Asymmetry Matters: A High-Frequency Risk-Reward Trade-Off*

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Abstract

Expected returns should not only include rewards for accepting the risk of a potential downside loss, but also discounts for potential upside gains. Since investors care differently about upside gains versus downside losses, they require a risk premium for bearing the relative downside risk. We validate this perception empirically as well as theoretically and show that conditional asymmetry forecasts equity market returns in the short run. The results hold not only for different return series and different data frequencies (daily and intra-daily) but also for various subsamples. Our short-term expected return predictor, the asymmetric realized volatility measure, captures more variation in equity returns than the variance risk premium, a forward-looking measure, or the price-earnings ratio, and can easily be extracted from realized return series. We formalize this intuition with a closed-form asset pricing model that incorporates disappointment aversion and time-varying macroeconomic uncertainty.

Keywords: realized variance, realized semivariance, stock market return predictability, asymmetric realized volatility, variance risk premium, high-frequency data, equilibrium asset pricing

JEL Classification: G1, G12, G11, C1, C5

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1 Introduction

The linear relation between the conditional variance and expected excess stock market return, as suggested by Merton (1973)'s intertemporal capital asset pricing model (ICAPM), has drawn a lot of attention in modern finance research. The systematic risk-return trade-off suggests that, conditional on the information available at each point in time, the conditional excess market return varies with its conditional variance. Even though there is a vast literature that investigates this relation, it has not been possible to find flawless empirical evidence. The literature finds mostly insignificant, and even negative, systematic risk-reward trade-offs.

We propose a risk measure that draws mainly from two important observations. First, the market price of risk is determined by two trends that have opposite pricing effects: the uncertain upside gain and the uncertain downside risk. It is a strong economic argument that an investor is willing to pay for potential upside returns but must be compensated for potential downside losses. Investigating this argument, we obtain empirical results that provide strong evidence that long-run upside uncertainty has a significant negative pricing effect, and long-run downside risk a significant positive pricing effect, on short-run equity market returns. To the best of our knowledge, this is the first paper to provide consistent evidence of the role of upside uncertainty. Second, following Kahneman and Tversky (1979)'s prospect theory, the finance literature shows that investors care differently about upside gains than downside losses, and that those who face downside risk require a relative downside risk premium (Ang et al. 2006). We confirm this insight empirically as well as theoretically and show that conditional asymmetry forecasts equity market returns. Our proposed measure of conditional asymmetry is the difference between the realized uncertainty of upside gains and the realized risk of downside losses: the asymmetric realized volatility (ARV) measure. The intuition behind this is that we exploit that returns are negatively skewed in particular in high frequency. Our long-run ARV measure is a significant predictor of short-run equity market returns. Computed using sixty-minute returns, it explains 1.18%, 4.00% and 10.66% of the total variation in expected one-month, three-month and six-month returns respectively. This predictive power increases to 3.65%, 10.95% and 21.68% respectively when the ARV measure is based on five-minute returns.

To strengthen our empirical results, we propose a closed-form consumption-based asset pricing model that explores the predictive power of the long-run ARV. The model features recursive utility with disappointment aversion preferences, and time-varying macroeconomic uncertainty measured by the volatility of aggregated consumption. We follow Bonomo et al. (2011), but calibrate the model at an intra-daily frequency to match the actual moments of annual consumption and equity dividend growths. The commonly used Epstein and Zin (1989) preferences, based on the expected utility certainty equivalent for timeless gambles, fail to generate the upside and downside pricing effects found in actual data. On the contrary, the Routledge and Zin (2010) preferences, based on a generalized disappointment aversion certainty equivalent, are able to reproduce the observed patterns.

An investor with generalized disappointment aversion preferences cares more about downside losses than upside gains, by assigning greater weights to outcomes that may realize less than the investor's certainty equivalent. Consequently, the investor is aware of tail risks. These tail risks are particularly important in intra-daily equity market returns, since high-frequency returns show high excess kurtosis and negative skewness. Consistent with this empirical evidence, the model requires a high excess kurtosis of consumption volatility at a higher frequency, to replicate the empirical findings. In addition to the observed first and second moments of the equity return, we are able not only to match the predictive power of the ARV, based on the sixty-minute equity market return series, but also the sign of the slope estimates and, to a large extent, their magnitude.

Turning to a more detailed review of related literature and related risk factors, we start with realized volatility, which is one of the most common risk factors. Whereas Bandi and Perron (2008) show that realized volatility has predictive power for the market return in the long run, others, such as Ghysels et al. (2005), develop new measures that produce reasonable predictions within a shorter period. The realized volatility-based risk measure is ambiguous, though, and current research is focusing on other possible, but related, risk measures. Since returns are negatively skewed, a possible approach is to disentangle realized volatility into upside volatility and downside volatility, implicitly incorporating the asymmetry of returns into the pricing model.

Despite the fact that the focus of most of the related literature is not on the asymmetry of returns, there has been growing interest in the asymmetry of equity market returns and their volatility over the last twenty years. Asymmetry is present in all major equity markets, since large negative market returns are more frequent than positive returns, implying a negative skewness. Supporting evidence comes from Jondeau and Rockinger (2003), who find that there is negative skewness in major equity markets. Harvey and Siddique (2000) also show that the skewness of returns is linked to the asymmetry of risk and thus demands a risk premium. Additionally, they show that the momentum effect is related to systematic skewness, which implies that an investor who faces a negatively skewed and volatile market requires a reward for accepting this downside risk.

Recent research by Feunou et al. (2010) models market downside volatility with a bi-normal GARCH model, deriving a time-varying market price of risk. We follow the non-parametric approach of Barndorff-Nielsen et al. (2008), who define downside and upside semivariances as the realized return deviation below and above a specified threshold, based on high-frequency data. Downside risk is thus the realized return deviation below this threshold, and upside uncertainty is the return deviation above this threshold. Recent research by Patton and Sheppard (2011) shows that disentangling downside and upside realized semivariance improves forecasts of future volatility, in particular because the downside realized semivariance is more important.

The price-earnings ratio serves traditionally as a well-established predictor of equity market returns. We find supporting evidence that the price-earnings ratio is a significant predictor of future equity returns, but becomes less significant once the ARV measure is added as a second factor in our predictive regression. The price-earnings ratio remains mostly significant, in particular for longer horizon predictions, but adds little explanatory power to our predictive regression. Our evidence here is based on daily returns, though, while the literature mainly focuses on monthly returns. Our results suggest that the ARV measure captures similar variation to that captured by the price-earnings ratio, but captures about twice as much of the variation in equity returns

Recent literature has focused on the variance risk premium (VRP), defined as the difference between the realized volatility and the implied volatility, and has shown that measuring risk this way improves return predictability significantly (Drechsler and Yaron 2011; Bollerslev et al. 2010). The VRP is a short-run forward-looking measure that extracts information from the prices of index options and captures additional information over that captured by traditional risk measures. Our backward-looking ARV measure, on the other hand, is easily extractable from realized return series, and outperforms the VRP, which captures the same but far less variation in equity returns. Not only the negative correlation between the ARV measure and the VRP, but also the fact that tail risk (high excess kurtosis) is generated by jumps, suggests that our results are consistent with Drechsler and Yaron (2011). They find that jumps in consumption volatility play an important role in generating the short-term predictability of excess returns by the VRP.

We carry out further robustness checks in this paper. We show the economic significance of our results by calculating the maximal achievable increase in the Sharpe ratio. The results suggest that the increase is quite large. Further, to be confident about our reported t-statistics, we provide bootstrapped standard errors.

The remainder of the paper proceeds as follows. First, Section 2 presents the methodology, showing how the asymmetric conditional risk-return trade-off is empirically investigated. In Section 3, we analyze the data source and provide detailed statistics regarding both the daily and intra-daily equity market returns. In Section 4, we develop a closed-form asymmetric asset pricing model that provides a theoretical foundation for our empirical findings, and discuss the model calibration and results. Finally, in Section 5, we provide conclusions.

2 Asymmetric Realized Volatility Tests of the Risk-Return Trade-off

In this section, we introduce the asymmetric realized volatility (ARV) measure, to test the relation between stock market risk and expected return, and present our empirical evidence.

2.1 Methodology

We use logarithmic (log) returns and realized volatility, aggregated over different periods, based on high-frequency equity log returns.[1] We define daily log returns and realized

[1] To ensure that our analysis is not driven by a few outliers, we winsorize the log returns at the 1% level, i.e. .5% from the top and .5% from the bottom. This is commonly done in related literature; for example, Ang et al. (2006) winsorize in their cross-sectional study on downside risk, and Drechsler and Yaron (2011) winsorize their exogenous variables at the 1% level.

variances by

$$r_{t,t+1} = \sum_{j=1}^{1/\Delta} r_{t+j\Delta} \quad \text{and} \quad \sigma^2_{t-1,t} = \sum_{j=1}^{1/\Delta} r^2_{t-1+j\Delta}, \tag{1}$$

where $1/\Delta$ is the number of high-frequency returns per day, i.e. $\Delta = 1/8$ for a sixty-minute return series and $\Delta = 1/85$ for a five-minute return series, $r_{t+j\Delta}$ denotes the jth intra-daily return of the current day, $r_{t,t+1}$ is the current day's return and $\sigma^2_{t-1,t}$ the previous day's realized volatility.

We aggregate future returns over h days and past realized variances over m days, simply defined as

$$r_{t,t+h} = \sum_{l=1}^{h} r_{t+l-1,t+l} \quad \text{and} \quad \sigma^2_{t-m,t} = \sum_{l=1}^{m} \sigma^2_{t-l,t-l+1}. \tag{2}$$

The risk-return trade-off, with realized volatility used as the measure of risk, on high-frequency data is:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{1,mh} \frac{\sigma^2_{t-m,t}}{m} + \epsilon^m_{t,t+h}. \tag{3}$$

In their study of the risk-reward trade-off, Feunou et al. (2010) examine the relationship between reward, as measured by expected returns or the mode of market returns, and risk, as measured by market volatility or the difference between the downside and upside volatilities with respect to the mode. The measures of the downside and upside volatilities are estimated using a binormal GARCH model. In contrast, Barndorff-Nielsen et al. (2008) introduce new measures of uncertainty, which they call "realized semivariances", based entirely on non-parametric high-frequency downward or upward movements in asset prices. We use the non-parametric method to define analogue daily measures as follows:

$$\left(\sigma^-_{t-1,t}\right)^2 = \sum_{j=1}^{1/\Delta} \left(r^-_{t-1+j\Delta}\right)^2 \quad \text{and} \quad \left(\sigma^+_{t-1,t}\right)^2 = \sum_{j=1}^{1/\Delta} \left(r^+_{t-1+j\Delta}\right)^2, \tag{4}$$

where $r^-_{t-1+j\Delta} = r_{t-1+j\Delta} I\left(r_{t-1+j\Delta} < \mu_r\right)$ and $r^+_{t-1+j\Delta} = r_{t-1+j\Delta} I\left(r_{t-1+j\Delta} \geq \mu_r\right)$, and where $I(\cdot)$ is an indicator function. The constant threshold μ_r is the unconditional mean of the intra-daily returns over the aggregation period.[2]

[2] In the following analysis, we use the mean as the threshold. For comparison, we redo our analysis for

Investors must be compensated for realizations below this threshold, but not for realizations above it. We define $\left(\sigma^{-}_{t-1,t}\right)^2$ as the measure of downside risk and $\left(\sigma^{+}_{t-1,t}\right)^2$ as the measure of upside uncertainty. We aggregate upside and downside realized semivariances over m days,

$$\left(\sigma^{-}_{t-m,t}\right)^2 = \sum_{l=1}^{m} \left(\sigma^{-}_{t-l,t-l+1}\right)^2 \quad \text{and} \quad \left(\sigma^{+}_{t-m,t}\right)^2 = \sum_{l=1}^{m} \left(\sigma^{+}_{t-l,t-l+1}\right)^2 . \tag{5}$$

This disentanglement of the realized volatility into upside and downside realized volatilities allows us to test a two-factor asymmetric asset pricing model:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{1,mh} \frac{\left(\sigma^{+}_{t-m,t}\right)^2}{m} + \beta_{2,mh} \frac{\left(\sigma^{-}_{t-m,t}\right)^2}{m} + \epsilon^{m}_{t,t+h}, \tag{6}$$

with an intercept estimate α_{mh} and a slope vector of estimates $\beta_{mh} = \begin{pmatrix} \beta_{1,mh} & \beta_{2,mh} \end{pmatrix}^{\top}$. This two-factor model has the advantage that we can directly see the impact of potential upside gain as well as that of potential downside loss. We would expect to see a negative pricing effect for upside uncertainty and a positive pricing effect for downside risk. Strong evidence will follow that supports our hypothesis that investors have to be compensated for downside risk but are willing to pay for upside potential.

