's implications for the disappointing event and the GDA factor risk premia. The definition of the disappointing event in (12) shows that it relies on two parameters: a governs the relative importance of the market return and changes in volatility, while b is the disappointment threshold. Panel D of Table 1 shows that for the GDA3 model $a = 0$ and $b = 0$. That is, the disappointing event is simply when the market return is negative. Note that model-implied unconditional probability of disappointment is 17.43%. For the GDA5 model $a = 1.38$, that is, a rise in market volatility is slightly more likely to trigger disappointment than a fall in the market return. The corresponding threshold is negative, $b = -0.1\%$. The unconditional disappointment probability for the GDA5 model is 16.06%.

The model-implied monthly factor risk premia are also reported in Panel D. The market risk premium is equal to $\lambda_W = 0.0065$ in the GDA3 model, and $\lambda_W = 0.0042$ for GDA5, while the volatility risk premium is $\lambda_X = 0$ for GDA3, and $\lambda_X = -1.38 \times 10^{-6}$ in the GDA5 model. Likewise, the market downside risk premium is equal to $\lambda_{W\mathcal{D}} = 0.0038$ in the GDA3 model, and $\lambda_{W\mathcal{D}} = 0.0023$ for GDA5, while the volatility downside risk premium is $\lambda_{X\mathcal{D}} = 0$ for GDA3, and $\lambda_{X\mathcal{D}} = -1.16 \times 10^{-6}$ in the GDA5 model. Finally the downside risk premium is $\lambda_{\mathcal{D}} = -0.3494$ in the GDA3 model, and $\lambda_{\mathcal{D}} = -0.3010$ for GDA5. While Table 1 presents the model's implications for our benchmark calibration, in the Online Appendix we show how sensitive these results are when we vary some of the preference parameters.

In the next section, we proceed with an empirical assessment of the model's cross-sectional implications and estimate the factor risk premia using a large class of asset returns. Let us keep in mind that the true market return (the return of a claim on aggregate consumption) and market volatility are not observable and have to be proxied by stock market and volatility indexes. This, in particular, may alter the magnitudes of the estimated risk premia.

4 Empirical assessment

Our empirical tests are based on the linear beta representation (13). We use the aggregate stock index return and aggregate stock index volatility to proxy for market return and market volatility, and we also fix the parameters a and b defining the disappointing event so that all factors are known and not to be estimated. We do not estimate the underlying preference parameters, but instead we estimate the risk premiums which are functions of both the underlying parameters and parameters governing the endowment dynamics in the economy.

Estimating the underlying preference parameters in the general setting has several challenges. First, since the indicator function appears in the GMM moment conditions, and a and b are functions of the preference parameters to estimate, the GMM objective function to minimize is not differentiable. This makes parameter inference unreliable in the GMM setting. Second, the market return is not observable and we use the return on the aggregate stock index as a proxy. This proxy is much more volatile, since it is a claim on the aggregate stock market dividend, whose growth rate is at least five times more volatile than the aggregate consumption growth rate. So, even if feasible, an attempt to estimate the underlying preference parameters with this proxy would induce large estimation bias. Third, the preference parameter estimates would be dependent on the dynamics assumed for the aggregate endowment in the economy. On the other hand, by estimating the reduced-form risk premia in the linear beta representation (13) we are able to circumvent these challenges. This approach also makes our results comparable to existing cross-sectional tests of models with downside risks (e.g., Ang, Chen and Xing, 2006; and Lettau et al., 2014).

In order to estimate the reduced-form risk premia, we fix the parameters a and b defining the disappointing event so that all factors are known. For the main part of the analysis we set $a = 0$ and $b = -0.03$. That is, the disappointing event is defined as $\mathcal{D}_t = \{r_{W,t} < -0.03\}$. Those months are considered disappointing when the market falls by more than three percent. Setting $a = 0$ is in line with the theoretical implications in case of the GDA3 model. However, in the more general (GDA5) setting theory implies that both market return and changes in market volatility should enter the definition of the disappointing event ($a > 0$). Nevertheless, we choose to use the above definition also for the GDA5 model for the main analysis. There are several reasons for using this simpler definition.

First, decreasing market return and increasing market volatility tend to coincide empirically, which is also known as the leverage effect. That is, even if increasing market volatility is not explicitly included in the definition, disappointment tend to be accompanied with increasing volatility. Table 2 shows that the unconditional correlation between $r_{W,t}$ and $\Delta\sigma^2_{W,t}$ in our sample is -0.25. Moreover, the conditional correlation (conditional on being in the disappointing state) is even stronger, -0.46. Extreme volatility increases also happen in disappointing months when disappointment is defined as $r_{W,t} < -0.03$. Nine of the largest ten $\Delta\sigma^2_{W,t}$ values in our sample is realized in disappointing months (and 16 of the largest 20).

Second, this simpler definition makes the results more comparable to previous literature, as all studies analyzing downside risk define the downstate in terms of the market return only. Ang, Chen and Xing (2006) define the disappointing event as the market return falling below its sample average, while Lettau et al. (2014) define it to be months when the market return is more than one standard deviation below its sample average. Considering the mean and standard deviation of the market return from Table 2, we see that the definition used by Lettau et al. (2014) is very close to our definition.

Nevertheless, in the robustness section we also consider a definition for the disappointing event that explicitly takes into account changes in market volatility. Our calibration results suggest that the model-implied value of the parameter a is slightly above one for the GDA5. Therefore, we consider $a = 1$ in our robustness checks. The general conclusions remain unchanged when that definition is used.

We set the disappointment threshold to -0.03 for the empirical analysis to match the disappointment probability implied by the theoretical model. As shown in Table 1, our calibration suggests that the model-implied disappointment probability is around 16% for both the GDA3 and the GDA5 models. Table 2, on the other hand, shows that if disappointing months are defined to be the ones when the market falls by more than three percent, the disappointment probability is also close to 16%. Note that we also consider the robustness of our results to setting the disappointment threshold to values closer to zero.

4.1 Data and estimation method

We test our model using monthly returns on various asset classes including stocks, index options, and currencies. Monthly returns on US stock portfolios are obtained from Kenneth French's data library.[3] The sample covers the period from July 1964 to December 2013. Index option returns are from Constantinides et al. (2013).[4] They construct a panel of S&P 500 index option portfolios. The data set contains leverage-adjusted (that is, with a targeted market beta of one) monthly returns of 54 option portfolios split across types (call and put), each with a targeted time to maturity (30, 60, or 90 days), and a targeted moneyness level (9 different levels). The option data is available from April 1986 to January 2012. Finally, currency returns are from Lettau et al. (2014), who use monthly data on 53 currencies to create six portfolios by sorting them in ascending order of their respective interest rates.[5] The sample period is from January 1974 to March 2010. In our main analysis, we estimate different asset pricing models on four sets of portfolios:

1. 25 (5×5) US stock portfolios sorted on size and momentum,

2. 24 (2×3×4) index option portfolios sorted on type, maturity, and moneyness,

3. 10 US stock portfolios sorted on size and 10 short maturity (30 days) index option portfolios,

[3]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[4]Return data on the option portfolios is obtained from the website of Alexi Savov at http://pages.stern.nyu.edu/asavov/alexisavov/Alexi_Savov.html

[5]Return data on the currency portfolios is obtained from Michael Weber's website at http://faculty.chicagobooth.edu/michael.weber

4. 6 (2×3) stock portfolios sorted on size and momentum, 6 short maturity index option portfolios, and 6 currency portfolios.

