Downside Risks and the Cross-Section of Asset Returns*

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Abstract

In an intertemporal equilibrium asset pricing model featuring disappointment aversion and changing macroeconomic uncertainty, we show that besides the market return and market volatility, three disappointment-related factors are also priced: a downstate factor, a market downside factor, and a volatility downside factor. We find that expected returns on various asset classes reflect premiums for bearing undesirable exposures to these factors. The signs of estimated risk premiums are consistent with the theoretical predictions. Our most general, five-factor model is very successful in jointly pricing stock, option, and currency portfolios, and provides considerable improvement over nested specifications previously discussed in the literature.

Keywords: Generalized Disappointment Aversion, Downside Risks, Cross-Section
JEL Classification: G12, C12, C31, C32

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1. Introduction

Downside risk refers to the risk of an asset or portfolio in case of an adverse economic scenario. Upside uncertainty is the analogue if the scenario is favorable. The asymmetric treatment of downside risk versus upside uncertainty by investors has long been accepted among practitioners and academic researchers (see, e.g., Roy, 1952; Markowitz, 1959), and has led to the development of new concepts in asset pricing and risk management, like the value-at-risk and the expected shortfall. Theories of rational behavior have been developed, where investors place greater weights on adverse market conditions in their utility functions. These include the lower-partial moment framework of Bawa and Lindenberg (1977), the loss aversion of Kahneman and Tversky (1979) in their prospect theory, and the disappointment aversion of Gul (1991), which has been generalized by Routledge and Zin (2010). These theories suggest priced downside risks in the capital market equilibrium.

We derive and test the cross-sectional predictions of a consumption-based asset pricing model where the representative investor has generalized disappointment aversion (GDA) preferences and macroeconomic uncertainty is time-varying. In a setting without disappointment aversion, two factors are priced in the cross-section: the market return (r_W) and changes in market volatility ($\Delta\sigma_W^2$). That is, investors require two premiums to invest in an asset. The first one is a compensation for covariation with the market return, $Cov\left(R_i^e, r_W\right)$, which is line with the prediction of the CAPM. The second premium is a compensation for covariation with changes in market volatility, $Cov\left(R_i^e, \Delta\sigma_W^2\right)$. It has been shown by previous empirical studies that volatility risk is priced in the cross-section (see, e.g., Ang et al., 2006b; Adrian and Rosenberg, 2008).

Our main theoretical contribution is to show that when disappointment aversion is added to the framework, investors require three additional premiums as compensation for exposures to disappointment-related risk factors. The first premium is a compensation for the covariance with the downstate factor, $Cov\left(R_i^e, I\left(\mathcal{D}\right)\right)$. The downstate factor, $I\left(\mathcal{D}\right)$, takes the value 1 if disappointment sets in and 0 otherwise. The model suggests that disappointment ($\mathcal{D}$) may set in due to two reasons: a sufficiently large fall in market return or rise in market volatility. The second premium is a compensation for the covariance with the interaction of the market return and the downstate factor, $Cov\left(R_i^e, r_W I\left(\mathcal{D}\right)\right)$. This factor represents movements of the market return in the downstate and we refer to it as the market downside factor. The third premium is a compensation for the covariance with the interaction of changes in market volatility and the downstate factor, $Cov\left(R_i^e, \Delta\sigma_W^2 I\left(\mathcal{D}\right)\right)$. This factor represents changes in market volatility in the downstate and we refer to it as the volatility downside factor.

In the general case, our setting thus leads to a five-factor model. Although there are five factors in the model, only two time series, the market return (r_W) and changes in market volatility ($\Delta\sigma_W^2$) are needed to construct these factors: the downstate factor is constructed as a function of these two series, and the two downside factors are simply interactions with the downstate factor. We also show that if the representative investor has infinite elasticity of intertemporal substitution, then market volatility has no role in the model, and the disappointing event reduces to a fall of the market return below a reference threshold. This special case corresponds to a three-factor model with the market, the downstate, and the market downside factors.