Similar to Barndorff-Nielsen et al. (2008), we define downside realized semivariance and upside realized semivariance such that their sum equates to the realized volatility itself:

$$\sigma^2_{t-m,t} = \left(\sigma^{-}_{t-m,t}\right)^2 + \left(\sigma^{+}_{t-m,t}\right)^2 . \tag{7}$$

The difference between upside and downside realized semivariance is the ARV measure of risk. It is defined by

$$s_{t-m,t} = \frac{\left(\sigma^{+}_{t-m,t}\right)^2}{m} - \frac{\left(\sigma^{-}_{t-m,t}\right)^2}{m}. \tag{8}$$

The asymmetric capital asset pricing model with the ARV risk measure becomes:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{2,mh} s_{t-m,t} + \epsilon^{m}_{t,t+h}. \tag{9}$$

As will be shown in the empirical part of this paper, the slope coefficient, $\beta_{2,mh}$, is negative as expected and has a significant pricing effect. The realized asymmetric volatility plays

both the median and the zero threshold, but find no qualitative differences.

a significant role and represents risk for which an investor has to be compensated. It is interesting to note that equation 6 can be rewritten as a two-factor model with realized volatility and the ARV measure as regressors.[3]

We run linear regressions and correct for heteroskedasticity and autocorrelation (heteroskedasticity and autocorrelation-consistent (HAC) standard errors). We use different lags for the HAC estimator ($m + h$ or $\max(m, h)$), but find they have no impact on our results.

2.2 Empirical Analysis

We estimate the asymmetric risk-return trade-off using high-frequency stock market returns from February 1986 to September 2010. The intra-daily equity market return series comes from Olsen Financial Technologies and is their longest available high-frequency return sample. We use this series of opening S&P 500 index prices as a proxy for the stock market return. To obtain the backward-looking risk measures, we create measures of the realized semivariances and the ARV measure.[4]

The analysis does not qualitatively depend on the frequency of intra-daily trades, but predictive power increases with frequency despite an increase in microstructure noise. To illustrate this, we perform our analysis on two different high-frequency returns series, a sixty-minute and a five-minute series. Table 1 displays summary statistics in the upper panel for both intra-daily and daily market return series on a non-aggregated level. We report summary statistics for the full sample and for a subsample from January 1990 to December 2007. We chose our subsample so that our results would be as comparable as possible to related literature [5]. The intra-daily annualized sixty-minute market return has a mean of 7.85% and an annualized standard deviation of 7.97%, and is negatively skewed as well as highly leptokurtic over the entire sample period. The intra-daily annualized five-

[3] The risk-return trade-off (equation 6) can be rewritten as

$$\frac{R_{t,t+h}}{h} = \alpha_{mh} + \beta_{1,mh} \frac{\sigma^2_{t-m,t}}{m} + \beta_{2,mh} s_{t-m,t} + \epsilon^m_{t,t+h}$$

but this would not add anything to the analysis, due to the nature of the linear dependence between realized volatility and its semivariances.

[4] We follow the literature (e.g. Drechsler and Yaron (2011) or Bollerslev et al. (2010)) and treat the overnight returns or returns over the weekend as one high-frequency return. For comparison, we also exclude these, but find this has no impact on our analysis.

[5] See for example Drechsler and Yaron (2011)

minute market return has a mean of 8.36% and an annualized standard deviation of 26.41%, is negatively skewed and has an even higher kurtosis of about 60. The subsample reflects similar properties, but with a mean return of over 9% for both frequencies and somewhat less kurtosis. This is not surprising since this sample excludes events such as Black Monday in 1987 and the recent financial crisis. The lower panel shows the annualized mean and standard deviation of the long-run realized backward-looking risk measures, for the sixty-minute return series, at a sample aggregation level of five years.[6] Annualized statistics are provided in squared percentages for the realized volatility, the realized semivolatilities, and the ARV measure. It is worth mentioning that the squared percentage return for downside volatility is twice as large as that for upside uncertainty. Further, there is no big difference between the sample periods; only the ARV measure is less negative in the subsample, which is not surprising given that some negative shocks are excluded.

[Table 1 about here!]

The main results of this paper are presented in Table 2. The return predictability regression results for the one-factor asset pricing model with the ARV measure (equation 4) are presented for the longest available respective samples of sixty-minute and five-minute equity market returns. There are two sets of rows and columns providing the OLS estimates of a regression with HAC standard errors.[7] The rows provide estimates for the sixty-minute and five-minute return series, for different levels of aggregation: one month (1M), two months (2M), three months (3M) and six months (6M). The columns show estimates of the ARV risk measure for two backward-looking aggregation periods. All slope coefficient estimates are significantly negative and increase in magnitude with an increasing level of aggregation, as well as with an increasing frequency. An increase in the asymmetry in returns has a positive pricing effect since it is defined as the difference between upside uncertainty and downside risk. We will show that the coefficient's sign and the R^2 are consistent with the theoretical model. The ARV measure explains 3.65% of the total

[6] For comparison, we vary the aggregation level and find no qualitative difference within a range of three to six years. For relatively short periods of aggregation, for example up to two years, the results become sample-dependent (small-sample property) and are therefore excluded from the analysis.

[7] We run an OLS regression on a daily rolling window. The results are robust for other rolling windows, such as a weekly rolling window. A monthly rolling window results in a substantial reduction in the number of observations, and inconsistent results over some samples and data frequencies.

variation over a monthly (1M) return aggregation period for the five-minute return series. This figure increases with an increase in the aggregation level of returns; for example, for the three-month (3M) return aggregation period, we are able to forecast 10.95%. While the R^2 (adjusted) is sensitive to the aggregation period, the significance of the positive pricing effect of our ARV measure is persistent.

[Table 2 about here!]

Table 3 is structured in the same way as the previous table and contains the regression results of a two-factor asset pricing model with upside uncertainty and downside risk (equation 6). The upside uncertainty has a significant negative coefficient, implying that upside potential has a negative pricing effect. Investors pay for upside potential. The downside risk has a significant positive coefficient, suggesting that downside risk has a positive pricing effect. Investors are compensated for bearing downside risk. These results are very persistent and hold for both different aggregation and sample periods. We will show that the coefficients' signs and R^2 valuesare consistent with the theoretical model. The two-factor semivariance predictive regression forecasts 4.88% of the total variation for a monthly (1M) return aggregation period, for the five-minute return series. This increases with increasing aggregation of returns; for example, for the quarterly (3M) return aggregation period, we are able to forecast 14.04%.

[Table 3 about here!]

Note that the realized variance is, by construction, the sum of the upside and downside semivariance. Since these have opposite pricing effects, we would expect, as can be observed in the empirical literature, to see an underestimation of the coefficient of relative risk aversion in the traditional risk-reward trade-off model. The empirical results presented suggest that the impact of the discount is virtually equal to, if not even bigger than, the price per standard deviation of additional risk. This could serve as an explanation of why the literature has found even negative relationships between risk and reward.

An R^2 of around 4% for the monthly return prediction is much bigger than the R^2 of the predictive variables investigated by Campbell and Thompson (2008), who state that the variables they investigate are potentially useful for investors. We provide further evidence addressing the natural question of whether an R^2 of the magnitude we obtain is economically significant. We follow Cochrane (1999), who uses a theorem by Hansen and Jagannathan (1991) to derive a relation between the maximum unconditional Sharpe ratio achievable by a predictive regression and its R^2. The relation is

$$S_{max} = \sqrt{\frac{S^2 + R^2}{1 - R^2}}$$

with S being the unconditional Sharpe ratio. Table 6 gives a detailed overview of all maximal Sharpe ratios for the ARV predictive regression (Model I) and the two-factor semivariance predictive regression (Model II). At a monthly frequency, based on the five-minute equity return series, the annualized unconditional Sharpe ratio, S, is 0.743. Given that the ARV predictive regression has an R^2 of 3.65%, the maximal Sharpe ratio is 1.013. We observe a similar picture for the bivariate predictive regression model and different return horizons. Putting this differently, the potential increase is fairly large and economically meaningful.

[Table 6 about here!]

2.3 Robustness Checks

This section provides evidence regarding the robustness of our results and relates this to other common predictors.

2.3.1 Related Risk Measures

Recent research on equity return predictability has focused on the forward-looking VRP as a measure of risk. This is the difference between implied volatility and realized volatility. We download the daily CBOE S&P 500 Volatility Index (VIX) series from January 1990 to December 2007, construct realized variances from our five-minute equity return series and compute first a daily and then, by taking the mean, a monthly VRP time series. Drechsler and Yaron (2011) find that the VRP accounts for about 1.46% – 4.07% of total variation for a monthly return aggregation level and 5.92% for a quarterly return aggregation level. Using the same sample period as them, from 1990 to 2007, we find that the VRP predicts 1.75% of quarterly return variation, but nothing for the monthly aggregation level. The ARV measure, however, forecasts 4.39% of monthly return variation and 12.47% of quarterly, for the five-minute return series, as shown in Table 4.

[Table 4 about here!]

Adding the ARV measure as a second factor, the VRP becomes insignificant, and predictability increases only marginally compared to a regression using the ARV measure only (Table 5). The price-earnings (P/E) ratio traditionally serves as a well-established predictor of equity market returns.[8] To construct a daily (P/E) ratio, we download daily S&P 500 value-weighted and equal-weighted returns, with and without distributions, from the CRSP database. The (P/E) ratio is given by the following relation:

$$\frac{P_t}{E_t} \approx \frac{P_t}{D_t} = \left(\frac{r_t^{with} - r_t^{without}}{1 + r_t^{without}} \right)^{-1}$$

The upper two panels of Table 5 give the estimation results for the (P/E) ratio only, as well as for the two-factor predictive regression using the (P/E) ratio and the ARV measure as regressors. The P/E ratio predicts 2.17% of monthly and 8.21% of quarterly total equity market return variation. Including our ARV measure as a second factor decreases the significance of the P/E ratio and the explanatory power increases only a little compared to the predictive regression using the ARV measure as the only factor. For the monthly return prediction, the P/E ratio becomes insignificant and predictability increases from 4.39% to 4.55%. For the quarterly, predictability increases from 12.47% to 13.71%.

[Table 5 about here!]

2.3.2 Alternative Dataset

Since our results and implications should not be driven by our equity return sample, we have tried to be as general as possible by applying the realized asymmetric volatility risk-return trade-off not only for different frequencies (sixty minutes and five minutes) and to a different subsample, but also to an entire different dataset. The daily value-weighted market return series from Kenneth French's Data Library (all NYSE, AMEX, and NASDAQ stocks

[8]The (P/E) ratio is highly persistent and skewed to the left. The literature shows that the (P/E) ratio predicts expected excess equity returns (see, e.g., Campbell and Shiller (1988)) if it does not predict dividend growth, which indeed it does not (Cochrane (2008)).

minus the one-month Treasury bill rate), from July 1963 to September 2010, serves as a second sample based on daily observations.

Bandi and Perron (2008) investigate the risk-return trade-off using daily realized volatility as their measure of risk and find significant evidence starting from a return aggregation period of six years.[9] In our asymmetric capital asset pricing model framework, we find that with this daily equity return series there are significant pricing effects, even using an aggregation period of just three months. Asymmetry matters in daily equity returns as well and taking it into consideration can increase predictive power markedly. Table 8 shows the estimation results for the daily return series. The estimation results for the ARV one-factor asset pricing model are presented in the top panel (MODEL I) and those for the semi-variances two-factor model in the bottom panel (MODEL II). We show results for return aggregation periods from one month (1M) up to three years (3Y). The estimates increase in magnitude as the level of return aggregation increases. The ARV measure forecasts nothing on a monthly return aggregation level, 0.19% on a quarterly return aggregation level and 9.76% on a three-year return aggregation level. Note that the two-factor model forecasts even less than the one-factor model for the daily data. To sum up, the same qualitative pattern for both the ARV one-factor asset pricing model and the realized semivariances two-factor asset pricing model is preserved for this alternative datasetThe advantage of using intra-daily data is that, with increasing data frequency, predictive power, particularly in the short run, also increases.

[Table 8 about here!]

2.3.3 Bootstrapping

Bandi and Perron (2008) show that an incorrect specification of HAC error terms can lead to over-rejections of the null, and spuriously increasing R^2 values. To provide further confidence that the stated t-statistics reflect the correct level, and no size distortions are leading to over-rejections of the null of zero slope, we show bootstrapped standard errors in Table 7. We perform the bootstrap 10,000 times and find, throughout the process, results that fully reflect the empirical findings presented earlier.

[9] We replicated Bandi and Perron (2008)'s results and found a very similar pattern.

[Table 7 about here!]