Table 3 provides a detailed description of these sets. We choose to represent US stocks with the portfolios sorted on size and momentum because this set provides the largest variation in average returns from all the possible 5×5 sorts available in Kenneth French's data library. In the robustness section, however, we consider several other US stock sorts, including the 25 portfolios sorted on size and book-to-market, and show that the results are very similar. When constructing the other sets of portfolios, we had two goals in mind. First, the total number of portfolios should be similar across the four sets. Second, if multiple asset classes are included at the same time, each asset class should be represented with the same number of portfolios. We show estimation results for several other sets of portfolios in the Online Appendix, which are omitted from the main text for the sake of brevity.[6]

To facilitate the empirical analysis, we also need data on the risk-free rate and the GDA factors. The risk-free rate is the one-month US Treasury bill rate from Ibbotson Associates, while the market return is the value-weighted average return on all CRSP stocks. Both series were obtained from Kenneth French's data library. Empirical tests of the GDA5 model require a measure of market volatility. Several approaches have been used for measuring market volatility in cross-sectional asset pricing studies. Ang et al. (2006) use the VIX, Adrian and Rosenberg (2008) estimate volatility from a GARCH-type model, while Bandi et al. (2006) use realized volatility computed from high-frequency index returns. In our main analysis, we measure monthly volatility as the realized volatility of the daily market

[6] In the Online Appendix we show that the results do not change much if we do not adjust the number of portfolios when including multiple asset classes, but use all the available portfolios instead. We also show that the results on currencies are robust to using an alternative source for the return on currency portfolios provided by Lustig et al. (2011). Furthermore, we consider additional asset classes, including corporate bonds, sovereign bonds, and commodity futures.

returns during the month. There are several advantages of this measure compared to other alternatives. First, it is very easy to construct as it requires only daily market return data. Second, it allows us to use a longer sample period. Note that we also use alternative measures, including the VIX, realized volatility calculated from intra-daily market returns, and a parametric model implied volatility in our robustness checks.

The portfolio betas and the factor premia from (13) are estimated jointly using the generalized method of moments (GMM) with moment conditions as in Cochrane (2000):

$$\begin{cases} E\left[R^e_{it} - \alpha_i - F_t\beta_{iF}\right] = 0 & i = 1, ..., N \\ E\left[\left[R^e_{it} - \alpha_i - F_t\beta_{iF}\right] f_{jt}\right] = 0 & i = 1, ..., N \ , \quad j = 1, ..., K \\ E\left[R^e_{it} - \beta_{iF}\lambda_F\right] = 0 & i = 1, ..., N \end{cases} , \tag{16}$$

where R^e_{it} is the excess return on portfolio i, f_{jt} denotes factor j, F_t is the vector of all factors in the model, β_{iF} is the vector of factor betas for portfolio i, and λ_F is the vector of factor risk premia.[7] The first two sets of moment conditions from (16) directly correspond to the formula for estimating the β-s in (14), while the last set of moment conditions represents the model in (13). The identity weighting matrix is used in the GMM estimation, so the β and λ estimates are numerically equivalent to the ones that would be obtained using the Fama-MacBeth (1973) estimation. The advantage of using the GMM is that the standard errors account for the "generated regressors" problem, i.e., the fact that the β-s are also

[7] In an earlier version of the paper that has been circulated under the title "Volatility Downside Risk", we follow the same empirical approach as Ang, Chen and Xing (2006) to test the multivariate beta representation in (13) using individual stocks (instead of portfolios). We use all common stocks traded on the NYSE, AMEX and NASDAQ markets covering the period from July 1963 to December 2010. The empirical methodology is based on cross-sectional regressions of Fama and MacBeth (1973) to estimate the factor risk premia, where monthly factor exposures are measured from realized daily returns. The results from this analysis are comparable to those in the current version of the paper and are available upon request.

estimated.[8]

4.2 Results

The main empirical results of the paper are presented in Table 4. Risk premia estimates for four different models are presented. We also report the root mean squared pricing error (RMSPE) expressed in basis points (bps) per month, to facilitate model comparison. Furthermore, we test if the pricing errors are jointly zero using the χ^2-test during the GMM estimation (the test statistic is denoted as J-stat, and the p-value is in parenthesis).

In Panel A the test assets are the 25 size-momentum portfolios. The first model is the CAPM. The price of market risk, λ_W, is positive and significant. The RMSPE is a quite sizable 39 bps per month, and the χ^2-test rejects that these pricing errors are jointly zero. The next column ads volatility risk to the model (we refer to this model as VOL). Volatility risk is also significantly priced in the cross-section of stock returns and carries a negative premium. The RMSPE barely decreases and the model is also rejected by the χ^2-test.

The next model is the GDA3. All three factors are statistically significant and the signs are as expected: the market (λ_W) and market downside ($\lambda_{W\mathcal{D}}$) factors have a positive premium, while the disappointment ($\lambda_{\mathcal{D}}$) factor has a negative premium. In terms of model fit, the GDA3 should be compared to the CAPM. First, they use the same information: although the GDA3 has three factors, the only information needed to construct those factors is the return on the market. Second, the CAPM arises as a restricted version of the GDA3 if the representative agent is not disappointment averse. The GDA3 provides a considerable improvement, as the RMSPE is 22 bps, almost half of that of the CAPM. Moreover, the

[8]It is showed by Cochrane (2000), for example, that the correction due to Shanken (1992) can be recovered as a special case of the GMM standard errors. During the GMM estimation we use the Newey-West estimator with 3 lags for the covariance matrix of the moment conditions.

χ^2-test cannot reject the null that the GDA3 pricing errors are jointly zero at the 5% level.

The fourth model is the GDA5. All risk premia are statistically significant at least at the 5% level, and the signs are as expected. The premia on both the volatility (λ_X) and the volatility downside ($\lambda_{X\mathcal{D}}$) factors are negative. In terms of model fit, the appropriate benchmark is the VOL model. For both models the market return and changes in market volatility are used to construct the factors, and the VOL arises as a restricted version of the GDA5 when the representative agent is not disappointment averse. The GDA5 provides substantial improvement with a RMSPE of 16 bps per month, less than half of the 36 bps associated with the VOL model. The χ^2-test rejects the null that the GDA5 pricing errors are jointly zero. Note, however, that this happens only for the stock portfolios. In all the other panels of Table 4 the GDA5 cannot be rejected.

Panels B to D present the results for the three other sets of portfolios. All factors in all specifications are statistically significant at least at the 10% level and the respective signs are the same as in Panel A. The magnitudes of the risk premia are fairly similar across the different panels. This is remarkable since the test assets in separate panels represent different asset classes. In alternative models, like the four-factor model of Carhart (1997), risk premia can change considerably when different asset classes are used for the estimation (see later results in the paper). Considering model fit, the GDA3 and GDA5 models provide even bigger improvement compared to the CAPM in Panels B to D then in the case of the stock portfolios. In Panel D, for example, the RMSPE of the CAPM is 50 bps per month, while that of the GDA3 and GDA5 are 19 and 13 bps, respectively. Note also, that the χ^2-test cannot reject the null that the GDA5 pricing errors are jointly zero at the 5% level in Panel B, and even at the 10% level in Panels C and D.

To visually assess the fit of different models, the top row in Figure 1 shows scatter plots of actual versus predicted returns corresponding to Panel D of Table 4, where all the asset classes are included in the estimation.[9] Panel A of Figure 1 highlights the failure of the CAPM to price these portfolios. Within each asset class, the actual return of the portfolios vary considerably, but the CAPM predicts similar returns for all portfolios. Consequently, portfolios within each asset class line up close to a vertical line. The improvement in fit is evident as we move from the CAPM towards the GDA5. For the GDA5, the portfolios line up almost perfectly along the 45 degree line.