The cross-sectional implications of downside risk have already been studied, most notably, by Ang et al. (2006a) and Lettau et al. (2014). Our three-factor model nests the models from both of these studies, with different restrictions on the premium corresponding to the downstate factor. We explicitly derive these restrictions and confront them with the data. Our results suggest that the restrictions imposed by the downside risk models of Ang et al.

(2006a) and Lettau et al. (2014) are not supported empirically. Therefore, our three-factor model provides considerable improvement in explaining the cross-section of different asset returns, even though all three models use exactly the same information.

The more general five-factor model emphasizes the role of volatility in understanding downside risks. To our knowledge, little or no attention has been paid to volatility downside risk in the literature. We argue that volatility downside risk is also an important factor in explaining the cross-section of asset returns, as the five-factor model provides further improvement compared to the three-factor model.

We use the generalized method of moments (GMM) to empirically investigate the performance of our three- and five-factor models. Our benchmark test assets are various portfolios formed from US stocks, index option portfolios sorted on type, maturity, and moneyness, and currency portfolios sorted on their respective interest rates. These portfolios exhibit large heterogeneity in their average returns, and thus are ideal for cross-sectional asset pricing tests. The main empirical results of the paper relate to the pricing of the disappointment-related risk factors.

All the disappointment-related factors have significant risk premiums and the signs on the risk prices are in line with the theoretical predictions. In terms of pricing errors, when tested on all asset classes jointly, our three-factor model with a root-mean-squared-pricing error (RMSPE) of 20 basis points (bps) per month provides a significant improvement over the CAPM with a RMSPE of 50bps. The corresponding pricing errors of the downside risk models of Ang et al. (2006a) (28bps) and Lettau et al. (2014) (33bps) are considerably higher than that of the three-factor model. Our five-factor model, with a RMSPE of 17bps, largely outperforms a two-factor model with market return and changes in market volatility with a RMSPE of 27bps. Moreover, the five-factor GDA model also outperforms the four-factor model of Carhart (1997) on all asset classes except for stock portfolios. Also, the GDA model has the benefit of being motivated by dynamic consumption-based equilibrium asset pricing and behavioral decision theories, rather than being motivated by asset pricing anomalies themselves. These findings suggest the importance of disappointment-related risk in the cross-section of asset returns. Our results are robust to using additional asset classes and test portfolios, to alternative specifications of the disappointing event, and to alternative measures of market volatility.

This paper contributes to the developing literature that attempts to provide empirical support for the recent generalization by Routledge and Zin (2010) of the axiomatic disappointment aversion framework of Gul (1991). In the literature, GDA preferences have appeared in consumption-based equilibrium models mainly with the goal of explaining the time series behavior of the aggregate stock market, and rarely in cross-sectional asset pricing studies.[1] One exception is Delikouras (forthcoming), who also studies the cross-sectional implications of a consumption-based model with disappointment aversion preferences. There are several differences between our study and that of Delikouras (forthcoming). First, he uses annual and quarterly consumption data. In contrast, we substitute out consumption in a way similar to Campbell (1993), and rely on the market return. We can then avoid potential measurement problems in consumption data (of the types advocated, for example, by Wilcox, 1992), or delayed responses of consumption to financial market news (as discussed, for example, by Parker and Julliard, 2005), and test the model at the monthly frequency using market return data. Second, he uses the original version of disappointment aversion as introduced by Gul (1991), while we use the generalized version of Routledge and Zin (2010). Our results, when considering different disappointment thresholds, suggest that the

[1] For instance, Bonomo et al. (2011) show that persistent shocks to consumption volatility are sufficient when coupled with GDA preferences to produce moments of asset prices and predictability patterns that are in line with the data. Schreindorfer (2014) aims at explaining properties of index option prices, equity returns, variance, and the risk-free rate using the GDA model and a heteroscedastic random walk for consumption with the multifractal process of Calvet and Fisher (2007). Delikouras (2014) uses the GDA model to explain the credit spread puzzle.

generalized version is more appropriate in a representative agent setting. Third, Delikouras (forthcoming) assumes constant volatility of aggregate consumption, while our setting also allows for time-varying macroeconomic uncertainty. This feature is supported empirically (see for example Bansal et al., 2005) and it gives rise to the volatility related premiums in our cross-sectional model. Finally, since we derive the cross-sectional implications in the form of a factor model and rely on market return rather than consumption, our results are directly comparable to recent cross-sectional studies on downside risks such as Ang et al. (2006a) and Lettau et al. (2014).