3 Rationalizing the Empirical Facts

In this section, we further strengthen our previous empirical results by showing that they are not a statistical fluke, and nor are they contrary to the asset pricing model, but in fact they reflect a rational economic model where agents care about consumption levels and volatility, and are aware of lower tail risk in consumption growth. In other words, in this section, we rationalize, in the context of a consumption-based reduced-form general equilibrium setting, the empirical evidence on return predictability by the ARV measure, as presented and discussed in the previous section. Our model borrows from Bonomo et al. (2011), who use a similar framework to analyze stock market behavior and the predictability of excess returns and growth rates by the dividend-price ratio. To be coherent and consistent with the intra-daily data frequency used previously, we construct model dynamics at intra-daily frequencies. We assume that there are $1/\Delta$ equally-spaced trading periods during a day, and that day t contains the periods $t - j\Delta, j = 0, 1, \ldots, 1/\Delta - 1$. For example, $\Delta = 1/8$ for sixty-minute and $1/85$ for five-minute periods. We base our model analysis on the frequency $\Delta = 1/8$, but our approach and model results are valid and hold for other intra-daily frequencies as well.

3.1 Model Setup, Assumptions and Asset Pricing Solution

3.1.1 Investor Preferences

We consider an endowment economy where the representative investor has the generalized disappointment aversion (GDA) preferences described in Routledge and Zin (2010). Following Epstein and Zin (1989), the investor derives utility from consumption, recursively, as follows:

$$\begin{aligned} V_t &= \left\{ (1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t \left(V_{t+\Delta} \right) \right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \\ &= C_t^{1-\delta} \left[\mathcal{R}_t \left(V_{t+\Delta} \right) \right]^{\delta} \quad \text{if } \psi = 1. \end{aligned} \tag{10}$$

The current period lifetime utility V_t is a combination of current consumption C_t, and $\mathcal{R}_t \left(V_{t+\Delta} \right)$, a certainty equivalent of next period lifetime utility. With GDA preferences, the risk-adjustment function $\mathcal{R}(\cdot)$ is implicitly defined by

$$\frac{\mathcal{R}^{1-\gamma}-1}{1-\gamma}=\int_{-\infty}^{\infty}\frac{V^{1-\gamma}-1}{1-\gamma}dF(V)-\left(\frac{1}{\alpha}-1\right)\int_{-\infty}^{\theta\mathcal{R}}\left(\frac{(\theta\mathcal{R})^{1-\gamma}-1}{1-\gamma}-\frac{V^{1-\gamma}-1}{1-\gamma}\right)dF(V), \tag{11}$$

where $0<\alpha\leq 1$ and $0<\theta\leq 1$. When α is equalto one, $\mathcal{R}$ becomes the Kreps and Porteus (1978) (henceforth KP) preferences, while V_t represents Epstein and Zin (1989)'s recursive utility. When $\alpha<1$, outcomes lower than $\theta\mathcal{R}$ receive an extra weight $(1/\alpha-1)$, which decreases the certainty equivalent. Therefore, the parameter α is interpreted as a measure of disappointment aversion, while the parameter θ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing.[10]. By overweighting outcomes below the disappointment threshold, the representative agent is downside risk sensitive. With KP preferences, the stochastic discount factor in terms of the continuation value of utility of consumption, is given by

$$M^{*}_{t,t+\Delta}=\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+\Delta}}{\mathcal{R}_t(V_{t+\Delta})}\right)^{\frac{1}{\psi}-\gamma}=\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}Z_{t+\Delta}^{\frac{1}{\psi}-\gamma}, \tag{12}$$

where

$$Z_{t+\Delta}=\frac{V_{t+\Delta}}{\mathcal{R}_t(V_{t+\Delta})}=\left(\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}R_{c,t+\Delta}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{13}$$

where the second equality in equation (13) implies an equivalent representation of the stochastic discount factor given by equation (12), based on consumption growth and the gross return $R_{c,t+\Delta}$ on a claim on the future aggregate consumption stream. In general, this return is unobservable.

For GDA preferences, the stochastic discount factor may be written as

$$M_{t,t+\Delta}=M^{*}_{t,t+\Delta}\left(\frac{I(Z_{t+\Delta}<\theta)+\alpha I(Z_{t+\Delta}\geq\theta)}{\eta E_t[I(Z_{t+\Delta}<\theta)]+\alpha E_t[I(Z_{t+\Delta}\geq\theta)]}\right), \tag{14}$$

where $\eta=\alpha+(1-\alpha)\theta^{1-\gamma}$ and $I(\cdot)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise.

[10]Notice that the certainty equivalent, besides being decreasing in γ, is increasing in α (for $0<\alpha\leq 1$), and decreasing in θ (for $0<\theta\leq 1$). Thus, α andθ are also measures of risk aversion, but of different types than γ.

3.1.2 Equilibrium Consumption and Dividend Growth Dynamics

Intra-daily consumption and dividend growth rates are assumed to be unpredictable and heteroskedastic, and their conditional variance and correlation change according to a Markov variable s_t, which takes N values, $s_t \in \{1, 2, \ldots, N\}$, when the economy is assumed to have N states of nature. The sequence s_t evolves according to a transition probability matrix P defined as follows:

$$P^\top = [p_{ij}]_{1 \le i,j \le N} \quad \text{and} \quad p_{ij} = Prob\,(s_{t+\Delta} = j \mid s_t = i)\,. \tag{15}$$

Following Hamilton (1994), let $\zeta_t = e_{s_t}$, where e_j is the$N \times 1$ vector with all components equal to zero except for the jth component which is equal to one.

Then, the dynamics of consumption and dividend growth are given by the following:

$$\begin{aligned} g_{c,t+\Delta} &= \ln\left(\frac{C_{t+\Delta}}{C_t}\right) = \mu_c + \sigma_t \varepsilon_{c,t+\Delta} \\ g_{d,t+\Delta} &= \ln\left(\frac{D_{t+\Delta}}{D_t}\right) = \mu_d + \sigma_{d,t} \varepsilon_{d,t+\Delta} \end{aligned} \tag{16}$$

where $\sigma_t = \sqrt{\omega_c^\top \zeta_t}$ and $\sigma_{d,t} = \nu_d \sigma_t = \sqrt{\omega_d^\top \zeta_t}$, and where

$$\begin{pmatrix} \varepsilon_{c,t+\Delta} \\ \varepsilon_{d,t+\Delta} \end{pmatrix} \mid \langle \varepsilon_{c,j\Delta}, \varepsilon_{d,j\Delta}, j \le t; \zeta_{m\Delta}, m \in \mathbb{Z} \rangle \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right). \tag{17}$$

The scalars μ_c, μ_d, ν_d and ρ are the expected consumption and dividend growth rates, the ratio of dividend volatility to consumption volatility, and the conditional correlation between consumption and dividend growths respectively. The vectors ω_c and $\omega_d = \nu_d^2 \omega_c$ contain the conditional variances of the consumption and dividend growth rates respectively, where the component j of a vector refers to the value in state $s_t = j$.

3.1.3 Asset Pricing Solution

Asset prices, for example, the price-dividend ratio $P_{d,t}/D_t$ (where $P_{d,t}$is the price of the portfolio that pays off an amount equal to the equity dividend), the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off an amount equal to consumption) and the risk-free simple gross return $R_{f,t+1}$, can be derived analytically in this model. To obtain these asset prices, we need expressions for $\mathcal{R}_t\,(V_{t+\Delta})\,/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and V_t/C_t, the ratio of lifetime utility to current consumption. The Markov property of the model is crucial for deriving analytical formulas for these expressions, and we adopt the following notation:

$$\frac{\mathcal{R}_t\left(V_{t+\Delta}\right)}{C_t} = \lambda_{1z}^{\top}\zeta_t, \quad \frac{V_t}{C_t} = \lambda_{1v}^{\top}\zeta_t, \quad \frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t \ \text{ and } \ R_{f,t+\Delta} = \frac{1}{\lambda_{1f}^{\top}\zeta_t}. \tag{18}$$

Solving these ratios amounts to characterizing the vectors λ_{1z}, λ_{1v}, λ_{1d} and λ_{1f} as functions of the parameters of the consumption and dividend dynamics and of the recursive utility function defined above. In Appendix A, we provide explicit analytical expressions for these ratios.

In particular, excess log equity return over the risk-free rate can also be written as

$$r_{t+\Delta} = \zeta_t^{\top}\Lambda\zeta_{t+\Delta} + \sqrt{\omega_d^{\top}\zeta_t}\varepsilon_{d,t+\Delta}, \tag{19}$$

where the components of matrix Λ are explicitly defined by

$$\nu_{ij} = \ln\left(\frac{\lambda_{1d,j}+1}{\lambda_{1d,i}}\right) + \mu_d + \ln\lambda_{1f,i}. \tag{20}$$

Given these endogenous high-frequency intra-daily returns, we derive analytical formulas to assess the model-implied univariate predictability of returns by the asymmetric realized variance, as well as the bivariate predictability by the combined upside and downside realized variances.

3.2 The Model-Implied Risk-Return Trade-off

3.2.1 Analytical Formulas for Assessing the Risk-Return Tradeoff

Population values of the drift coefficient α_{mh}, the slope coefficients $\beta_{1,mh}$ and $\beta_{2,mh}$, and the R^2 values of the predictive regressions (6) and (4) can also be derived analytically in this model. In particular, for specification (6), population values of the intercept α_{mh}, the slope vector $\beta_{mh} = \begin{pmatrix} \beta_{1,mh} & \beta_{2,mh} \end{pmatrix}^{\top}$ and the coefficient of determination R^2_{mh} are given by

$$\begin{aligned} \alpha_{mh} &= \frac{E\left[r_{t,t+h}\right]}{h} - \beta_{1,mh}\frac{E\left[\left(\sigma_{t-m,t}^{+}\right)^2\right]}{m} - \beta_{2,mh}\frac{E\left[\left(\sigma_{t-m,t}^{-}\right)^2\right]}{m} \\ \beta_{mh} &= \frac{m}{h}\Sigma_m^{-1}\Upsilon_{mh} \ \text{ and } \ R^2_{mh} = \frac{\Upsilon_{mh}^{\top}\Sigma_m^{-1}\Upsilon_{mh}}{Var\left[r_{t,t+h}\right]}, \end{aligned} \tag{21}$$

where the 2×2 symmetric matrix Σ_m and the 2×1 vector Υ_{mh} are defined by

$$\Sigma_m = \begin{bmatrix} Var\left[\left(\sigma^+_{t-m,t}\right)^2\right] & Cov\left(\left(\sigma^+_{t-m,t}\right)^2, \left(\sigma^-_{t-m,t}\right)^2\right) \\ Cov\left(\left(\sigma^+_{t-m,t}\right)^2, \left(\sigma^-_{t-m,t}\right)^2\right) & Var\left[\left(\sigma^-_{t-m,t}\right)^2\right] \end{bmatrix}$$

$$\Upsilon_{mh} = \begin{pmatrix} Cov\left(\left(\sigma^+_{t-m,t}\right)^2, r_{t,t+h}\right) \\ Cov\left(\left(\sigma^-_{t-m,t}\right)^2, r_{t,t+h}\right) \end{pmatrix}.$$

In the context of the reduced-form general equilibrium asset pricing model previously described, we provide analytical formulas for the population values defined in equation (21). These quantities are relevant for assessing the risk-return relation through the predictability regressions (6) and (4). The expected values in equation (21), as well as the components of the matrix Σ_m and of the vector Υ_{mh}, may be expressed in terms of the components of the mean vector, μ^X, and the autocovariance matrices, $\Gamma^X(l)$, of the stationary vector process

$$X_t = \begin{pmatrix} r_{t-1,t} & \sigma^2_{t-1,t} & \left(\sigma^-_{t-1,t}\right)^2 & \left(\sigma^+_{t-1,t}\right)^2 \end{pmatrix}^\top.$$

The components of the vector μ^X and the matrices $\Gamma^X(l)$ may in turn be expressed in terms of those of the mean and autocovariance matrices of the stationary vector process

$$Y_t = \begin{pmatrix} r_t & r_t^2 & \left(r_t^-\right)^2 & \left(r_t^+\right)^2 \end{pmatrix}^\top.$$

Finally, knowledge of the mean vector and the autocovariance matrices of the process Y_t is sufficient for analyzing the risk-return trade-off implied by equations (6) and (4). To avoid a lengthy mathematical exposition at this stage, these moments are explicitly derived in the appendix.