4.2.1 Comparison with alternative models

The cross-sectional implications of market downside risk has already been studied, for example, by Ang, Chen and Xing (2006) and Lettau et al. (2014). These authors estimate slightly different, but related models to quantify the effect of market downside risk. More importantly, it can be shown that our GDA3 specification,

$$E\left[R_{it}^{e}\right] = \lambda_{W}\beta_{iW} + \lambda_{\mathcal{D}}\beta_{i\mathcal{D}} + \lambda_{W\mathcal{D}}\beta_{iW\mathcal{D}} \ , \tag{17}$$

nests the models from both of these studies, with different restrictions on the value of $\lambda_{\mathcal{D}}$.[10] Ang, Chen and Xing (2006) specify the model for expected returns as

$$\begin{aligned} &E\left[R_{it}^{e}\right] = \lambda^{+}\beta_{i}^{+} + \lambda^{-}\beta_{i}^{-} \ , \quad \text{with} \\ &\beta_{i}^{+} = \frac{Cov\left(R_{it}^{e}, r_{Wt} \mid \mathcal{U}_{t}\right)}{Var\left(r_{Wt} \mid \mathcal{U}_{t}\right)} \quad \text{and} \quad \beta_{i}^{-} = \frac{Cov\left(R_{it}^{e}, r_{Wt} \mid \mathcal{D}_{t}\right)}{Var\left(r_{Wt} \mid \mathcal{D}_{t}\right)} \ , \end{aligned} \tag{18}$$

[9] The Online Appendix contains scatter plots similar to the ones in Figure 1 for the three other sets of portfolios.

[10] Appendix A contains detailed derivations of the results summarized below.

where $\mathcal{U}$ refers to the upside event, which is the complement of the disappointing event $\mathcal{D}$. The model in (18) is equivalent to the GDA3 specification in (17) with

$$\lambda_W = \lambda^+ + \lambda^- \ , \qquad \lambda_{\mathcal{D}} = 0 \ , \qquad \lambda_{W\mathcal{D}} = \lambda^- \ . \tag{19}$$

That is, the model proposed by Ang, Chen and Xing (2006) imposes the restriction $\lambda_{\mathcal{D}} = 0$. On the other hand, Lettau et al. (2014) propose

$$\begin{aligned} &E\left[R^e_{it}\right] = \lambda\beta_i + \lambda^-\left(\beta^-_i - \beta_i\right) \ , \quad \text{with} \\ &\beta_i = \frac{Cov\left(R^e_{it}, r_{Wt}\right)}{Var\left(r_{Wt}\right)} \quad \text{and} \quad \beta^-_i = \frac{Cov\left(R^e_{it}, r_{Wt} \mid \mathcal{D}_t\right)}{Var\left(r_{Wt} \mid \mathcal{D}_t\right)} \ , \end{aligned} \tag{20}$$

where β_i is the CAPM beta and β^-_i is the same downside beta that Ang, Chen and Xing (2006) use. The specification in (20) is equivalent to the GDA3 specification in (17) with

$$\lambda_W = \lambda \ , \qquad \lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}\left(\lambda_W - \lambda_{W\mathcal{D}}\right) \ , \qquad \lambda_{W\mathcal{D}} = \gamma_1\lambda + \left(1-\gamma_1\right)\lambda^- \ , \tag{21}$$

where

$$\gamma_1 \equiv \frac{Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)} \ , \qquad \gamma_2 \equiv \frac{Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)} \ .$$

That is, the model proposed by Lettau et al. (2014) imposes $\lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}\left(\lambda_W - \lambda_{W\mathcal{D}}\right)$. Since $\mathcal{D}_t = \{r_{W,t} < -0.03\}$, i.e., $I\left(\mathcal{D}_t\right)$ takes the value of one when the market return is low, we have $\gamma_2 < 0$. It is also easy to see that $1 - \gamma_1 > 0$. Therefore, if $\lambda_{W\mathcal{D}} > \lambda_W$, which is true for all asset classes in Table 4, then the model of Lettau et al. (2014) imposes $\lambda_{\mathcal{D}} > 0$.

Table 5 presents risk premia estimates for the model of Ang, Chen and Xing (2006) in Panel A and for the model of Lettau et al. (2014) in Panel B. These two models are estimated using the Fama-MacBeth (1973) procedure instead of the GMM, because according to the definitions in (18) and (20) only a subset of the sample should be used to calculate the downside- and upside beta of a portfolio. Note that the different estimation procedure does not have an effect on the point estimates, only on their standard errors. All factors from both models are statistically significant for all sets of portfolios. More importantly, we also report implied values of λ_W, $\lambda_{\mathcal{D}}$, and $\lambda_{W\mathcal{D}}$ calculated using the formulas in (19) for Panel A and in (21) for Panel B. This enables a direct comparison with the GDA3 model and allows us to assess whether the restrictions imposed by the models are supported in the data. The model used by Ang, Chen and Xing (2006) imposes $\lambda_{\mathcal{D}} = 0$. This is rejected as the $\lambda_{\mathcal{D}}$ estimates are significantly different from zero for the GDA3 model for all four sets of portfolios. Another consequence of restricting $\lambda_{\mathcal{D}} = 0$ is that implied prices of market downside risk, $\lambda_{W\mathcal{D}}$, are lower than the values estimated for the GDA3 model. Comparing model fit, the GDA3 model is associated with lower pricing errors than the model of Ang, Chen and Xing (2006), and the difference can be quite substantial, as in the case of stock portfolios (22 bps for the GDA3 and 31 bps for the model of Ang, Chen and Xing, 2006) and in the case when all three asset classes are included (19 bps versus 27 bps). The model of Lettau et al. (2014) imposes a different restriction on the premium associated with the disappointment factor. As we pointed out earlier and can be seen in Panel B of Table 5, the implied $\lambda_{\mathcal{D}}$ values are positive. This restriction is also rejected by the data since the $\lambda_{\mathcal{D}}$ estimates are negative and significantly different from zero for the GDA3 model. Moreover, the implied $\lambda_{W\mathcal{D}}$ values are even lower than in the case of the Ang, Chen and Xing (2006) model. Consequently, the Lettau et al. (2014) model provides a poorer fit, in terms of RMSPE, than the other two models. In general, the models used by Ang, Chen and Xing (2006) and Lettau et al. (2014) impose restrictions, compared to the GDA3 model, that are not supported by the data. Panels E and F in Figure 1 show scatter plots of actual versus predicted returns when all the asset classes are included in the estimation. These plots provide a visual evidence that the GDA3 has a better fit than the two nested models.

The last model in Table 5 is the four-factor model of Carhart (1997). We include this model in our analysis as it is an important benchmark in the literature. The Carhart (1997) model does a good job in pricing the 25 stock portfolios sorted on size and momentum with a RMSPE of 14 bps per month. This is not surprising, as the four-factor model was tailor made to price these stock portfolios correctly. It is more surprising that the GDA5 provides a very similar fit with a RMSPE of 16 bps, since the GDA5 was not explicitly constructed with the size and momentum anomalies in mind. When we consider other asset classes, the success of the Carhart (1997) model is not that evident any more. When estimating the model using option portfolios, the pricing error is higher than that of the GDA5 model, but even more importantly, the estimated risk premia change considerably compared to the first column. The estimate on λ_{HML} switches sign, and the magnitudes of the other estimates also change a lot. In other words, the estimated risk premia are very different, when different asset classes are used. Consequently, the Carhart (1997) model performs badly when we estimate it on the stock and option portfolios jointly. From all the models in Table 4 and Table 5, only the CAPM has worse performance in terms of pricing error when stocks and options are jointly considered. Similar conclusion arises when currency portfolios are included in the test assets. In general, the four-factor model works well for pricing our stock portfolios, but it is less successful in pricing portfolios from other asset classes. This is also illustrated in Panel G of Figure 1. The stock portfolios line up along the 45 degree line, but the portfolios from other asset classes do not.

4.2.2 Robustness checks

Alternative disappointment regions

For our main results the disappointing event was defined as the market falling more than three percent in a given month. Table 6 presents risk premia estimates for the GDA3 and GDA5 models using different definitions. In Panel A disappointment sets in if the market falls more than 1.5% in a given month. With this definition, the probability of having a disappointing month between 1964 and 2013 is 26.7%, ten percentage points higher than in the main specification. The results are similar to those obtained from the benchmark specification in Table 4. The signs on the estimated risk premia are as expected and the estimate are typically statistically significant. The only exception is the premium on the disappointment factor, $\lambda_{\mathcal{D}}$, which is insignificant in most of the specifications.