The remainder of this paper is organized as follows. In Section 2, we present the theoretical setup from which we derive the implied cross-sectional model. Section 3 contains the empirical analysis with several robustness checks. Section 4 concludes, while the Appendix contains the description of the data sources and some technical derivations. An Online Appendix contains additional details that are omitted from the main text for brevity.

2. Theoretical motivation

We consider an economy where the representative investor has recursive utility as in Epstein and Zin (1989) and Weil (1989)

$$V_{t-1} = \begin{cases} \left[(1-\delta)\, C_{t-1}^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_{t-1}\left(V_t\right) \right]^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} & \text{if } \psi > 0 \;\; \text{and} \;\; \psi \neq 1 \\ C_{t-1}^{1-\delta} \left[\mathcal{R}_{t-1}\left(V_t\right) \right]^{\delta} & \text{if } \psi = 1 \end{cases}, \tag{1}$$

with $t = 1, 2, ...$, and where $0 < \delta < 1$ is the parameter of time preference and $\psi > 0$ is the elasticity of intertemporal substitution. The lifetime utility at $t-1$, V_{t-1}, is a function of the period's consumption, C_{t-1}, and the certainty equivalent of next period's lifetime utility, $\mathcal{R}_{t-1}\left(V_t\right)$. Routledge and Zin (2010) embed generalized disappointment aversion (GDA) into this framework by assuming that the certainty equivalent $\mathcal{R}_{t-1}$ is implicitly defined by

$$U\left(\mathcal{R}_{t-1}\right)=E_{t-1}\left[U\left(V_t\right)\right]-\ell E_{t-1}\left[\left(U\left(\kappa \mathcal{R}_{t-1}\right)-U\left(V_t\right)\right) I\left(V_t<\kappa \mathcal{R}_{t-1}\right)\right] \ , \tag{2}$$

where $E_t\left[\cdot\right]$ denotes the expectation conditional on information up to time t. The utility function, U, is defined as

$$U\left(x\right)=\begin{cases}\frac{x^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \geq 0 \ \text{ and } \ \gamma \neq 1 \\ \ln x & \text{if } \gamma = 1\end{cases} \ , \tag{3}$$

where the parameter $\gamma \geq 0$ is the coefficient of relative risk aversion. When ℓ is equal to zero, $\mathcal{R}_{t-1}$ reduces to expected utility (EU) preferences and V_{t-1} represents the Epstein and Zin (1989) recursive utility. GDA preferences are a two-parameter extension of the EU framework. When $\ell > 0$, outcomes lower than $\kappa\mathcal{R}_{t-1}$ receive an extra weight, decreasing the certainty equivalent. The larger weight given to these bad outcomes implies an aversion to losses. The parameter $\ell \geq 0$ is interpreted as the degree of disappointment aversion, while the parameter $0 < \kappa \leq 1$ is the percentage of the certainty equivalent such that outcomes below it are considered disappointing. The special case $\kappa = 1$ corresponds the original disappointment aversion preferences of Gul (1991).

The investor maximizes the lifetime utility subject to the budget constraint

$$W_t=\left(W_{t-1}-C_{t-1}\right) R_{Wt} \ , \tag{4}$$

where W_{t-1} is the wealth in period $t-1$ and R_{Wt} is the simple gross return on wealth, which we refer to as the market return. Following Hansen et al. (2007), Routledge and Zin (2010), and Bonomo et al. (2011), the stochastic discount factor (SDF) between periods $t-1$ and t

in the model with generalized disappointment aversion is

$$M_{t-1,t}^{GDA} = M_{t-1,t}\left(\frac{1+\ell I\left(\mathcal{D}_t\right)}{1+\kappa^{1-\gamma}\ell E_{t-1}\left[I\left(\mathcal{D}_t\right)\right]}\right) \ , \tag{5}$$