3.2.2 Model Calibration and Basic Asset Pricing Implications

Bonomo et al. (2011) calibrate the consumption process in each monthly decision interval to match the actual sample mean and volatility of real annual US consumption growth from 1930 to 2007. In each monthly decision interval, the mean of consumption growth is calibrated to $\mu_c^M = 0.15 \times 10^{-2}$ and its volatility, which is equal to $\sqrt{\mu_\sigma^M}$, where μ_σ^M is the mean of consumption volatility, is calibrated to $\sqrt{\mu_\sigma^M} = 0.7305 \times 10^{-2}$. The volatility of consumption volatility is $\sigma_\sigma^M = 0.6263 \times 10^{-4}$, and its persistence is $\phi_\sigma^M = 0.995$. The mean of monthly dividend growth is calibrated to $\mu_d^M = \mu_c^M = 0.15 \times 10^{-2}$ and its volatility, which is equal to ν_d^M times the volatility of consumption growth, is calibrated to $\nu_d^M = 6.42322$.

First, we consider the monthly calibration carried out by Bonomo et al. (2011), but we set the persistence of consumption volatility to $\phi_\sigma^M = 0.96$, which is lower than their value of 0.995, but matches closely the value estimated from US monthly real per capita consumption growth data from 1959 to 2010. From the monthly calibration, we derive a daily calibration that matches the monthly first and second moments, assuming 22 trading days per month. The mapping that expresses the daily parameters in terms of the monthly parameters is given in Appendix D.

Next, analogously, from the daily calibration we derive an intra-daily calibration that matches the daily first and second moments, assuming $1/\Delta$ trading periods per day. That is, we use an analogue mapping to express the intra-daily parameters in terms of the daily parameters. Finally, we vary the excess kurtosis of consumption volatility and discuss the sensitivity of the model results to this parameter. The model-implied annualized (time-averaged) mean, volatility and first-order autocorrelation of consumption growth are respectively 1.80%, 2.04% and 0.25%, and are consistent with the observed annual values of 1.88%, 2.21% and 0.46%, respectively. The corresponding values for dividend growth are respectively 1.80%, 13.24% and 0.25%, also consistent with the observed annual values of 1.57%, 13.69% and 0.14%, respectively. Finally, the model-implied correlation between consumption and dividend growth is 0.40, which matches the observed value of 0.59.

Following Routledge and Zin (2010), asset pricing implications are analyzed for an investor who exhibits GDA. This warrants the choice of relevant preference parameters, which are adopted from Bonomo et al. (2011). The latter authors were able to match stylized facts about the equity markets. We decided to restrict ourselves to their choice of parameters, as we wanted to see how our model would reproduce the empirical predictability pattern in Tables 2 and 3, conditional on matching the moments of the equity excess returns and the risk-free rate. The constant coefficient of relative risk aversion is set to $\gamma = 2.5$. The parameter of disappointment aversion α is equal to 0.3, implying that the ratio of the investor's marginal utility of wealth from non-disappointing to disappointing outcomes is 30%. In addition, θ, which defines the fraction of the certainty equivalent below which disappointment kicks in, is equal to 0.997. Bonomo et al. (2011) use a level of θ equal to 0.989, to match the stylized fact regarding asset prices. However, their decision interval is monthly. In order to remain consistent with the calibration results based on a sixty-minute interval, we need to adjust this parameter. The elasticity of intertemporal substitution (EIS) ψ is equal to 1.50, implying that the investor prefers the early resolution of uncertainty. The one-period subjective discount factor is kept constant at $\delta = 0.9989$ for a monthly frequency.

[Table 9 about here!]

To reproduce the empirical results, an important factor that the models must incorporate is the high excess kurtosis of (sixty-minute) consumption volatility, consistent with the high excess kurtosis found in high-frequency equity returns. By increasing the kurtosis of consumption volatility, however, we generate a lower and more volatile risk-free rate, as shown in Table 9, as well as a higher equity premium and more volatile excess equity returns. Overall, Table 9 shows that the model-implied moments of asset prices are comparable to their data counterparts. High kurtosis in consumption volatility is also consistent with large jumps in volatility, as in Drechsler and Yaron (2011), who show that this feature is necessary in the asset pricing model in order to reproduce the short-run predictability of excess returns by the VRP. Our benchmark calibration corresponds to a large jump in the sixty-minute consumption volatility, generated by an excess kurtosis of $\kappa_\sigma = 100$. We further compare our previous empirical findings to the model results corresponding to this scenario. We also show results for three alternative scenarios, corresponding to $\kappa_\sigma = 75$, $\kappa_\sigma = 50$ and $\kappa_\sigma = 25$.

3.2.3 Model Implications for the High-Frequency Risk-Return Trade-Off

Figure 1 shows the model-implied regression coefficients and R^2 values for the univariate predictability of one-, three- and six-month excess returns by the ARV. We compare the numbers in the figure to the numbers in the top panel of Table 2, which correspond to realized semivariance calculations based on sixty-minute returns. First, the slope coefficient is negative, as in the data, so that an increase in the long-run ARV forecasts low future returns. Second, the pattern and the magnitudes of the regression coefficients and the R^2 values are consistent with the empirical findings. The three top graphs in Figure 1 show that the model-implied regression coefficients increase with the maturity h of the predicted returns, just as in the data. Empirically, the five-year ARV forecasts one-, three- and six-month returns with regression coefficients of -0.11, -0.20 and -0.33 respectively. The corresponding numbers in the model are approximately -0.12 , -0.20 and -0.27. Meanwhile, the R^2 values are 1.18%, 4.00% and 10.66% in the data, with model-implied values of approximately 1.15%, 4.00% and 7.50%, as shown in the three bottom graphs in Figure 1, again matching the empirical values.

[Figure 1 about here!]

Similarly, Figure 2 shows the model-implied regression coefficients and R^2 values for the bivariate predictability of one-, three- and six-month excess returns by the upside and the downside semivariances. The numbers in the figure are compared to the numbers in the top panel of Table 3, corresponding to the realized semivariance calculations based on sixty-minute returns. First, the coefficient of upside realized semivariance is negative and that of downside realized semivariance is positive, as in the data, so that expected returns reflect a premium for downside variance and a discount for upside variance. Second, the pattern and magnitudes of the regression coefficients and R^2 values are also consistent with the empirical findings. The three top and three bottom graphs in Figure 2 show that the model-implied regression coefficients increase with the maturity h of the predicted returns, just as in the data. The R^2 values are 1.89%, 5.71% and 13.14% in the data, matched once again by their model-implied values of approximately 1.80%, 5.00% and 9.00%, as shown in the three bottom graphs of Figure 2. Overall, our empirical findings are validated by the theoretical predictions of our proposed model.

[Figure 2 about here!]

It is important to notice that a sufficiently high value for the excess kurtosis of consumption volatility is an important ingredient, ensuring the model generates the required empirical pattern and magnitude of regression coefficients and R^2. Another important feature of the model is (generalized) disappointment aversion. We present model results corresponding to an economy where the representative investor does not have an aversion to downside losses as he/she would with (generalized) disappointment aversion preferences. We consider a KP investor ($\alpha = 1$), with a risk aversion parameter $\gamma = 15$ and an elasticity of intertemporal substitution $\psi = 1.5$. Basic asset pricing implications in such an economy are given in Table 10. Overall, the KP preferences match the mean of the price-dividend ratio and the risk-free rate, and the mean and volatility of excess equity returns, for a sufficiently large value of excess kurtosis. However, the implied R^2 of the predictability of excess equity returns by the price-dividend ratio is less than 1%, inconsistent with the empirical findings. These model results corroborate earlier findings by Beeler and Campbell (2012) and Bonomo et al. (2011) in their empirical assessments of the long-run risk model of Bansal and Yaron (2004), based on KP preferences.

[Figure 3 about here!]

For the KP preferences, we plot the regression coefficients and R^2 values of the bivariate predictability of excess returns by the realized semivariances, in Figure 3. The two top graphs show that the coefficient of upside realized semivariance is negative and that of downside realized semivariance is positive, as in the data. However, the maximum model-implied R^2 in the bottom-right graph is approximatley 0.40% for all horizons of excess returns and realized semivariances. This shows that the KP preferences cannot reproduce the pattern and magnitudes found empirically, and that aversion to downside losses is an essential feature in making consistent theoretical predictions. The bottom-left graph of Figure 3 shows that the magnitude of the regression coefficient for the univariate predictability by the ARV is too small compared to the empirical values. Finally, we observe that jumps in volatility have an opposite effect on the theoretical prediction than they do in the case of (generalized) disappointment aversion preferences. Higher excess kurtosis of volatility generates a lower predictability R^2 when using KP preferences.

4 Conclusion

This paper offers evidence of the relationship between expected market returns and realized semivariances. The fundamental intuition behind our approach is based on the strong economic argument that expected returns should not only include rewards for accepting the risk of a potential loss, but also discounts for potential upside gains. In other words, an investor is willing to accept a lower expected return for upside potential, but requires a higher expected return for downside risk. We investigate this insight empirically with intra-daily high-frequency equity market returns, and find that upside volatility has a negative effect on expected returns, whereas downside volatility has a positive one. Economists know that investors care differently about potential downside losses versus potential upside gains. Investors require additional compensation for downside market movements. We show that asymmetry improves the predictive power of equity returns significantly. Asymmetry in returns does matter. We find evidence of this both using various subsamples and different high-frequency equity return series. With increasing frequency, short-run predictability also increases. We formalize our findings in a closed-form asset pricing model that incorporates disappointment aversion and time-varying consumption volatility. To reproduce the empirical results, the model should be able to match the high excess kurtosis found in high-frequency data, consistent with the possibility of jumps in volatility put forward in earlier studies. The model produces qualitatively similar results to our empirical studies. We propose a backward-looking ARV measure, which can easily be extracted from realized return series, and which outperforms the price-earnings ratio, as well as the forward-looking VRP, in capturing equity return variation.

Appendix

A Markov Chain, Stochastic Discount Factor and Valuation Ratios

The Markov chain is stationary with ergodic distribution and second moments given by:

$$\begin{aligned} &E\left[\zeta_t\right] = \Pi \in \mathbb{R}_+^N, \\ &E\left[\zeta_t \zeta_t^\top\right] = Diag\left(\Pi_1, .., \Pi_N\right) \text{ and } Var\left[\zeta_t\right] = Diag\left(\Pi_1, .., \Pi_N\right) - \Pi\Pi^\top, \end{aligned} \tag{A.1}$$

where $Diag\left(u_1, .., u_N\right)$ is the $N \times N$ diagonal matrix whose diagonal elements are u_1,..,u_N.

We show that the stochastic discount factor $M_{t,t+\Delta}$ can also be written as

$$M_{t,t+\Delta} = \delta^*_{t,t+\Delta} \exp\left(-\gamma g_{c,t+\Delta}\right) \left[1 + \left(\frac{1}{\alpha} - 1\right) I\left(g_{c,t+\Delta} < -g_{v,t+\Delta} + \ln\theta\right)\right] \tag{A.2}$$

where

$$\ln \delta^*_{t,t+\Delta} = \zeta_t^\top A \zeta_{t+\Delta} \quad \text{and} \quad g_{v,t+\Delta} = \zeta_t^\top B \zeta_{t+\Delta} \tag{A.3}$$

and where the components of matrices A and B are explicitly defined by

$$\begin{aligned} a_{ij} &= \ln\delta + \left(\frac{1}{\psi} - \gamma\right) b_{ij} - \ln\left[1 + \left(\frac{1}{\alpha} - 1\right)\theta^{1-\gamma} \sum_{j=1}^{N} p_{ij} \Phi\left(q_{ij}\right)\right] \\ b_{ij} &= \ln\left(\frac{\lambda_{1v,j}}{\lambda_{1z,i}}\right) \quad \text{and} \quad q_{ij} = \frac{-b_{ij} + \ln\theta - \mu_{c,i}}{\sqrt{\omega_{c,i}}}. \end{aligned} \tag{A.4}$$

Proposition A.1 ***Characterization of Welfare Valuation Ratios.*** *Let*

$$\frac{\mathcal{R}_t\left(V_{t+\Delta}\right)}{C_t} = \lambda_{1z}^\top \zeta_t \text{ and } \frac{V_t}{C_t} = \lambda_{1v}^\top \zeta_t$$

respectively denote the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. The components of the vectors λ_{1z} *and* λ_{1v} *are given by:*

$$\lambda_{1z,i} = \exp\left(\mu_{c,i} + \frac{1-\gamma}{2}\omega_{c,i}\right) \left(\sum_{j=1}^{N} p^*_{ij} \lambda_{1v,j}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{A.5}$$

$$\lambda_{1v,i} = \left\{(1-\delta) + \delta\lambda_{1z,i}^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \text{ if } \psi \neq 1 \text{ and } \lambda_{1v,i} = \lambda_{1z,i}^{\delta} \text{ if } \psi = 1, \tag{A.6}$$

where the matrix $P^{*\top} = \left[p^*_{ij}\right]_{1 \le i,j \le N}$ *is defined in A.10.*