In Panel B all months with a negative market return are considered disappointing. The probability of observing a negative market return between 1964 and 2013 is 38.5%. Note that this definition is the closest to the one used by Ang, Chen and Xing (2006).[11] Four out of the five risk premia are statistically significant and have the expected sign in all specifications. The exception is the disappointment factor, which is not significantly negative in any of the specifications and often has a positive sign.

Panels A and B of Table 6 show that as the disappointment threshold increases, the premium on the disappointment factor becomes insignificant. That is, disappointing events should be sufficiently out in the left tail so that the disappointment factor is priced in the cross-section. It is also important to know which threshold is more appropriate to describe the

[11] For their main results, Ang, Chen and Xing (2006) define the downside event as the market return falling below its mean. However, they also use the same definition as Panel B of Table 6 in one of their robustness checks. They report that the average cross-sectional correlation of their downside beta measure using the two definitions for stocks listed on the NYSE is above 0.96.

cross-section of different asset returns. Comparing the model fit for our benchmark definition and the alternative definitions in Panel A and B, in seven out of eight specifications the lowest RMSPE value is obtained for our benchmark definition, when disappointment corresponds to the market falling more than three percent in a month.

Finally, changes in market volatility explicitly enter the definition of the disappointing event in Panel C. The term $(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1}$ may be viewed as the return on a volatility index with same standard deviation as the market return. Disappointment occurs if the return of holding a long position in the market index combined with a short position in the volatility index falls below three percent in a given month. The unconditional probability of a disappointing month between 1964 and 2013 is 17.3% using this definition. Recall that the unconditional probability of disappointment using our benchmark definition is very close to this, 16.3%. For further evidence that our benchmark definition for the disappointing event ($\mathcal{D}_1$) and the definition in Panel C ($\mathcal{D}_2$) are empirically close to each other, note that the probability of $\mathcal{D}_1$ conditional on $\mathcal{D}_2$ is 81.6% and the probability of $\mathcal{D}_2$ conditional on $\mathcal{D}_1$ is 86.6% between 1964 and 2013. All the risk premia in Panel C have the expected sign and most of them are statistically significant. The only notable exception is the premium on the volatility factor (λ_X), which becomes insignificant in all specifications.

Alternative measures of market volatility

We also explore how the estimates for the GDA5 model change if a different measure of market volatility is considered. Our alternative measures are the option-implied volatility index (VIX), realized volatility calculated from intra-daily market returns, and a model implied volatility calculated using an EGARCH specification. Appendix B contains a detailed description of how these alternative measures are calculated. Note that the measures are available for different time periods. The VIX data is available starting from 1986 and our intra-daily return data covers only the period from February 1986 to September 2010. The model implied volatility is available for the entire sample period. For each specification we used the longest sample possible, i.e., the sample period is the intersection of the availability of the volatility data and the respective sample period from Table 3.

The results are presented in Table 7. The conclusion is similar across the panels. The signs on the risk premia are as expected and, apart from a few cases, the risk premia estimates are statistically significant. Looking at the RMSPE values, it is not clear which volatility measure provides the best fit. In fact, for each portfolio set, a different volatility measure provides the lowest pricing error (RMSPE). While there is no clear winner among alternative volatility measures in terms of model fit, it is important to point out that the GDA5 model considerably improves upon the GDA3 and its nested specifications, regardless of which measure is used.

Alternative portfolios as test assets

Finally, we consider alternative sets of US stock portfolios. It has a long tradition in asset pricing to study portfolios formed on size and book-to-market. Fama and French (2015) show that sorting stocks based on size-profitability or on size-investments creates similar patterns. Therefore, Table 8 shows estimated risk premia for the GDA5 model when the test assets are (i) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum portfolios, (ii) 25 (5×5) portfolios formed on size and book-to-market, (iii) 25 (5×5) portfolios formed on size and operating profitability, and (iv) 25 (5×5) portfolios formed on size and investment. The results show that all the risk premia are significant and have the expected sign. The only exception is the disappointment premium in column 4, which is not statistically significant.

That is, our results are robust to the choice of stock portfolios.

5 Conclusion

This paper provides an analysis of downside risks in asset prices. Our empirical tests are motivated by a the cross-sectional implications of a dynamic consumption-based general equilibrium model where the representative investor has generalized disappointment aversion preferences and macroeconomic uncertainty is time-varying. We explicitly characterise the factors that are valued by such an investor and show empirically that they are significantly priced. We demonstrate that in addition to a fall in the market return, downside risk may also be associated with a rise in market volatility, and that in consequence investors demand a volatility downside risk premium to invest in risky securities. Our empirical tests confirm the presence of a significant volatility downside risk premium in the cross-section of stocks, options and currencies.

The related literature has mainly focused on the time series implications of this general equilibrium setting, discussing the preference parameter values necessary to match empirical regularities in equity returns, risk-free rate, variance premium and options. As we have discussed, estimating these preference parameter values to jointly target both the time series and the cross-section of asset returns remains challenging and constitutes an interesting avenue for future research.

A The GDA3 and nested models

To calculate betas in the GDA3 model, the following regression is estimated:

$$R_{it}^{e} = \alpha_i + \beta_{iW} r_{Wt} + \beta_{i\mathcal{D}} I\left(\mathcal{D}_t\right) + \beta_{iW\mathcal{D}} r_{Wt} I\left(\mathcal{D}_t\right) + \varepsilon_{it} \tag{A.1}$$

The mechanics of the OLS implies $E\left[e_{it}\right] = E\left[e_{it} r_{Wt}\right] = E\left[e_{it} I\left(\mathcal{D}_t\right)\right] = E\left[e_{it} r_{Wt} I\left(\mathcal{D}_t\right)\right] = 0$, where e_{it} denote realized errors after the estimation. Then, with the estimated α_i and β_i-s,

$$\begin{aligned}
E\left[R_{it}^{e}\right] &= \alpha_i + \beta_{iW} E\left[r_{Wt}\right] + \beta_{i\mathcal{D}} \pi + \beta_{iW\mathcal{D}} E\left[r_{Wt} \mid \mathcal{D}_t\right] \pi && \text{(A.2)}\\
E\left[R_{it}^{e} r_{Wt}\right] &= \alpha_i E\left[r_{Wt}\right] + \beta_{iW} E\left[r_{Wt}^2\right] + \beta_{i\mathcal{D}} E\left[r_{Wt} \mid \mathcal{D}_t\right] \pi + \beta_{iW\mathcal{D}} E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] \pi && \text{(A.3)}\\
E\left[R_{it}^{e} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{Wt} \mid \mathcal{D}_t\right] && \text{(A.4)}\\
E\left[R_{it}^{e} r_{Wt} \mid \mathcal{D}_t\right] &= \left(\alpha_i + \beta_{i\mathcal{D}}\right) E\left[r_{Wt} \mid \mathcal{D}_t\right] + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] \ , && \text{(A.5)}
\end{aligned}$$

where $E\left[I\left(\mathcal{D}_t\right)\right] \equiv \pi$ is the unconditional probability of disappointment. Also note that the occurence of the upside event, the complement of the disappointing event can be written as $I\left(\mathcal{U}_t\right) = 1 - I\left(\mathcal{D}_t\right)$, hence (A.1) can be rewritten as

$$\begin{aligned}
R_{it}^{e} &= \alpha_i + \beta_{iW} r_{Wt} + \beta_{iW\mathcal{D}} r_{Wt} \cdot \left[1 - I_t\left(\mathcal{U}\right)\right] + \beta_{i\mathcal{D}} \left[1 - I_t\left(\mathcal{U}\right)\right] + \varepsilon_{it}\\
&= \left(\alpha_i + \beta_{i\mathcal{D}}\right) + \left(\beta_{iW} + \beta_{iW\mathcal{D}}\right) r_{Wt} - \beta_{iW\mathcal{D}} r_{Wt} \cdot I_t\left(\mathcal{U}\right) - \beta_{i\mathcal{D}} I_t\left(\mathcal{U}\right) + \varepsilon_{it} \ .
\end{aligned} \tag{A.6}$$