where $I\left(\cdot\right)$ denotes the indicator function taking the value 1 if the condition is met and 0 otherwise, and

$$M_{t-1,t} = \delta\left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\psi}}\left(\frac{V_t}{\mathcal{R}_{t-1}\left(V_t\right)}\right)^{\frac{1}{\psi}-\gamma} \quad \text{and} \quad \mathcal{D}_t = \left\{V_t < \kappa\mathcal{R}_{t-1}\left(V_t\right)\right\} \ , \tag{6}$$

where $M_{t-1,t}$ is the SDF without disappointment aversion ($\ell = 0$), and $\mathcal{D}_t$ denotes the disappointing event. The logarithm of $M_{t-1,t}$ and $\mathcal{D}_t$ may be written as

$$\ln M_{t-1,t} = \ln\delta - \gamma\Delta c_t - \left(\gamma - \frac{1}{\psi}\right)\Delta z_{Vt} \quad \text{and} \quad \mathcal{D}_t = \left\{\Delta c_t + \Delta z_{Vt} < \ln\kappa\right\}, \tag{7}$$

where $\Delta c_t \equiv \ln\left(\frac{C_t}{C_{t-1}}\right)$ and $\Delta z_{Vt} \equiv \ln\left(\frac{V_t}{C_t}\right) - \ln\left(\frac{\mathcal{R}_{t-1}(V_t)}{C_{t-1}}\right)$ represent the change in the log consumption level (or consumption growth) and the change in the log welfare valuation ratio (or welfare valuation ratio growth), respectively.

For every asset i, optimal consumption and portfolio choice by the representative investor induces a restriction on the simple excess return R_{it}^e that is implied by the Euler condition:

$$E_{t-1}\left[M_{t-1,t}^{GDA}R_{it}^e\right] = 0 \tag{8}$$

In the special case when $\ell = 0$ and $\gamma = 1/\psi$, the moment condition (8) is readily testable by GMM using actual data on aggregate consumption growth and asset returns. Earlier results for this test of the standard model are presented in Hansen and Singleton (1982, 1983). In the general case, however, the moment condition (8) is not directly testable by GMM since the continuation value is not observable from the data. Following the long-run risks asset pricing literature pioneered by Bansal and Yaron (2004), an assumed endowment dynamics can be exploited, together with the utility recursion (1) and the certainty equivalent definition (2), to express welfare valuation ratios in terms of economic state variables such as aggregate volatility, which may be measured or estimated from the data.

2.1. Cross-sectional implications

In order to obtain the cross-sectional implications that form the basis of our empirical investigation, we make two substitutions in the expressions for $\ln M_{t-1,t}$ and $\mathcal{D}_t$ in (7). First, we substitute out consumption growth following Epstein and Zin (1989), Hansen et al. (2007) and Routledge and Zin (2010) who show that in equilibrium, the market return is related to consumption growth and the welfare valuation ratio growth through[2]

$$r_{Wt} = -\ln\delta + \Delta c_t + \left(1 - \frac{1}{\psi}\right)\Delta z_{Vt} \ . \tag{9}$$

Second, assuming that aggregate consumption growth is heteroscedastic and unpredictable as in Bollerslev et al. (2009), Tauchen (2011) and Bonomo et al. (2011), and consistent with the empirical evidence presented in Beeler and Campbell (2012) among many others, we can solve for the welfare valuation ratio endogenously and express the welfare valuation ratio growth, $\Delta z_{V,t}$, in terms of changes in the volatility of the market return, which we refer to as market volatility.