Proposition A.2 ***Characterization of Asset Prices.*** *Let*

$$\frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_{1c}^{\top}\zeta_t \quad \text{and} \quad R_{f,t+\Delta} = \frac{1}{\lambda_{1f}^{\top}\zeta_t}$$

respectively denote the price-dividend ratio, the price-consumption ratio and the risk-free rate. The components of the vectors λ_{1d}, λ_{1c}, and λ_{1f} are given by:

$$\lambda_{1d,i} = \delta \left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma} \exp\left(\mu_{cd,i} + \frac{\omega_{cd,i}}{2}\right) \left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^{\top} P^{**} \left(Id - \delta A^{**}\left(\mu_{cd} + \frac{\omega_{cd}}{2}\right)\right)^{-1} e_i \tag{A.7}$$

$$\lambda_{1c,i} = \delta \left(\frac{1}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma} \exp\left(\mu_{cc,i} + \frac{\omega_{cc,i}}{2}\right) \left(\lambda_{1v}^{\frac{1}{\psi}-\gamma}\right)^{\top} P^{*} \left(Id - \delta A^{*}\left(\mu_{cc} + \frac{\omega_{cc}}{2}\right)\right)^{-1} e_i \tag{A.8}$$

$$\lambda_{1f,i} = \delta \exp\left(-\gamma\mu_{c,i} + \frac{\gamma^2}{2}\omega_{c,i}\right) \sum_{j=1}^{N} \tilde{p}_{ij}^{*} \left(\frac{\lambda_{1v,j}}{\lambda_{1z,i}}\right)^{\frac{1}{\psi}-\gamma} \tag{A.9}$$

where the vectors $\mu_{cd} = -\gamma\mu_c + \mu_d$, $\omega_{cd} = \omega_c + \omega_d - 2\gamma\rho \odot \sqrt{\omega_c} \odot \sqrt{\omega_d}$, $\mu_{cc} = (1-\gamma)\,\mu_c$, $\omega_{cc} = (1-\gamma)^2\,\omega_c$, and the matrices $P^{**\top} = \left[p_{ij}^{**}\right]_{1\leq i,j\leq N}$ and $\tilde{P}^{*\top} = \left[\tilde{p}_{ij}^{*}\right]_{1\leq i,j\leq N}$ as well as the matrix functions $A^{**}(u)$ and $A^{*}(u)$ are defined in (A.12), (A.13), (A.11) and (A.14), respectively. The vector e_i denotes the $N \times 1$ vector with all components equal to zero but the ith component is equal to one.

The components of the matrix $P^{*\top} = \left[p_{ij}^{*}\right]_{1\leq i,j\leq N}$ in (A.5) and (A.8), and the matrix function $A^{*}(u)$ also in (A.8) are defined by:

$$p_{ij}^{*} = p_{ij} \frac{1 + (1/\alpha - 1)\,\Phi\left(q_{ij} - (1-\gamma)\sqrt{\omega_{c,i}}\right)}{1 + (1/\alpha - 1)\,\theta^{1-\gamma} \sum_{j=1}^{N} p_{ij}\Phi\left(q_{ij}\right)} \tag{A.10}$$

$$A^{*}(u) = Diag\left(\exp\left(\left(\frac{1}{\psi} - \gamma\right) b_{11} + u_1\right), ..., \exp\left(\left(\frac{1}{\psi} - \gamma\right) b_{NN} + u_N\right)\right) P^{*}, \tag{A.11}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal random variable.

The matrix $P^{**\top} = \left[p_{ij}^{**}\right]_{1\leq i,j\leq N}$ in (A.7), and the matrix $\tilde{P}^{*\top} = \left[\tilde{p}_{ij}^{*}\right]_{1\leq i,j\leq N}$ in (A.9) have their components given by:

$$p_{ij}^{**} = p_{ij} \frac{1 + (1/\alpha - 1)\,\Phi\left(q_{ij} - \left(\rho_i \sqrt{\omega_{d,i}} - \gamma\sqrt{\omega_{c,i}}\right)\right)}{1 + (1/\alpha - 1)\,\theta^{1-\gamma} \sum_{j=1}^{N} p_{ij}\Phi\left(q_{ij}\right)} \tag{A.12}$$

$$\tilde{p}_{ij}^{*} = p_{ij} \frac{1 + (1/\alpha - 1)\,\Phi\left(q_{ij} + \gamma\sqrt{\omega_{c,i}}\right)}{1 + (1/\alpha - 1)\,\theta^{1-\gamma} \sum_{j=1}^{N} p_{ij}\Phi\left(q_{ij}\right)}. \tag{A.13}$$

The matrix function $A^{**}(u)$ in (A.7) is defined by:

$$A^{**}(u) = Diag\left(\exp\left(\left(\frac{1}{\psi} - \gamma\right) b_{11} + u_1\right), ..., \exp\left(\left(\frac{1}{\psi} - \gamma\right) b_{NN} + u_N\right)\right) P^{**}. \tag{A.14}$$

B Population Moments of the Daily Vector Process X

The autocovariance matrices of the vector process X_t are defined by

$$\Gamma^X(l) = Cov\left(X_t, X_{t+l}\right) = \begin{bmatrix} \gamma_{11}^X(l) & \gamma_{12}^X(l) & \gamma_{13}^X(l) & \gamma_{14}^X(l) \\ \gamma_{21}^X(l) & \gamma_{22}^X(l) & \gamma_{23}^X(l) & \gamma_{24}^X(l) \\ \gamma_{31}^X(l) & \gamma_{32}^X(l) & \gamma_{33}^X(l) & \gamma_{34}^X(l) \\ \gamma_{41}^X(l) & \gamma_{42}^X(l) & \gamma_{43}^X(l) & \gamma_{44}^X(l) \end{bmatrix}. \tag{A.15}$$

The variances of long-horizon returns, long-horizon realized variance and long-horizon realized semivariances, as well as their covariances, can be expressed as follows:

$$Var\left[\begin{pmatrix} r_{t,t+h} \\ \sigma_{t,t+h}^2 \\ \left(\sigma_{t,t+h}^-\right)^2 \\ \left(\sigma_{t,t+h}^+\right)^2 \end{pmatrix}\right] = Var\left[\begin{pmatrix} r_{t-h,t} \\ \sigma_{t-h,t}^2 \\ \left(\sigma_{t-h,t}^-\right)^2 \\ \left(\sigma_{t-h,t}^+\right)^2 \end{pmatrix}\right] = h\Gamma^X(0) + \sum_{l=1}^{h-1}(h-l)\left(\Gamma^X(l) + \Gamma^X(l)^\top\right). \tag{A.16}$$

The covariance of future long-horizon returns with past long-horizon realized variance can be expressed as follows:

$$Cov\left(\sigma_{t-m,t}^2, r_{t,t+h}\right) = \min(m,h) \sum_{l=\min(m,h)}^{\max(m,h)} \gamma_{21}^X(l) + \sum_{l=1}^{\min(m,h)-1} l\left(\gamma_{21}^X(l) + \gamma_{21}^X(m+h-l)\right), \tag{A.17}$$

and similar formulas are obtained for the covariances of future long-horizon returns with past long-horizon semivariances, $Cov\left(\left(\sigma_{t-m,t}^-\right)^2, r_{t,t+h}\right)$ and $Cov\left(\left(\sigma_{t-m,t}^+\right)^2, r_{t,t+h}\right)$, by replacing γ_{21}^X with γ_{31}^X and γ_{41}^X, respectively.

The covariance of past long-horizon returns with future long-horizon realized variance can be expressed as follows:

$$Cov\left(r_{t-m,t}, \sigma_{t,t+h}^2\right) = h \sum_{l=\min(m,h)}^{\max(m,h)} \gamma_{12}^X(l) + \sum_{l=1}^{\min(m,h)-1} l\left(\gamma_{12}^X(l) + \gamma_{12}^X(m+h-l)\right), \quad \text{(A.18)}$$

and similar formulas are obtained for the covariances of past long-horizon returns with future long-horizon semivariances, $Cov\left(r_{t-m,t}, \left(\sigma_{t,t+h}^-\right)^2\right)$ and $Cov\left(r_{t-m,t}, \left(\sigma_{t,t+h}^+\right)^2\right)$, by replacing γ_{12}^X with γ_{13}^X and γ_{14}^X, respectively.

We also have that $\forall l$ and $\forall n, q \in \{1,2,3,4\}$,

$$\gamma_{nq}^X(l) = \frac{1}{\Delta}\gamma_{nq}^Y\left(\frac{l}{\Delta}\right) + \sum_{j=1}^{1/\Delta-1}\left(\frac{1}{\Delta} - j\right)\left(\gamma_{nq}^Y\left(\frac{l}{\Delta}+j\right) + \gamma_{nq}^Y\left(\frac{l}{\Delta}-j\right)\right). \quad \text{(A.19)}$$

C Population Moments of the Intra-Daily Vector Process Y

The autocovariance matrices of the vector process Y_t are defined by

$$\Gamma^Y(j) = Cov\left(Y_t, Y_{t+j\Delta}\right) = \begin{bmatrix} \gamma_{11}^Y(j) & \gamma_{12}^Y(j) & \gamma_{13}^Y(j) & \gamma_{14}^Y(j) \\ \gamma_{21}^Y(j) & \gamma_{22}^Y(j) & \gamma_{23}^Y(j) & \gamma_{24}^Y(j) \\ \gamma_{31}^Y(j) & \gamma_{32}^Y(j) & \gamma_{33}^Y(j) & \gamma_{34}^Y(j) \\ \gamma_{41}^Y(j) & \gamma_{42}^Y(j) & \gamma_{43}^Y(j) & \gamma_{44}^Y(j) \end{bmatrix}. \quad \text{(A.20)}$$

We recall the property $\forall j \geq 0,\ E_t\left[\zeta_{t+j\Delta}\right] = P^j\zeta_t$. Let $Y_t^{(n)}$ denotes the nth component of the vector process Y_t, for example $Y_t^{(3)} \equiv \left(r_t^-\right)^2$.

We also recall that for a standard normal random variable ε and a real scalar z, one has $E\left[\varepsilon^n I(\varepsilon < z)\right] = f_n(z)$, where the sequence $\{f_n(z)\}_{n\in\mathbb{N}}$ is given by the linear recursion $f_n(z) = (n-1)f_{n-2}(z) - z^{n-1}\phi(z)$, with the two initial conditions $f_0(z) = \Phi(z)$ and $f_1(z) = -\phi(z)$, and where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the PDF and the CDF of a standard normal.

We now adopt the following notations, $\forall n, q \in \{1,2,3,4\}$:

$$\begin{aligned} E_t\left[Y_{t+\Delta}^{(n)} \mid \zeta_{m\Delta}, m \in \mathbb{Z}\right] &= \zeta_t^\top U^{(n)}\zeta_{t+\Delta}, \\ E_t\left[Y_{t+\Delta}^{(n)}Y_{t+\Delta}^{(q)} \mid \zeta_{m\Delta}, m \in \mathbb{Z}\right] &= \zeta_t^\top U^{(nq)}\zeta_{t+\Delta}. \end{aligned} \quad \text{(A.21)}$$

We show that:

$$
\begin{aligned}
&U^{(1)} = \Lambda \quad \text{and} \quad U^{(2)} = (\Lambda \odot \Lambda) + \omega_d e^{\top} \quad \text{and} \quad U^{(4)} = U^{(2)} - U^{(3)} \\
&U^{(3)} = (\Lambda \odot \Lambda) \odot f_0(Z) + 2\Lambda \odot \left(\sqrt{\omega_d} e^{\top}\right) \odot f_1(Z) + \left(\omega_d e^{\top}\right) \odot f_2(Z)
\end{aligned}
\tag{A.22}
$$

and where the matrix Z is defined by

$$
Z = \left(\mu_r e e^{\top} - \Lambda\right) \oslash \left(\sqrt{\omega_d} e^{\top}\right).
$$

We also show that:

$$
\begin{aligned}
U^{(11)} &= (\Lambda \odot \Lambda) + \omega_d e^{\top} \\
U^{(12)} &= U^{(21)} = (\Lambda \odot \Lambda \odot \Lambda) + 3\Lambda \odot \left(\omega_d e^{\top}\right) \\
U^{(13)} &= U^{(31)} = (\Lambda \odot \Lambda \odot \Lambda) \odot f_0(Z) + 3(\Lambda \odot \Lambda) \odot \left(\sqrt{\omega_d} e^{\top}\right) \odot f_1(Z) \\
&\qquad + 3\Lambda \odot \left(\omega_d e^{\top}\right) \odot f_2(Z) + \left(\left(\omega_d \odot \sqrt{\omega_d}\right) e^{\top}\right) \odot f_3(Z) \\
U^{(14)} &= U^{(41)} = U^{(12)} - U^{(13)} \\
U^{(22)} &= (\Lambda \odot \Lambda \odot \Lambda \odot \Lambda) + 6(\Lambda \odot \Lambda) \odot \left(\omega_d e^{\top}\right) + 3(\omega_d \odot \omega_d) e^{\top} \\
U^{(23)} &= U^{(32)} = U^{(33)} = (\Lambda \odot \Lambda \odot \Lambda \odot \Lambda) \odot f_0(Z) + 4(\Lambda \odot \Lambda \odot \Lambda) \odot \left(\sqrt{\omega_d} e^{\top}\right) \odot f_1(Z) \\
&\qquad + 6(\Lambda \odot \Lambda) \odot \left(\omega_d e^{\top}\right) \odot f_2(Z) + 4\Lambda \odot \left(\left(\omega_d \odot \sqrt{\omega_d}\right) e^{\top}\right) \odot f_3(Z) \\
&\qquad + \left((\omega_d \odot \omega_d) e^{\top}\right) \odot f_4(Z) \\
U^{(24)} &= U^{(42)} = U^{(44)} = U^{(22)} - U^{(23)} \\
U^{(34)} &= U^{(43)} = 0.
\end{aligned}
\tag{A.23}
$$

The matrix operators $\odot$ and $\oslash$ denote the component-by-component multiplication and division, respectively. Furthermore, functions of matrices are component-wise.