Again, the mechanics of the OLS, namely $E\left[e_{it} I\left(\mathcal{U}_t\right)\right] = E\left[e_{it} r_{Wt} I\left(\mathcal{U}_t\right)\right] = 0$, gives us

$$\begin{aligned}
E\left[R_{it}^{e} | \mathcal{U}_t\right] &= \alpha_i + \beta_{iW} E\left[r_{Wt} | \mathcal{U}_t\right] && \text{(A.7)}\\
E\left[R_{it}^{e} r_{Wt} | \mathcal{U}_t\right] &= \alpha_i E\left[r_{Wt} | \mathcal{U}_t\right] + \beta_{iW} E\left[r_{Wt}^2 | \mathcal{U}_t\right] \ . && \text{(A.8)}
\end{aligned}$$

Using (A.4) and (A.5), it can be shown that the market downside beta is

$$\beta_i^- \equiv \frac{Cov\left(R_{it}^e, r_{Wt}|\mathcal{D}_t\right)}{Var\left(r_{Wt}|\mathcal{D}_t\right)} = \frac{E\left[R_{it}^e r_{Wt}|\mathcal{D}_t\right] - E\left[R_{it}^e|\mathcal{D}_t\right]E\left[r_{Wt}|\mathcal{D}_t\right]}{Var\left(r_{Wt}|\mathcal{D}_t\right)} = \beta_{iW} + \beta_{iW\mathcal{D}} \tag{A.9}$$

Using (A.7) and (A.8), the upside beta is

$$\beta_i^+ \equiv \frac{Cov\left(R_{it}^e, r_{Wt}|\mathcal{U}_t\right)}{Var\left(r_{Wt}|\mathcal{U}_t\right)} = \frac{E\left[R_{it}^e r_{Wt}|\mathcal{U}_t\right] - E\left[R_{it}^e|\mathcal{U}_t\right]E\left[r_{Wt}|\mathcal{U}_t\right]}{Var\left(r_{Wt}|\mathcal{U}_t\right)} = \beta_{iW} \tag{A.10}$$

Finally, using (A.2) and (A.3) it can be shown that

$$Cov\left(R_{it}^e, r_{Wt}\right) = \beta_{iW} Var\left(r_{Wt}\right) + \beta_{iW\mathcal{D}} Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right) + \beta_{i\mathcal{D}} Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right) . \tag{A.11}$$

Hence, the CAPM beta is

$$\beta_i \equiv \frac{Cov\left(R_{it}^e, r_{Wt}\right)}{Var\left(r_{Wt}\right)} = \beta_{iW} + \beta_{iW\mathcal{D}} \underbrace{\frac{Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)}}_{\equiv \gamma_1} + \beta_{i\mathcal{D}} \underbrace{\frac{Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)}}_{\equiv \gamma_2} \tag{A.12}$$

Using (A.9) and (A.10), the model proposed by Ang, Chen and Xing (2006) can be written as

$$E\left[R_{it}^e\right] = \lambda^+ \beta_i^+ + \lambda^- \beta_i^- = \left(\lambda^+ + \lambda^-\right) \beta_{iW} + \lambda^- \beta_{iW\mathcal{D}} . \tag{A.13}$$

Using (A.9) and (A.12) the model proposed by Lettau et al. (2014) can be written as

$$\begin{aligned} E\left[R_i^e\right] &= \lambda \beta_i + \lambda^- \left(\beta_i^- - \beta_i\right) \\ &= \lambda \beta_{iW} + \left(\gamma_1 \lambda + \left(1 - \gamma_1\right) \lambda^-\right) \beta_{iW\mathcal{D}} + \gamma_2 \left(\lambda - \lambda^-\right) \beta_{i\mathcal{D}} . \end{aligned} \tag{A.14}$$

B Measures of market volatility

For the main part of the analysis, monthly volatility is measured as the realized volatility of the daily market returns during the month:

$$\sigma_{W,t}^2 = \sum_{\tau=1}^{N_t} \left(r_{W,t,\tau} - \mu_{W,t}\right)^2 \ , \tag{B.1}$$

where $r_{W,t,\tau}$ is the daily market return on the τ-th trading day of month t, $\mu_{W,t}$ is the mean of the daily market returns in month t, and N_t is the number of trading days in month t.

The options-implied monthly volatility is calculated as

$$\sigma_{W,t}^{2,VIX} = \frac{1}{N_t} \sum_{\tau=1}^{N_t} \left(\frac{VIX_{t,\tau}}{100 \cdot \sqrt{12}} \right)^2 \ , \tag{B.2}$$

where $VIX_{t,\tau}$ is the value of the VIX index on the τ-th trading day of month t. The daily value of the VIX index is obtained from CBOE through the WRDS service.

Monthly realized volatility from intra-daily market returns is calculated as

$$\sigma_{W,t}^{2,RV} = \sum_{\tau=1}^{N_t} \sum_{j=1}^{N_\tau} r_{W,t,\tau,j}^2 \ , \tag{B.3}$$

where $r_{W,t,\tau,j}$ denotes the 10-minute log return series on the τ-th trading day of month t and N_τ is the number intra-daily returns within a trading day. We use intra-daily return series of the S&P 500. The data comes from Olsen Financial Technologies and covers the period between February 1986 and September 2010.

Finally, in the model based approach, we fit a model with conditional heteroskedasticity to the daily log market return series $r_{W,\tau}$. We consider the EGARCH(1,1,1) by Nelson (1991),

$$\begin{aligned} r_{W,\tau} &= \mu + \sigma_{W,\tau}\varepsilon_\tau \ , \quad \text{with} \ \ \varepsilon_\tau \overset{\text{iid}}{\sim} \mathcal{N}\left(0,1\right) \\ \ln\left(\sigma^2_{W,\tau}\right) &= \omega + \nu\left(\left|\varepsilon_\tau\right| - \sqrt{2/\pi}\right) + \theta\varepsilon_\tau + \phi\ln\left(\sigma^2_{W,\tau-1}\right) \end{aligned} \tag{B.4}$$

Then the model-implied monthly volatility is calculated as

$$\sigma^{2,EGARCH}_{W,t} = \sum_{\tau=1}^{N_t} \hat{\sigma}^2_{W,t,\tau} \ , \tag{B.5}$$

where $\hat{\sigma}^2_{W,t,\tau}$ is the estimated daily variance on the τ-th trading day of month t.

Change in monthly volatility for all of the above measures is calculated as

$$\Delta\sigma^{2,\cdot}_{W,t} = \sigma^{2,\cdot}_{W,t} - \sigma^{2,\cdot}_{W,t-1} \ . \tag{B.6}$$

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Table 1: Model calibration

A. Endowment parameters

$\mu = 0.15\%$, $\sqrt{\omega_c(L)} = 0.46\%$, $\sqrt{\omega_c(H)} = 1.32\%$, $\nu_d = 6.42$, $\rho = 0.3$, $p_{HH} = 0.9961$, $p_{LL} = 0.9989$

C. Preference parameters

$\delta = 0.998$, $\gamma = 2.5$, $\ell = 2.33$, $\kappa = 0.998$

B. Endowment and asset pricing moments

	Sample	GDA3	GDA5
$E[\Delta c_t]$ (%)	1.84	1.80	1.80
$\sigma[\Delta c_t]$ (%)	2.20	2.07	2.07
$AC1(\Delta c_t)$	0.48	0.25	0.25
$E[\Delta d_t]$ (%)	1.05	1.80	1.80
$\sigma[\Delta d_t]$ (%)	13.02	13.29	13.29
$AC1(\Delta d_t)$	0.11	0.25	0.25
$Corr(\Delta c_t, \Delta d_t)$	0.52	0.30	0.30
$E[pd]$ (%)	3.33	2.72	2.89
$\sigma[pd]$ (%)	0.44	0.20	0.11
$E[r_f]$ (%)	0.57	0.46	0.76
$\sigma[r_f]$ (%)	3.77	0.15	1.55
$E[r - r_f]$ (%)	5.50	8.06	6.61
$\sigma[r - r_f]$ (%)	20.25	17.65	16.84

D. Downside event and factor risk premia

	GDA3	GDA5
ψ	∞	1.5
a	0.00	1.38
b (%)	0.00	-0.10
$Prob(\mathcal{D})$ (%)	17.43	16.09
λ_W	0.0065	0.0042
λ_X		-1.38E-6
$\lambda_{\mathcal{D}}$	-0.3494	-0.3010
$\lambda_{W\mathcal{D}}$	0.0038	0.0023
$\lambda_{X\mathcal{D}}$		-1.16E-6

The entries of the table are, in Panel (A), the first and second moments of consumption and dividend growth rates, the first and second moments of the log price-dividend ratio, and the log risk-free rate and excess log equity returns, in Panel (B), the characteristics of the downside event, and in Panel (C), the factor risk premia, as implied by the GDA model. The first column represents annual data counterparts over the period from January 1930 to December 2012.