Making these substitutions, and after some algebraic manipulation, the Euler equation

[2]The true market return is unobservable and an emprical proxy is later used in asset pricing tests consistent with the literature. The usual proxy is the return on a stock market index which shall be more volatile than the true market return since stock market dividends are at least five times more volatile than consumption. Therefore, properties of a consumption proxy backed out through equation (9) using the stock market index return may be different from those of the observed consumption.

in (8) may be written as a cross-sectional linear factor model

$$E\left[R_{it}^{e}\right] = p_W \sigma_{iW} + p_{\mathcal{D}} \sigma_{i\mathcal{D}} + p_{W\mathcal{D}} \sigma_{iW\mathcal{D}} + p_X \sigma_{iX} + p_{X\mathcal{D}} \sigma_{iX\mathcal{D}} \ , \tag{10}$$

with

$$\begin{aligned} \sigma_{iW} &\equiv Cov\left(R_{it}^{e}, r_{Wt}\right) \\ \sigma_{i\mathcal{D}} &\equiv Cov\left(R_{it}^{e}, I\left(\mathcal{D}_t\right)\right) \\ \sigma_{iW\mathcal{D}} &\equiv Cov\left(R_{it}^{e}, r_{Wt} I\left(\mathcal{D}_t\right)\right) \\ \sigma_{iX} &\equiv Cov\left(R_{it}^{e}, \Delta\sigma_{Wt}^{2}\right) \\ \sigma_{iX\mathcal{D}} &\equiv Cov\left(R_{it}^{e}, \Delta\sigma_{Wt}^{2} I\left(\mathcal{D}_t\right)\right) \ , \end{aligned} \tag{11}$$

where r_{Wt} is the log-return on the market and $\Delta\sigma_{Wt}^{2}$ is the change in the variance of the market return (we are going to refer to this as the volatility factor).[3] Equation (10) corresponds to a linear multifactor representation of expected excess returns in the cross-section. This is a five-factor model which we refer to as GDA5 throughout the rest of the paper. It states that in addition to the market (r_{Wt}) and volatility ($\Delta\sigma_{Wt}^{2}$) factors, three additional factors command a risk premium: the *downstate factor* $I\left(\mathcal{D}_t\right)$, the *market downside factor* $r_{Wt} I\left(\mathcal{D}_t\right)$, and the *volatility downside factor* $\Delta\sigma_{Wt}^{2} I\left(\mathcal{D}_t\right)$.

The covariance risk prices $p_W \geq 0$, $p_{\mathcal{D}} \leq 0$, $p_{W\mathcal{D}} \geq 0$, $p_X \leq 0$, and $p_{X\mathcal{D}} \leq 0$ are functions of the preference parameters δ, γ, ψ, ℓ and κ, as well as functions of the parameters governing the endowment dynamics. Let us have a detailed look at the signs of the covariance risk prices. First, as $p_W \geq 0$, investors require a premium for a security that has positive

[3] Details of the derivation are outlined in the Online Appendix, where we also derive sign restrictions on the covariance risk prices, the definition of disappointment in (12), and the cross-price restrictions in (13). We use "W" in subscript to refer to quantities (e.g., risk measures or risk prices) related to the factor r_{Wt}. Similarly, we use "$\mathcal{D}$" to refer to the factor $I\left(\mathcal{D}\right)$, "$W\mathcal{D}$" for the factor $r_W I\left(\mathcal{D}\right)$, "$X$" for the factor $\Delta\sigma_W^{2}$, and "$X\mathcal{D}$" for the factor $\Delta\sigma_W^{2} I\left(\mathcal{D}\right)$, respectively.

covariance with the market return. This is in line with the CAPM theory of Sharpe (1964) and Lintner (1965). Second, as $p_X \leq 0$, investors are willing to pay a premium for a security that has positive covariance with $\Delta\sigma^2_{Wt}$. This is consistent with the existing empirical literature (see, e.g., Ang et al., 2006b; Adrian and Rosenberg, 2008). The third factor in (10), $I(\mathcal{D}_t)$, indicates periods when the economy is in the disappointing state. We refer to it as the downstate factor throughout the paper. The associated risk price is $p_{\mathcal{D}} \leq 0$, showing that disappointment-averse investors are willing to pay a premium for a security that has a positive covariance with the downstate indicator. Note that $\sigma_{i\mathcal{D}} = Prob(\mathcal{D}_t)\left(E\left[R^e_{it} \mid \mathcal{D}_t\right] - E\left[R^e_{it}\right]\right)$, i.e., assets with $\sigma_{i\mathcal{D}} > 0$ are desirable because they have a higher expected return in the downstate. The fourth factor is $r_{Wt}I(\mathcal{D}_t)$, and it represents changes in the market index when the economy is in the downstate. We refer to it as the market downside factor throughout the paper. The associated risk price is non-negative, $p_{W\mathcal{D}} \geq 0$. Investors require a premium for a security that has positive covariance with $r_{Wt}I(\mathcal{D}_t)$, since such an asset tends to have a negative return when there is a low market return in the downstate. The fifth and final factor is $\Delta\sigma^2_{Wt}I(\mathcal{D}_t)$, representing changes in market volatility when the economy is in the downstate. We subsequently refer to it as the volatility downside factor. The associated risk price is non-positive, $p_{X\mathcal{D}} \leq 0$. Investors are willing to pay a premium for a security that has positive covariance with the volatility downside factor. Such an asset tends to have positive returns when market volatility increases in a downstate.