We also adopt the following notations, $\forall n, q \in \{1, 2, 3, 4\}$:

$$
\begin{aligned}
E_t\left[Y^{(n)}_{t+\Delta+j\Delta}\right] &= \left(\Psi^{(n)}_0\right)^{\top} P^j \zeta_t, \\
E_t\left[Y^{(n)}_{t+\Delta} Y^{(q)}_{t+\Delta+j\Delta}\right] &= \left(\Psi^{(nq)}_j\right)^{\top} \zeta_t, \quad \forall j \geq 0.
\end{aligned}
\tag{A.24}
$$

We show that, $\forall n, q \in \{1,2,3,4\}$:

$$\begin{aligned}
&\Psi_0^{(n)} \text{ is the diagonal of the matrix } U^{(n)}P, \\
&\Psi_0^{(nq)} \text{ is the diagonal of the matrix } U^{(nq)}P, \\
&\Psi_j^{(nq)} \text{ is the diagonal of the matrix } \left(U^{(n)} \odot \left(e\left(\Psi_0^{(q)}\right)^\top P^{j-1}\right)\right)P, \quad \forall j \geq 1.
\end{aligned} \tag{A.25}$$

Finally we have that, $\forall n, q \in \{1,2,3,4\}$:

$$\begin{aligned}
\mu_n^Y &= E\left[Y_t^{(n)}\right] = \left(\Psi_0^{(n)}\right)^\top \Pi, \\
\gamma_{nq}^Y(j) &= \left(\left(\Psi_j^{(nq)}\right)^\top \Pi\right) - \left(\left(\Psi_0^{(n)}\right)^\top \Pi\right)\left(\left(\Psi_0^{(q)}\right)^\top \Pi\right), \quad \forall j \geq 0.
\end{aligned} \tag{A.26}$$

D Model Calibration and Implications for the Risk-Return Tradeoff

Our calibration of the dynamics (16) is as follows. We first consider the random walk model, written at a monthly decision interval as:

$$\begin{aligned}
g_{c,t+1}^M &= \mu_c^M + \sigma_t^M \epsilon_{c,t+1}^M \\
g_{d,t+1} &= \mu_d^M + \nu_d^M \sigma_t \epsilon_{d,t+1}^M \\
\left(\sigma_{t+1}^M\right)^2 &= \left(1 - \phi_\sigma^M\right)\mu_\sigma^M + \phi_\sigma^M \left(\sigma_t^M\right)^2 + \nu_\sigma^M \epsilon_{\sigma,t+1}^M
\end{aligned} \tag{A.27}$$

where

$$\begin{pmatrix} \epsilon_{c,t+1}^M \\ \epsilon_{d,t+1}^M \\ \epsilon_{\sigma,t+1}^M \end{pmatrix} \mid J_t^M \sim \mathcal{NID}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho^M & 0 \\ \rho^M & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right).$$

Next, we consider an analogue model at a daily decision interval. This analogue is given by:

$$\begin{aligned}
g_{c,t+\Delta}^D &= \mu_c^D + \sigma_t^D \epsilon_{c,t+\Delta}^D \\
g_{d,t+\Delta}^D &= \mu_d^D + \nu_d^D \sigma_t^D \epsilon_{d,t+\Delta}^D \\
\left(\sigma_{t+\Delta}^D\right)^2 &= \left(1 - \phi_\sigma^D\right)\mu_\sigma^D + \phi_\sigma^D \left(\sigma_t^D\right)^2 + \nu_\sigma^D \epsilon_{\sigma,t+\Delta}^D
\end{aligned} \tag{A.28}$$

where $\Delta = 1/22$ and where

$$\begin{pmatrix} \epsilon^D_{c,t+\Delta} \\ \epsilon^D_{d,t+\Delta} \\ \epsilon^D_{\sigma,t+\Delta} \end{pmatrix} \mid J^D_t \sim \mathcal{NID}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho^D & 0 \\ \rho^D & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right).$$

Assuming that the aggregate daily processes $\sum_{j=1}^{1/\Delta} g^D_{c,t+j\Delta}$, $\sum_{j=1}^{1/\Delta} g^D_{d,t+j\Delta}$ and $\sum_{j=1}^{1/\Delta} \left(\sigma^D_{t+j\Delta}\right)^2$ have the same first and second moments as the monthly processes $g^M_{c,t+1}$, $g^M_{d,t+1}$ and $\left(\sigma^M_{t+1}\right)^2$, respectively, we show that parameters for the daily decision interval are defined in terms of the monthly parameters as:

$$\mu^D_c = \Delta\mu^M_c, \;\; \mu^D_d = \Delta\mu^M_d, \;\; \mu^D_\sigma = \Delta\mu^M_\sigma, \;\; \nu^D_d = \nu^M_d, \;\; \rho^D = \rho^M,$$
$$\phi^D_\sigma = \left(\phi^M_\sigma\right)^\Delta, \;\; \nu^D_\sigma = \nu^M_\sigma\sqrt{\Delta}\sqrt{\left(\frac{1-(\phi^M_\sigma)^{2\Delta}}{1-(\phi^M_\sigma)^2}\right) \Big/ \left(1 + \frac{2\phi^M_\sigma}{1-\phi^M_\sigma} - \frac{2\Delta\phi^M_\sigma\left(1-(\phi^M_\sigma)^{1/\Delta}\right)}{(1-\phi^M_\sigma)^2}\right)}, \tag{A.29}$$

with $\Delta = 1/22$.

Next, we use the same procedure (mapping) to express the intra-daily parameters in terms of the daily parameters, assuming $\Delta = 1/8$ (hourly frequency). Finally, we approximate the continuous volatility AR(1) dynamics with a discrete two-state Markov chain dynamics as described in Garcia et al. (2008). This leads to the high-frequency endowment dynamics (16).

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Table 1: Summary Statistics of High Frequency Returns.

This table of summary statistics shows the mean, standard deviation, skewness, kurtosis and number of observations for daily and intra-daily annualized expected returns, $r_{t,t+252}$. The upper panel provides statistics for daily, sixty-minute and five-minute returns for the full sample and a commonly used subsample. The lower panel shows the mean and standard deviation for the realized volatility, $\sigma^2_{t-m,t}$, upside volatility, $\left(\sigma^+_{t-m,t}\right)^2$, downside volatility, $\left(\sigma^-_{t-m,t}\right)^2$, and the asymmetric volatility measure, $s_{t-m,t}$, as squared percentages. The samples are computed on a five-year aggregation level.

		Mean	Std.	Skewness	Kurtosis	N
INTRA DAILY -60 MINUTES						
1986:02-2010:09	$r_{t,t+252}$	7.854	7.974	-0.274	34.145	43090
1990:01-2007:12	$r_{t,t+252}$	9.149	6.822	-0.101	13.922	31528
INTRA DAILY -5 MINUTES						
1986:02-2010:09	$r_{t,t+252}$	8.363	26.405	-0.260	64.254	484256
1990:01-2007:12	$r_{t,t+252}$	9.701	22.805	0.118	61.036	355370
DAILY						
1964:01-2010:06	$r_{t,t+252}$	3.668	2.487	-0.843	23.512	11831

		1986:02-2010:09		1990:01-2007:12	
RISK MEASURES		Mean	Std.	Mean	Std.
	$\sigma^2_{t-m,t}$	469.712	199.171	457.819	190.834
	$\left(\sigma^+_{t-m,t}\right)^2$	224.753	100.457	225.734	98.827
	$\left(\sigma^-_{t-m,t}\right)^2$	244.959	101.216	232.086	92.228
	$s_{t-m,t}$	-20.207	31.680	-6.352	11.302

Table 2: Estimates for Different Levels of Aggregation. 1986:02-2010:09

This table provides an overview of the regression results for the following one-factor asymmetric asset pricing model:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{2,mh} s_{t-m,t} + \epsilon^{m}_{t,t+h}.$$

where $s_{t-m,t}$ is the backward-looking realized asymmetric risk measure for an aggregation period m. The backward-looking aggregation period of the risk measures is either five years (Aggregation Level I) or four years (Aggregation Level II). The equity return aggregation periods, h, are one, two, three and six months. The upper panel provides estimates for sixty-minute high-frequency data and the lower panel provides estimates for five-minute high-frequency data.

		Aggregation Level I				Aggregation Level II			
		1M	2M	3M	6M	1M	2M	3M	6M
60 minutes returns									
	α_{hm}	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000
		(-0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)	(0.00)	(0.00)	(0.00)
	β_{hm}	-0.110*	-0.159*	-0.201*	-0.327*	-0.070†	-0.097‡	-0.125‡	-0.212*
		(-2.99)	(-3.01)	(-2.97)	(-3.22)	(-1.83)	(-1.97)	(-2.08)	(-2.51)
	R^2	1.18	2.48	4.00	10.66	0.45	0.91	1.52	4.45
5 minutes returns									
	α_{hm}	-0.000	-0.000	0.000	0.000	-0.000	-0.000	0.000	0.000
		(-0.00)	(-0.00)	(0.00)	(0.00)	(-0.00)	(-0.00)	(0.00)	(0.00)
	β_{hm}	-0.192*	-0.272*	-0.332*	-0.466*	-0.127‡	-0.181‡	-0.219‡	-0.342*
		(-3.20)	(-3.22)	(-3.30)	(-3.34)	(-2.30)	(-2.27)	(-2.24)	(-2.35)
	R^2	3.65	7.38	10.95	21.68	1.58	3.24	4.76	11.69

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 3: Estimates for Different Levels of Aggregation. 1986:02-2010:09

This table provides an overview of the regression results for the following one-factor asymmetric asset pricing model:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{1,mh}\frac{\left(\sigma^{+}_{t-m,t}\right)^2}{m} + \beta_{2,mh}\frac{\left(\sigma^{-}_{t-m,t}\right)^2}{m} + \epsilon^{m}_{t,t+h}.$$

where $\left(\sigma^{+}_{t-m,t}\right)^2$ is the potential upside gain and $\left(\sigma^{-}_{t-m,t}\right)^2$ is the potential downside loss. The backward-looking aggregation of the semivariances is either five years (Aggregation Level I) or four years (Aggregation Level II). The equity return aggregation periods, h, are one, two, three and six months. The upper panel provides estimates for sixty-minute high-frequency data and the lower panel for five-minute high-frequency data.