Table 2: Summary statistics

Period	07/1964 – 12/2013	04/1986 – 01/2012
$E(r_{W,t})$	0.0080	0.0073
$Std(r_{W,t})$	0.0454	0.0474
$Corr(r_{W,t}, \Delta\sigma^2_{W,t})$	-0.25	-0.30
$Corr(r_{W,t}, \Delta\sigma^2_{W,t} \mid \mathcal{D}_t)$	-0.46	-0.51
$Prob(\mathcal{D}_t)$	16.3%	16.5%

The table reports some summary statistics about the monthly market return ($r_{W,t}$) and the disappointing event $\mathcal{D}_t = \{r_{W,t} < -0.03\}$ for two subperiods. We report the mean and standard deviation of the market return, the conditional and unconditional correlation between the market return and changes in market volatility ($\Delta\sigma^2_{W,t}$), and the unconditional probability of disappointment.

Table 3: Description of the portfolios

Set	#Port	Description	Sample period
Stocks	25	25 (5×5) value-weighted US stock portfolios sorted on size and momentum.	07/1964 – 12/2013
Options	24	24 (2×3×4) S&P 500 index option portfolios sorted on type (call and put), maturity (30, 60, or 90 days), and moneyness (5% ITM, ATM, 5% OTM, and 10% OTM)	04/1986 – 01/2012
Stocks and options	20	10 value-weighted US stock portfolios sorted on size and 10 (2×5) short maturity (30 days) S&P 500 index option portfolios sorted on type (call and put) and moneyness (10% ITM, 5% ITM, ATM, 5% OTM, and 10% OTM)	04/1986 – 01/2012
Stocks, options, and currencies	18	6 (2×3) value-weighted US stock portfolios sorted on size and momentum, 6 (2×3) short maturity (30 days) S&P 500 index option portfolios sorted on type (call and put) and moneyness (ATM, 5% OTM, and 10% OTM), and 6 currency portfolios sorted on their respective interest rates.	04/1986 – 03/2010

Table 4: Risk premia

	CAPM	VOL	GDA3	GDA5	CAPM	VOL	GDA3	GDA5
A. Stocks (N=25)					B. Options (N=24)			
λ_W	0.0059***	0.0048**	0.0079***	0.0082***	0.0056**	0.0055*	0.0056*	0.0091***
	(0.0022)	(0.0020)	(0.0022)	(0.0022)	(0.0028)	(0.0029)	(0.0032)	(0.0027)
$\lambda_{\mathcal{D}}$			-0.3614***	-0.2421**			-0.2535***	-0.4172***
			(0.1184)	(0.1016)			(0.0688)	(0.0642)
$\lambda_{W\mathcal{D}}$			0.0273***	0.0216***			0.0185***	0.0330***
			(0.0066)	(0.0062)			(0.0045)	(0.0063)
λ_X		-0.0032***		-0.0026***		-0.0029***		-0.0026***
		(0.0010)		(0.0008)		(0.0007)		(0.0008)
$\lambda_{X\mathcal{D}}$				-0.0031***				-0.0057***
				(0.0008)				(0.0017)
RMSPE	39.2	35.6	21.8	16.0	49.2	16.3	13.4	10.5
J-stat	111.7	124.0	32.3	53.5	141.5	96.3	71.4	28.9
	(0.00)	(0.00)	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)
C. Stocks and Options (N=20)					D. Stocks, Options, and Currencies (N=18)			
λ_W	0.0062**	0.0054*	0.0068**	0.0088***	0.0066**	0.0057*	0.0070**	0.0080***
	(0.0029)	(0.0028)	(0.0031)	(0.0029)	(0.0030)	(0.0029)	(0.0034)	(0.0029)
$\lambda_{\mathcal{D}}$			-0.2129**	-0.2321**			-0.2765***	-0.1939*
			(0.0935)	(0.1131)			(0.1018)	(0.1133)
$\lambda_{W\mathcal{D}}$			0.0197***	0.0227***			0.0210***	0.0156**
			(0.0043)	(0.0042)			(0.0044)	(0.0079)
λ_X		-0.0032***		-0.0036**		-0.0035***		-0.0034*
		(0.0007)		(0.0014)		(0.0008)		(0.0020)
$\lambda_{X\mathcal{D}}$				-0.0054***				-0.0042**
				(0.0021)				(0.0018)
RMSPE	41.9	17.6	15.8	8.5	48.9	26.5	19.3	12.8
J-stat	84.6	50.5	29.3	14.8	88.7	64.3	30.5	16.4
	(0.00)	(0.00)	(0.03)	(0.46)	(0.00)	(0.00)	(0.01)	(0.23)

The table shows risk premia estimates for four different models using four different sets of portfolios (in Panles; the compositions of different sets are described in Table 3). The factor premia are estimated using GMM with moment conditions as in (16) and the identity weighting matrix. Standard errors of the estimates are reported in parenthesis. The Newey-West estimator with 3 lags is used for the covariance matrix of the moment conditions. The row "RMSPE" presents the root mean squared pricing error of the model and is expressed in basis points (bps) per month. "J-stat" refers to the test statistic of the χ^2-test with the null that the pricing errors are jointly zero (the associated p-value is in parenthesis).

Table 5: Risk premia in alternative models

Stocks	✓		✓	✓
Options		✓	✓	✓
Currencies				✓
A. Ang, Chen and Xing (2006)				
λ^{+}	-0.0101***	-0.0072***	-0.0078***	-0.0092***
	(0.0030)	(0.0027)	(0.0028)	(0.0027)
λ^{-}	0.0168***	0.0162***	0.0156***	0.0174***
	(0.0032)	(0.0032)	(0.0031)	(0.0030)
(λ_W)	0.0067	0.0090	0.0078	0.0082
$(\lambda_{\mathcal{D}})$	0	0	0	0
$(\lambda_{W\mathcal{D}})$	0.0168	0.0162	0.0156	0.0174
RMSPE	30.7	15.9	22.2	27.1
B. Lettau et al. (2014)				
λ	0.0063***	0.0099***	0.0080***	0.0085***
	(0.0020)	(0.0031)	(0.0029)	(0.0029)
λ^{-}	0.0158***	0.0216***	0.0185***	0.0218***
	(0.0037)	(0.0045)	(0.0045)	(0.0042)
(λ_W)	0.0063	0.0099	0.0080	0.0085
$(\lambda_{\mathcal{D}})$	0.1035	0.1406	0.1272	0.1632
$(\lambda_{W\mathcal{D}})$	0.0113	0.0154	0.0129	0.0146
RMSPE	33.7	17.3	25.8	31.5
C. Carhart (1997)				
λ_W	0.0058***	0.0169***	0.0077***	0.0068**
	(0.0020)	(0.0035)	(0.0027)	(0.0028)
λ_{SMB}	0.0021	0.0646***	-0.0070**	0.0016
	(0.0015)	(0.0133)	(0.0035)	(0.0021)
λ_{HML}	0.0055***	-0.0989***	0.0381***	0.0096**
	(0.0021)	(0.0308)	(0.0143)	(0.0039)
λ_{WML}	0.0080***	0.0162	0.0669***	0.0076**
	(0.0018)	(0.0186)	(0.0170)	(0.0031)
RMSPE	13.7	12.4	31.0	41.9
J-stat	73.6	28.2	27.8	70.1
	(0.00)	(0.11)	(0.03)	(0.00)

The table shows risk premia estimates for three different models (in panels) using four different sets of portfolios. The factor premia are estimated using the Fama and MacBeth (1973) procedure in Panels A and B, and using GMM (same as in Table 4) in Panel C. Standard errors of the estimates are reported in parenthesis. Implied premia for the GDA3 model, (λ_W), $(\lambda_{\mathcal{D}})$, and $(\lambda_{W\mathcal{D}})$ are calculated using the formulas in (19) for Panel A and in (21) for Panel B. The row "RMSPE" presents the root mean squared pricing error of the model and is expressed in basis points (bps) per month.