We also show in the Online Appendix that the disappointing event may be written as

$$\mathcal{D}_t = \left\{ r_{Wt} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < b \right\}, \tag{12}$$

where $\sigma_W = Std\left[r_{Wt}\right]$ and $\sigma_X = Std\left[\Delta\sigma^2_{Wt}\right]$ are the standard deviations of market return and changes in market volatility, respectively. Similar to the covariance risk prices, the coefficients $a > 0$ and b are also functions of the preference parameters and the parameters governing the endowment dynamics. The term $(\sigma_W/\sigma_X)\,\Delta\sigma^2_{Wt}$ may be viewed as the return on a volatility index that has the same standard deviation as the market return. Disappointment occurs if the return on a portfolio consisting of a long position in the market index and a times a short position in the volatility index falls below a constant threshold b. In particular, if the coefficient a is equal to one, the long position in the market index is exactly balanced by the short position in the volatility index in determining disappointment. As a decreases from one towards zero, disappointment is more likely to occur due to a fall in the market index rather than an increase in the volatility index. Note also that the following two non-linear restrictions apply to the GDA5 model:

$$\begin{aligned} \frac{p_{W\mathcal{D}}}{p_W} &= \frac{p_{X\mathcal{D}}}{p_X} \\ p_{X\mathcal{D}} &= -a\frac{\sigma_W}{\sigma_X} p_{W\mathcal{D}}\ . \end{aligned} \tag{13}$$

There are two special cases of the model worth examining. First, if the elasticity of intertemporal substitution is infinite ($\psi = \infty$), then $a = 0$ and $p_X = p_{X\mathcal{D}} = 0$. That is, changes in market volatility disappear from the model. In this case, the cross-sectional model (10) reduces to a three-factor model with the market, the downstate, and the market downside factors, and the disappointing event has the simple form $\mathcal{D}_t = \{r_{Wt} < b\}$. We refer to this restricted model as GDA3 throughout the paper. Second, if the representative investor is not disappointment averse ($\ell = 0$), then $p_{\mathcal{D}} = p_{W\mathcal{D}} = p_{X\mathcal{D}} = 0$, i.e., all disappointment-related related factors disappear from the model. In this case, (10) reduces to a two-factor model where only market risk and volatility risk are priced.

Equation (10) may ultimately be expressed as a multivariate beta pricing model:

$$E\left[R^e_{it}\right] = \lambda_F^\top \beta_{iF} \tag{14}$$

where β_{iF} is the vector containing the multivariate regression coefficients of asset excess returns onto the factors, and λ_F is the vector of factor risk premiums, respectively given by

$$\beta_{iF} = \Sigma_F^{-1}\sigma_{iF} \quad \text{and} \quad \lambda_F = \Sigma_F p_F. \tag{15}$$