		Aggregation Level I				Aggregation Level II			
		1M	2M	3M	6M	1M	2M	3M	6M
60 minutes returns									
	α_{hm}	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000
		(0.00)	(0.00)	(0.00)	(0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)
	$\beta_{1,hm}$	-0.801*	-1.147*	-1.449*	-2.336*	-0.579‡	-0.791*	-0.983*	-1.519*
		(-3.31)	(-3.37)	(-3.35)	(-3.43)	(-2.28)	(-2.44)	(-2.59)	(-3.11)
	$\beta_{2,hm}$	0.738*	1.063*	1.350*	2.218*	0.510*	0.701*	0.879*	1.399*
		(3.34)	(3.21)	(3.09)	(3.06)	(2.36)	(2.49)	(2.62)	(2.88)
	R^2	1.89	3.73	5.71	13.14	1.47	2.68	3.94	8.13
5 minutes returns									
	α_{hm}	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000
		(0.00)	(0.00)	(0.00)	(0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)
	$\beta_{1,hm}$	-6.644*	-9.388*	-11.401*	-15.773*	-3.726*	-5.234*	-6.270*	-9.249*
		(-3.85)	(-3.78)	(-3.88)	(-3.96)	(-3.02)	(-2.95)	(-2.90)	(-2.88)
	$\beta_{2,hm}$	6.561*	9.272*	11.263*	15.603*	3.647*	5.125*	6.140*	9.096*
		(3.77)	(3.69)	(3.79)	(3.85)	(2.94)	(2.86)	(2.81)	(2.79)
	R^2	4.88	9.62	14.04	25.94	3.11	6.06	8.62	17.27

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 4: Estimates for Different Levels of Aggregation. 1990:01-2007:12

This table provides an overview of the regression results for the following one-factor asymmetric asset pricing model:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{2,mh} s_{t-m,t} + \epsilon^{m}_{t,t+h}.$$

where $s_{t-m,t}$ is the backward-looking realized asymmetric risk measure for aggregation period m. The backward-looking aggregation of the risk measures is five years. The equity return aggregation periods, h, are one, two, three and six months. The upper panel provides estimates for sixty-minute high-frequency data and the lower panel for five-minute high-frequency data.

		1M	2M	3M	6M
60 minutes returns					
	α_{hm}	0.000	-0.000	-0.000	-0.000
		(0.00)	(-0.00)	(-0.00)	(-0.00)
	β_{hm}	-0.173‡	-0.230‡	-0.271‡	-0.367*
		(-2.19)	(-2.16)	(-2.16)	(-2.43)
	R^2	2.93	5.22	7.29	13.43
5 minutes returns					
	α_{hm}	0.000	0.000	0.000	-0.000
		(0.00)	(0.00)	(0.00)	(-0.00)
	β_{hm}	-0.211*	-0.299*	-0.354*	-0.467*
		(-3.72)	(-3.98)	(-3.96)	(-4.09)
	R^2	4.39	8.89	12.47	21.79

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 5: Robustness Checks. 1990:01-2007:12

This table provides an overview of the regression results for the most commonly used predictors and the ARV measure. Panel 1 provides estimation results for the daily price-earnings ratio. Panel 2 gives results for the predictive regression using the price-earnings ratio and the ARV measure. Panel 3 contains regression results for the VRP. Finally, the bottom panel provides estimates for the two-factor predictive regression using the VRP and the ARV measure. The equity return aggregation periods, h, are one, two, three and six months. The table shows results for a five-minute return series.

		1M	2M	3M	6M
Price-Earnings-Ratio					
	α_{hm}	-0.000	-0.000	-0.000	-0.000
		(-0.00)	(-0.00)	(-0.00)	(-0.00)
	$\beta_{PE,hm}$	-0.149*	-0.216*	-0.288*	-0.373*
		(-4.48)	(-3.64)	(-4.21)	(-4.37)
	R^2	2.17	4.63	8.21	13.83
Price-Earnings-Ratio & ARV					
	α_{hm}	-0.000	-0.000	-0.000	-0.000
		(-0.00)	(-0.00)	(-0.00)	(-0.00)
	$\beta_{PE,hm}$	-0.051	-0.078†	-0.135†	-0.159†
		(-1.30)	(-1.72)	(-1.86)	(-1.93)
	$\beta_{ARV,hm}$	-0.184*	-0.257*	-0.280*	-0.377*
		(-2.96)	(-3.53)	(-2.97)	(-3.06)
	R^2	4.55	9.29	13.71	23.48
Variance Risk Premium					
	α_{hm}	0.000	0.000	0.000	-0.000
		(0.00)	(0.00)	(0.00)	(-0.00)
	$\beta_{VRP,hm}$	0.016	0.082	0.134†	0.219*
		(0.32)	(1.13)	(1.80)	(2.79)
	R^2	-0.04	0.61	1.75	4.72
Variance Risk Premium & ARV					
	α_{hm}	0.000	0.000	0.000	-0.000
		(0.00)	(0.00)	(0.00)	(-0.00)
	$\beta_{VRP,hm}$	-0.076	-0.037	-0.004	0.044
		(-1.23)	(-0.39)	(-0.05)	(0.49)
	$\beta_{ARV,hm}$	-0.240*	-0.313*	-0.356*	-0.450*
		(-3.28)	(-3.22)	(-3.23)	(-3.45)
	R^2	4.86	8.98	12.44	21.93

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 6: Economic Significance

This table shows the maximum achievable Sharpe ratios for the ARV trade-off (Model I) and the two-factor semivariance risk-reward trade-off (Model II), following Cochrane (1999), who uses a Hansen and Jagannathan (1991) theorem to derive a relation between the maximum unconditional Sharpe ratio achievable by a predictive regression, and its R^2. The relation is

$$S_{max} = \sqrt{\frac{S^2 + R^2}{1 - R^2}}$$

with S being the unconditional Sharpe ratio. The upper panel provides results for the 60 minutes return series and the lower panel for the 5 minutes return series.

		1M	2M	3M	6M
60 minutes returns					
	S	0.497	0.603	0.771	1.002
Model I	Smax	0.627	0.824	1.058	1.599
Model II	Smax	0.695	0.918	1.165	1.724
5 minutes returns					
	S	0.743	0.978	1.226	1.601
Model I	Smax	1.013	1.410	1.779	2.568
Model II	Smax	1.093	1.528	1.925	2.769

Table 7: Bootstrapping

This table shows bootstrap results for the ARV trade-off (Model I) and the two-factor semivariance risk-reward trade-off (Model II). We bootstrap $10,000$ times with the five-minute return series. The mean of all bootstraps, as well as the 95% confidence interval for each point estimate is provided.

		1M	[2.5% 97.5%]	2M	[2.5% 97.5%]	3M	[2.5% 97.5%]	6M	[2.5% 97.5%]
Model I									
	α_{hm}	0.000	[-0.000 0.000]	0.000	[-0.000 0.000]	0.000	[-0.000 0.000]	0.000	[-0.000 0.000]
		(0.00)		(0.00)		(0.00)		(0.00)	
	β_{hm}	-0.192*	[-0.217 -0.167]	-0.272*	[-0.297 -0.248]	-0.331*	[-0.354 -0.309]	-0.466*	[-0.485 -0.446]
		(-16.72)		(-23.14)		(-28.77)		(-40.80)	
	R^2	3.67		7.40		10.96		21.68	
Model II									
	α_{hm}	-0.000	[-0.000 0.000]	-0.000	[-0.000 0.000]	-0.000	[-0.000 0.000]	0.000	[-0.000 0.000]
		(-0.00)		(-0.00)		(-0.00)		(0.00)	
	$\beta_{1,hm}$	-6.646*	[-7.409 -5.862]	-9.393*	[-10.132 -8.634]	-11.402*	[-12.073 -10.732]	-15.775*	[-16.349 -15.191]
		(-18.21)		(-25.28)		(-31.48)		(-44.32)	
	$\beta_{2,hm}$	6.562*	[5.778 7.327]	9.278*	[8.517 10.013]	11.263*	[10.589 11.939]	15.605*	[15.016 16.183]
		(17.99)		(24.95)		(31.04)		(43.70)	
	R^2	4.92		9.66		14.07		25.96	

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 8: Estimates for Different Levels of Aggregation. 1964:01-2010:09

This table provides an overview of the regression results for the one-factor asymmetric asset pricing model (Model I) and for the two-factor upside-downside realized volatility asset pricing model (Model II) with daily equity market return data. The backward-looking aggregation, m, of the risk measures is seven years. The equity return aggregation periods, h, are one month, two months, three months, six months, one year, two years and three years.

Daily Data		1M	3M	6M	1Y	2Y	3Y
Realized Volatility							
	α_{hm}	-0.000	-0.000	0.000	-0.000	-0.000	-0.000
		(-0.00)	(-0.00)	(0.00)	(-0.00)	(-0.00)	(-0.00)
	β_{hm}	0.011	0.044	0.060	0.056	0.029	0.091
		(0.30)	(0.70)	(0.72)	(0.50)	(0.17)	(0.48)
	R^2	-0.42	-0.24	-0.07	-0.12	-0.37	0.36
Model I							
	α_{hm}	-0.000	-0.000	0.000	0.000	-0.000	-0.000
		(-0.00)	(-0.00)	(0.00)	(0.00)	(-0.00)	(-0.00)
	β_{hm}	-0.028	-0.079†	-0.114†	-0.146‡	-0.210‡	-0.319*
		(-1.21)	(-1.93)	(-1.88)	(-2.19)	(-2.22)	(-2.46)
	R^2	-0.35	0.19	0.88	1.71	3.97	9.76
Model II							
	α_{hm}	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
		(-0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)
	$\beta_{1,hm}$	-0.054	-0.124	-0.180	-0.246†	-0.385†	-0.515‡
		(-0.80)	(-1.42)	(-1.61)	(-1.69)	(-1.78)	(-1.97)
	$\beta_{2,hm}$	0.065	0.167‡	0.239‡	0.299*	0.408*	0.598*
		(1.24)	(2.14)	(2.11)	(2.43)	(2.42)	(2.55)
	R^2	-0.57	-0.02	0.66	1.51	4.14	9.76

Reported t-statistics are Newey-West (HAC) corrected. † indicates a 10% significance level, ‡ indicates a 5% significance level and $*$ indicates a 1% significance level.

Table 9: (GDA) Model Asset Pricing Implications

The entries in the top panel are the model preference parameters of the representative investor, and the excess kurtosis of the consumption volatility process. The entries in the bottom panel are the annualized (as percentages) population mean and volatility of the log price-dividend ratio, the risk-free return, and the equity log return in excess of the log risk-free rate. The last two rows show the slope coefficient and the R^2 of the predictability of five-year excess returns by the log price-dividend ratio.

	Data	GDA1	GDA2	GDA3	GDA4
Preferences					
δ		0.9989	0.9989	0.9989	0.9989
γ		2.5	2.5	2.5	2.5
ψ		1.5	1.5	1.5	1.5
α		0.3	0.3	0.3	0.3
θ		0.997	0.997	0.997	0.997
κ_σ		100	75	50	25
Asset Pricing Implications					
$E[pd]$	3.33	2.82	3.09	3.24	3.27
$\sigma[pd]$	0.44	0.22	0.21	0.18	0.13
$AC1[pd]$	0.94	0.73	0.73	0.73	0.73
$E[r_f]$	0.57	0.45	0.97	1.14	1.22
$\sigma[r_f]$	3.77	9.28	8.78	7.84	5.90
$E[r]$	5.50	7.74	5.64	4.72	4.44
$\sigma[r]$	20.25	30.52	28.16	25.47	21.72
$\beta(5Y)$	-0.08	-0.40	-0.36	-0.35	-0.34
$R^2(5Y)$	29.74	34.72	33.25	29.76	22.10

Table 10: (KP) Model Asset Pricing Implications

The entries in the top panel are the model preference parameters of the representative investor, and the excess kurtosis of the consumption volatility process. The entries in the bottom panel are the annualized (as percentages) population mean and volatility of the log price-dividend ratio, the risk-free return, and the equity log return in excess of the log risk-free rate. The last two rows show the slope coefficient and the R^2 of the predictability of five-year excess returns by the log price-dividend ratio.

	Data	KP1	KP2	KP3	KP4
Preferences					
δ		0.9989	0.9989	0.9989	0.9989
γ		15	15	15	15
ψ		1.5	1.5	1.5	1.5
α		1	1	1	1
θ		1	1	1	1
κ_σ		100	75	50	25
Asset Pricing Implications					
$E[pd]$	3.33	3.33	3.50	3.77	4.15
$\sigma[pd]$	0.44	0.02	0.02	0.02	0.02
$AC1[pd]$	0.94	0.73	0.73	0.73	0.73
$E[r_f]$	0.57	0.04	0.79	1.38	1.71
$\sigma[r_f]$	3.77	0.79	0.85	0.90	0.92
$E[r]$	5.50	5.33	4.03	2.70	1.67
$\sigma[r]$	20.25	16.32	16.34	16.37	16.32
$\beta(5Y)$	-0.08	-0.35	-0.34	-0.33	-0.37
$R^2(5Y)$	29.74	0.61	0.74	0.87	0.68

Figure 1: (GDA) Model-Implied Regression Slope and R^2 of the Predictability of Excess Returns by the ARV.

This figure plots the model-implied population slope and R^2 for the univariate predictability of excess returns by the asymmetric realized variance, as a function of the backward-looking aggregation of the risk measures (m, in years). The risk measures are based on sixty-minute returns. The three top graphs show the slope coefficient, for given equity return aggregation periods h of one month, three months and six months. The three bottom graphs show the corresponding predictability, R^2. The representative investor has GDA preferences.