Table 6: Risk premia for alternative definitions of the disappointing event

Stocks	✓	✓			✓	✓	✓	✓
Options			✓	✓	✓	✓	✓	✓
Currencies							✓	✓
A. $\mathcal{D}_t = \{r_{W,t} < -0.015\}$								
λ_W	0.0068***	0.0079***	0.0075**	0.0119***	0.0079***	0.0091***	0.0078**	0.0093***
	(0.0024)	(0.0022)	(0.0031)	(0.0026)	(0.0030)	(0.0027)	(0.0033)	(0.0028)
$\lambda_{\mathcal{D}}$	-0.2478**	-0.2213*	-0.0301	-0.1298*	0.1269	-0.2938	-0.0253	0.0305
	(0.1248)	(0.1190)	(0.0941)	(0.0757)	(0.0896)	(0.3554)	(0.1360)	(0.1166)
$\lambda_{W\mathcal{D}}$	0.0199***	0.0164***	0.0154***	0.0301***	0.0153***	0.0197***	0.0171***	0.0150**
	(0.0046)	(0.0046)	(0.0041)	(0.0047)	(0.0038)	(0.0035)	(0.0037)	(0.0058)
λ_X		-0.0018**		-0.0021***		-0.0045**		-0.0041*
		(0.0008)		(0.0007)		(0.0022)		(0.0022)
$\lambda_{X\mathcal{D}}$		-0.0025***		-0.0045***		-0.0069*		-0.0050**
		(0.0008)		(0.0014)		(0.0038)		(0.0022)
RMSPE	23.2	15.4	14.4	12.4	17.2	13.6	22.7	16.9
J-stat	38.1	47.0	104.0	57.9	35.1	10.6	38.7	25.4
	(0.02)	(0.00)	(0.00)	(0.00)	(0.01)	(0.78)	(0.00)	(0.02)
B. $\mathcal{D}_t = \{r_{W,t} < 0\}$								
λ_W	0.0064**	0.0078***	0.0076**	0.0120***	0.0088***	0.0087***	0.0080**	0.0098***
	(0.0026)	(0.0022)	(0.0030)	(0.0024)	(0.0029)	(0.0028)	(0.0032)	(0.0027)
$\lambda_{\mathcal{D}}$	-0.1409	-0.0988	0.1111	0.0490	0.3742**	0.2919*	0.1688	0.1720
	(0.1033)	(0.0964)	(0.0911)	(0.0785)	(0.1476)	(0.1697)	(0.1386)	(0.1210)
$\lambda_{W\mathcal{D}}$	0.0174***	0.0152***	0.0150***	0.0295***	0.0162***	0.0130***	0.0169***	0.0174***
	(0.0043)	(0.0043)	(0.0040)	(0.0044)	(0.0038)	(0.0046)	(0.0039)	(0.0045)
λ_X		-0.0017**		-0.0022***		-0.0038***		-0.0035*
		(0.0009)		(0.0006)		(0.0012)		(0.0018)
$\lambda_{X\mathcal{D}}$		-0.0023***		-0.0045***		-0.0043***		-0.0045**
		(0.0008)		(0.0014)		(0.0013)		(0.0019)
RMSPE	22.8	15.3	14.0	12.5	14.0	12.0	21.2	15.5
J-stat	40.6	54.2	99.6	58.4	23.2	23.7	30.7	21.7
	(0.01)	(0.00)	(0.00)	(0.00)	(0.14)	(0.07)	(0.01)	(0.06)
C. $\mathcal{D}_t = \{r_{W,t} - \frac{\sigma_W}{\sigma_{\Delta\sigma^2_{W,t}}}\Delta\sigma^2_W < -0.03\}$								
λ_W		0.0079***		0.0100***		0.0068**		0.0082***
		(0.0021)		(0.0038)		(0.0031)		(0.0031)
$\lambda_{\mathcal{D}}$		-0.1672*		-0.3931***		-0.2657***		-0.2454*
		(0.1008)		(0.0593)		(0.0831)		(0.1341)
$\lambda_{W\mathcal{D}}$		0.0094		0.0478***		0.0197***		0.0218***
		(0.0071)		(0.0089)		(0.0065)		(0.0065)
λ_X		-0.0004		-0.0010		-0.0012		-0.0010
		(0.0011)		(0.0006)		(0.0010)		(0.0016)
$\lambda_{X\mathcal{D}}$		-0.0020***		-0.0004		-0.0015		-0.0017***
		(0.0005)		(0.0005)		(0.0011)		(0.0007)
RMSPE		14.1		8.9		13.2		20.3
J-stat		45.4		18.9		19.4		20.6
		(0.00)		(0.46)		(0.20)		(0.08)

The table shows risk premia estimates for the GDA3 and GDA5 models using four different sets of portfolios (the compositions of different sets are described in Table 3). The panels of the table correspond to different definitions of the disappointing event. The estimation procedure is the same as in Table 4 (for details, see the caption of Table 4).

Table 7: Risk premia with alternative volatility measures

Stocks	✓		✓	✓
Options		✓	✓	✓
Currencies				✓
A. Implied Volatility (VIX)				
λ_W	0.0087***	0.0160***	0.0080**	0.0088***
	(0.0029)	(0.0044)	(0.0031)	(0.0030)
$\lambda_{\mathcal{D}}$	-0.0969	-0.4287***	-0.3801**	-0.2901**
	(0.1436)	(0.0770)	(0.1814)	(0.1153)
$\lambda_{W\mathcal{D}}$	0.0116*	0.0487***	0.0242***	0.0251***
	(0.0070)	(0.0094)	(0.0044)	(0.0063)
λ_X	-0.0012**	-0.0012***	-0.0010	-0.0007
	(0.0005)	(0.0004)	(0.0006)	(0.0009)
$\lambda_{X\mathcal{D}}$	-0.0018***	-0.0036***	-0.0020***	-0.0013*
	(0.0006)	(0.0010)	(0.0007)	(0.0007)
RMSPE	15.9	10.1	14.4	16.7
J-stat	36.6	25.0	10.3	15.5
	(0.01)	(0.16)	(0.80)	(0.28)
B. Realized Volatility (intra-daily)				
λ_W	0.0080**	0.0111***	0.0080***	0.0084***
	(0.0031)	(0.0039)	(0.0030)	(0.0030)
$\lambda_{\mathcal{D}}$	-0.2214	-0.2964***	-0.2098*	-0.2239*
	(0.1958)	(0.0499)	(0.1093)	(0.1313)
$\lambda_{W\mathcal{D}}$	0.0164*	0.0351***	0.0195***	0.0190**
	(0.0096)	(0.0066)	(0.0037)	(0.0088)
λ_X	-0.0015**	0.0004	-0.0020***	-0.0017*
	(0.0007)	(0.0006)	(0.0007)	(0.0010)
$\lambda_{X\mathcal{D}}$	-0.0019**	-0.0002	-0.0028***	-0.0022***
	(0.0008)	(0.0008)	(0.0010)	(0.0009)
RMSPE	17.1	13.2	10.9	11.3
J-stat	24.3	60.3	19.9	13.2
	(0.23)	(0.00)	(0.18)	(0.43)
C. Model Implied Volatility (EGARCH)				
λ_W	0.0089***	0.0077**	0.0082***	0.0082***
	(0.0021)	(0.0035)	(0.0030)	(0.0030)
$\lambda_{\mathcal{D}}$	-0.1671*	-0.4285***	-0.1553**	-0.1565*
	(0.0999)	(0.0538)	(0.0720)	(0.0902)
$\lambda_{W\mathcal{D}}$	0.0217***	0.0368***	0.0209***	0.0176**
	(0.0060)	(0.0088)	(0.0044)	(0.0078)
λ_X	-0.0006	0.0002	-0.0008**	-0.0010
	(0.0003)	(0.0005)	(0.0004)	(0.0007)
$\lambda_{X\mathcal{D}}$	-0.0010***	-0.0012***	-0.0015***	-0.0015**
	(0.0004)	(0.0004)	(0.0005)	(0.0006)
RMSPE	14.9	11.3	9.0	13.5
J-stat	51.1	27.7	23.2	26.0
	(0.00)	(0.09)	(0.08)	(0.02)