The vector σ_{iF} contains the covariances of the asset excess returns with the priced factors as shown in (11), the vector p_F contains the associated factor risk prices, and Σ_F is the factor covariance matrix. Since the risk premiums in λ_F are linear combinations of the risk prices in p_F, the restrictions in (13) can easily be translated into equivalent restrictions on the λ-s. Also note that if the covariance between the market return and changes in market volatility is negative, then the sign restrictions on the elements of p_F discussed earlier in this section imply the same sign restrictions on the corresponding elements of λ_F, i.e., $\lambda_W \geq 0$, $\lambda_{\mathcal{D}} \leq 0$, $\lambda_{W\mathcal{D}} \geq 0$, $\lambda_X \leq 0$, and $\lambda_{X\mathcal{D}} \leq 0$. The negative covariance, $Cov\left(r_{Wt}, \Delta\sigma^2_{Wt}\right) < 0$, is consistent with the leverage effect postulated by Black (1976) and documented by Christie (1982) and others, and it is also empirically supported in our data. The model specification in (14) is the basis of our empirical analysis.

3. Empirical assessment

In this section, we provide an empirical assessment of the GDA3 and GDA5 models. The GDA3 is a three-factor model with the market, the downstate, and the market downside factors:

$$E\left[R^e_{it}\right] = \lambda_W\beta_{iW} + \lambda_{\mathcal{D}}\beta_{i\mathcal{D}} + \lambda_{W\mathcal{D}}\beta_{iW\mathcal{D}}\ , \tag{16}$$

where the disappointing event has the simple form $\mathcal{D}_t = \{r_{Wt} < b\}$. The GDA5 is a five-factor model containing also the volatility related factors:

$$E\left[R_{it}^e\right] = \lambda_W \beta_{iW} + \lambda_{\mathcal{D}} \beta_{i\mathcal{D}} + \lambda_{W\mathcal{D}} \beta_{iW\mathcal{D}} + \lambda_X \beta_{iX} + \lambda_{X\mathcal{D}} \beta_{iX\mathcal{D}} \; . \tag{17}$$

Additionally for the GDA5, volatility enters the definition of the disappointing event as shown in (12), and the two cross-price restrictions in (13) should also be satisfied. The number of freely estimated λ-s decreases to three due to the two cross-price restrictions. For the GDA5, we also estimate the parameter a, which determines the relative importance of the market return and changes in volatility in the definition of disappointment. Altogether, there are four parameters to estimate in case of the GDA5 model. For both models, the disappointment threshold is set to $b = -0.03$ for the empirical analysis, but we also consider other values in the robustness section.

Finally, note that we do not estimate the underlying preference parameters, but instead we estimate the risk premiums, which are functions of both the preference parameters and the parameters governing the endowment dynamics. There are several reasons for focusing on the risk premiums. First, the market return is not observable and we use the return on the aggregate stock index as a proxy. This proxy is much more volatile, since it is a claim on the aggregate stock market dividend, whose growth rate is at least five times more volatile than the aggregate consumption growth rate. So, estimating the underlying preference parameters with this proxy would induce large estimation bias. Second, the preference parameter estimates would be dependent on the dynamics assumed for the aggregate endowment in the economy. By estimating the reduced-form risk premiums in the linear beta representations (16) and (17), the assumed endowment dynamics do not have a direct effect on our results. Third, estimating the reduced-form risk premiums makes our results comparable to existing cross-sectional tests of models with downside risks (e.g., Ang et al., 2006a; and Lettau et al., 2014).

3.1. Data and estimation method

Following Lewellen et al. (2010), we do not restrict our attention to pricing size/book-to-market portfolios. Instead, we estimate our models using various sets of stock portfolios, and also include additional asset classes like index options and currencies. Monthly returns on several sets of US stock portfolios are from Kenneth French's data library. Index option returns are from Constantinides et al. (2013), who construct a panel of leverage-adjusted (that is, with a targeted market beta of one) S&P 500 index option portfolios. Currency returns are from Lettau et al. (2014), who use monthly data on 53 currencies to create six portfolios by sorting them based on their respective interest rates. The detailed description of the data and sample periods can be found in Appendix A.