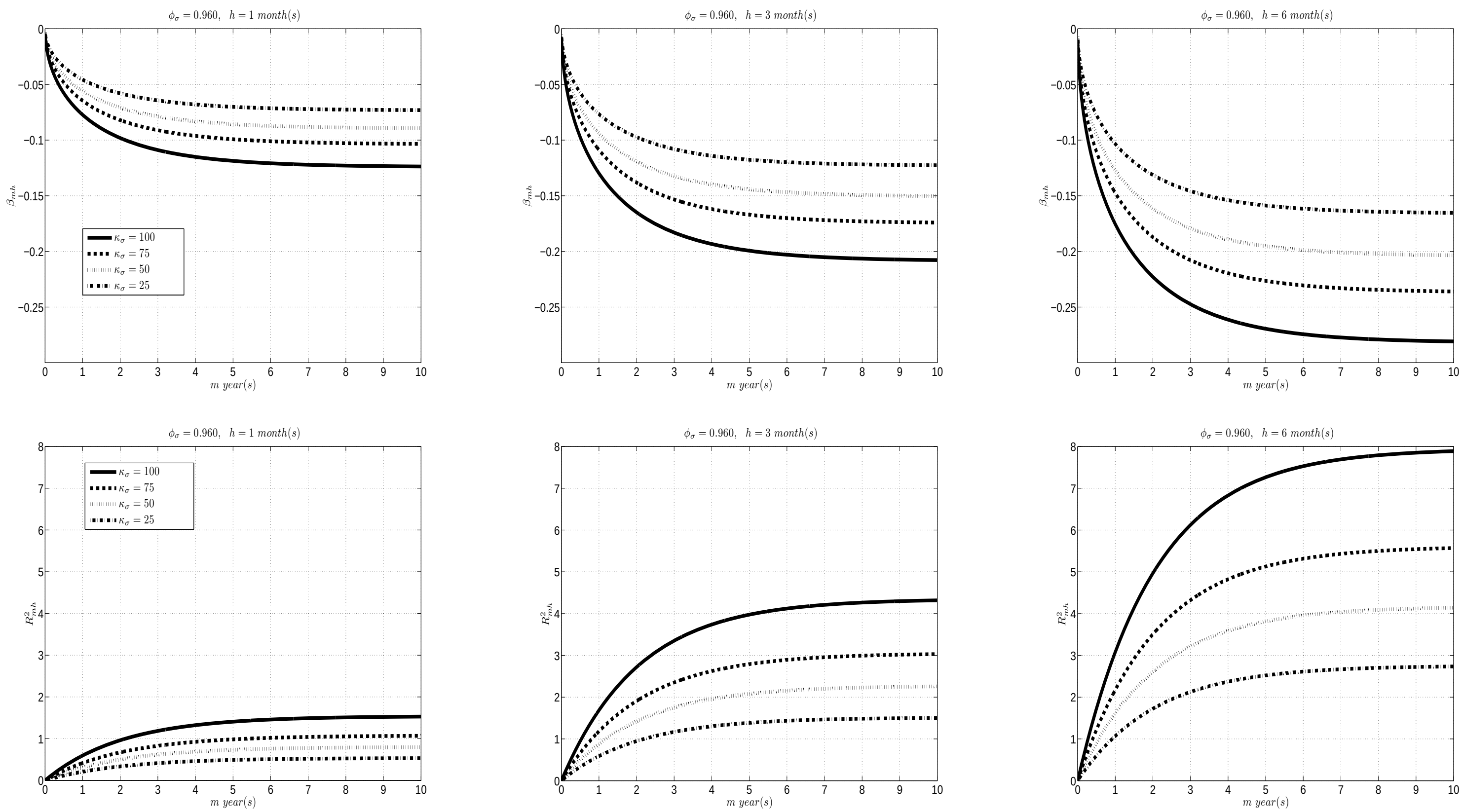


Figure 2: (GDA) Model-Implied Regression Slope and R^2 of the Predictability of Excess Returns by the Upside and Downside Realized Semivariances.

This figure plots the model-implied population slopes and R^2 values for the bivariate predictability of excess returns by the upside and downside realized semivariances (based on sixty-minute returns), as a function of the backward-looking aggregation of the risk measures (m, in years). The three top graphs show the coefficients of upside realized semivariance, for given equity return aggregation periods h of one, three and six months. The three middle graphs show the coefficients of downside realized semivariance, while the three bottom graphs show the corresponding predictability, R^2. The representative investor has GDA preferences.

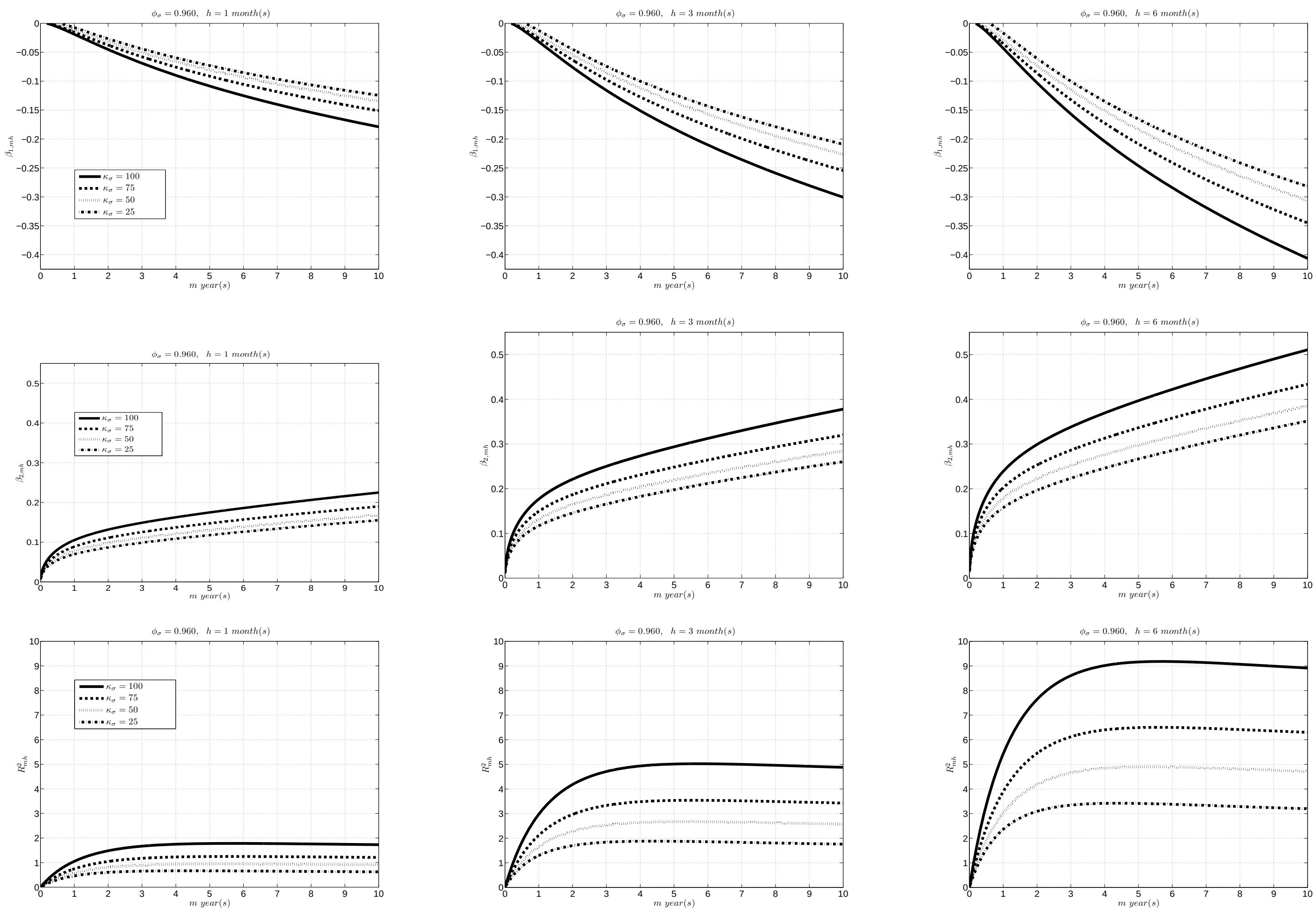
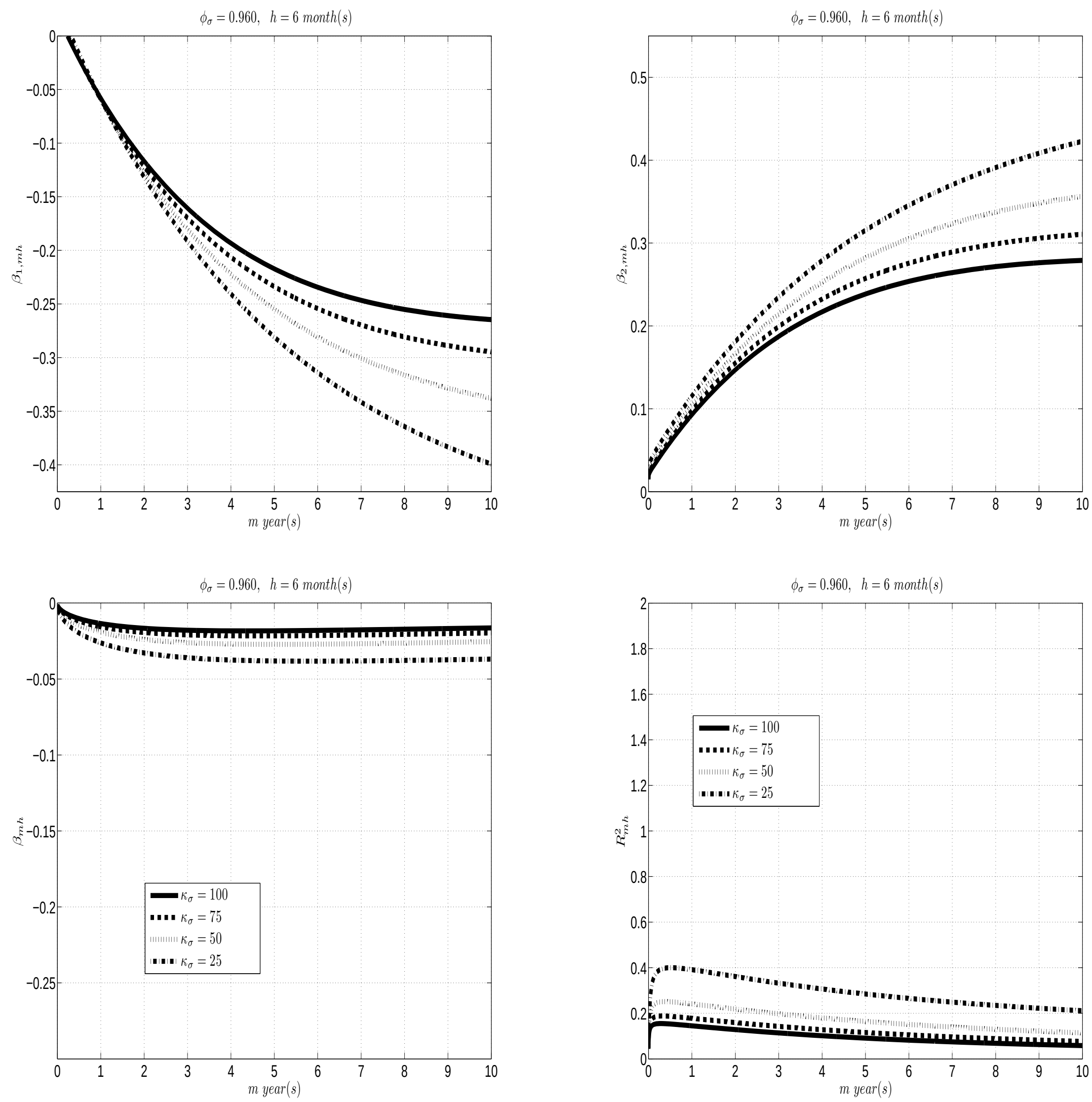


Figure 3: **(KP) Model-Implied Regression Slope and R^2 of the Predictability of Excess Returns by the Upside and Downside Realized Semivariances.**



This figure plots the model-implied population slopes for the bivariate predictability of excess returns by the upside semivariance (top-left graph) and the downside realized semivariance (top-right graph), as well as the corresponding R^2 (bottom-right graph) as a function of the backward-looking aggregation of the risk measures (m, in years), for a given equity return aggregation period h of six months. The model-implied population slope for the univariate predictability of excess returns by the ARV is shown in the bottom-left graph. The risk measures are based on sixty-minute returns. The representative investor has KP preferences.