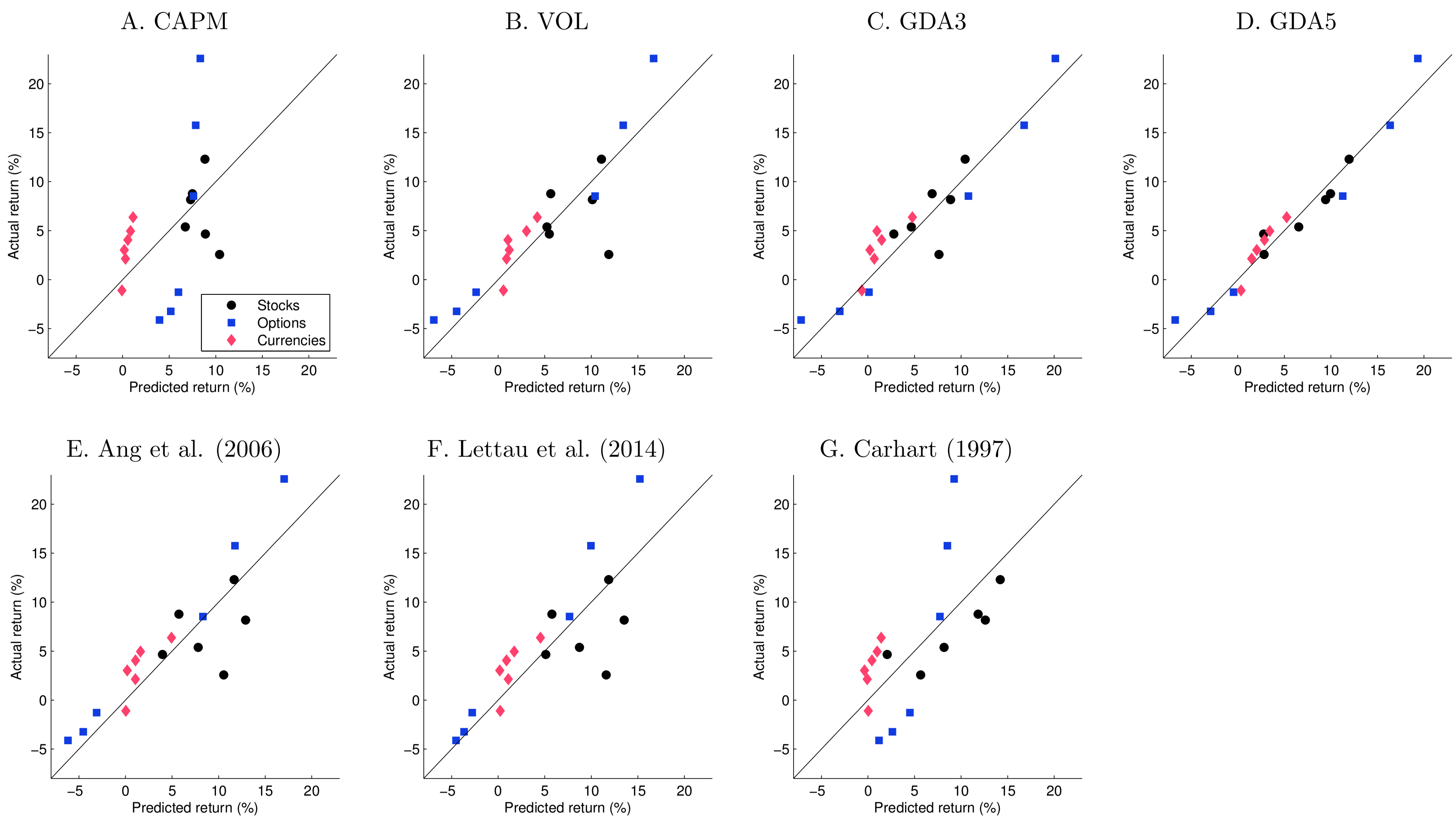
The table shows risk premia estimates for the GDA5 model using four different sets of portfolios (the compositions of different sets are described in Table 3). The panels of the table correspond to different measures of market volatility. Details on how the measures are calculated can be found in Appendix B. The estimation procedure is the same as in Table 4 (for details, see the caption of Table 4).

Table 8: Risk premia with alternative stock portfolios

	10 B,S,M	25 S-BM	25 S-OP	25 S-INV
A.GDA3				
λ_W	0.0072***	0.0068***	0.0070***	0.0068***
	(0.0022)	(0.0022)	(0.0023)	(0.0022)
$\lambda_{\mathcal{D}}$	-0.3018***	0.1569	-0.1798**	0.0970
	(0.1077)	(0.0998)	(0.0900)	(0.0795)
$\lambda_{W\mathcal{D}}$	0.0207***	0.0089	0.0155*	0.0078
	(0.0063)	(0.0059)	(0.0084)	(0.0063)
RMSPE	17.9	22.9	17.1	21.5
J-stat	30.4	70.7	24.0	90.9
p-value	(0.30)	(0.00)	(0.35)	(0.00)
B. GDA5				
λ_W	0.0075***	0.0092***	0.0081***	0.0108***
	(0.0021)	(0.0025)	(0.0023)	(0.0025)
$\lambda_{\mathcal{D}}$	-0.2440**	-0.2002*	-0.3562***	-0.0620
	(0.0998)	(0.1073)	(0.1325)	(0.0884)
$\lambda_{W\mathcal{D}}$	0.0202***	0.0298***	0.0293***	0.0362***
	(0.0061)	(0.0091)	(0.0101)	(0.0091)
λ_X	-0.0017**	-0.0036***	-0.0043***	-0.0053***
	(0.0007)	(0.0009)	(0.0013)	(0.0009)
$\lambda_{X\mathcal{D}}$	-0.0022***	-0.0048***	-0.0052***	-0.0072***
	(0.0007)	(0.0012)	(0.0016)	(0.0011)
RMSPE	16.7	17.9	13.9	15.6
J-stat	33.1	32.0	12.5	17.4
p-value	0.13	0.04	0.90	0.63

The table shows risk premia estimates for the GDA3 and GDA5 models using four different sets of US stock portfolios as test assets: (i) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum portfolios, (ii) 25 (5×5) portfolios formed on size and book-to-market, (iii) 25 (5×5) portfolios formed on size and operating profitability, and (iv) 25 (5×5) portfolios formed on size and investment. The estimation procedure is the same as in Table 4 (for details, see the caption of Table 4)

Figure 1: Actual versus predicted returns



The figure shows the realized average excess returns for selected stock, option, and currency portfolios, against the predicted average excess returns from models reported in Panel D of Table 4 (CAPM, VOL, GDA3, and GDA5) and the last column of Table 5 (the models of Ang et al., 2006; Lettau et al., 2014; and Carhart, 1997). The portfolios correspond to the last set in Table 3.