The risk-free rate is the one-month US Treasury bill rate from Ibbotson Associates, while the market return is the value-weighted average return on all CRSP stocks.[4] Both series were obtained from Kenneth French's data library. Empirical tests of the GDA5 model require a measure of market volatility. Several approaches have been used for measuring market volatility in cross-sectional asset pricing studies. Ang et al. (2006b) use the VIX, Adrian and Rosenberg (2008) estimate volatility from a GARCH-type model, while Bandi et al. (2006) use realized volatility computed from high-frequency index returns. In our main analysis, we measure monthly volatility as the realized volatility of the daily market returns during the month. The main advantage of this latter measure is that it is very easy to construct as it requires only daily market return data. Therefore, it allows us to use a longer sample period. Nevertheless, we also use alternative measures in our robustness checks, including the VIX, realized volatility calculated from intra-daily market returns, and

[4]We closely follow the predictions of the theoretical model by using the log return on the market (r_{Wt}) as the market factor and using simple excess returns on the portfolios (R^e_{it}) as the dependent variable.

GARCH volatility.

Portfolio betas and factor premiums from (14) are estimated jointly using GMM with moment conditions as in Cochrane (2000):

$$\begin{cases} E\left[R_{it}^{e}-\alpha_{i}-F_{t}\beta_{iF}\right]=0 & i=1,...,N \\ E\left[\left[R_{it}^{e}-\alpha_{i}-F_{t}\beta_{iF}\right]f_{jt}\right]=0 & i=1,...,N\ ,\quad j=1,...,K\ , \\ E\left[R_{it}^{e}-\beta_{iF}\lambda_{F}\right]=0 & i=1,...,N \end{cases} \tag{18}$$

where R_{it}^{e} is the excess return on portfolio i, f_{jt} denotes factor j, F_t is the row vector of all factors in the model, β_{iF} is the vector of factor betas for portfolio i, and λ_F is the vector of factor risk premiums. The first two sets of moment conditions from (18) directly correspond to the formula for estimating the β-s in (15), while the last set of moment conditions represents the model in (14). The advantage of using the GMM is that it allows us to impose the cross-price restrictions in the GDA5 model and that the standard errors account for the "generated regressors" problem, i.e., the fact that the β-s are also estimated.[5]

When estimating the factor risk premiums, we always apply the additional restriction that the market portfolio should be perfectly priced. This additional restriction reduces the number of free parameters in all the models by one. As a consequence, there are two free parameters to estimate for the GDA3, and three free parameters to estimate for the GDA5, which makes the models more parsimonious. As it can bee seen from (10) and (11), the return to be explained in our cross-sectional models is in the form of simple excess return (R_{it}^{e}), while the market factor is the log-return on the market (r_{Wt}).[6] Thus, when the test

[5]It is shown by Cochrane (2000), for example, that the correction due to Shanken (1992) can be recovered as a special case of the GMM standard errors. During the GMM estimation we use the identity weighting matrix, and we use the Newey-West estimator with 3 lags for the covariance matrix of the moment conditions. Delikouras (forthcoming) shows that the GMM estimators are consistent and asymptotically normal even when the GMM moment conditions include indicator functions as in the case of the GDA models.

[6]It is shown in the Online Appendix that deviating from the theoretical predictions and using R_{Wt}^{e} instead of r_{Wt} as the market factor does not change our empirical results considerably.

asset is the market portfolio, the return to be explained and the market factor are not exactly the same. Therefore, imposing the restriction that the market is priced perfectly is not equivalent to setting the market premium equal to the expected excess return on the market, but it imposes a linear restriction on the λ-s. This restriction is discussed in detail in the Online Appendix. Essentially, we have to pick one of the premiums, whose value is implied by the other risk premiums through the market restriction. We pick the downstate premium ($\lambda_{\mathcal{D}}$) to be imposed for the GDA models, but the risk premium estimates would be exactly the same if we chose another one instead (e.g., λ_W or $\lambda_{W\mathcal{D}}$).

3.2. Results

Table 1 presents risk premium estimates for the CAPM, GDA3, and GDA5 models using several sets of US stock portfolios: (i) 25 (5×5) portfolios formed on size and book-to-market, (ii) 25 (5×5) portfolios formed on size and momentum, (iii) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum portfolios, (iv) 25 (5×5) portfolios formed on size and operating profitability, and (v) 25 (5×5) portfolios formed on size and investment.

Panel A corresponds to the CAPM, which also arises as a restricted version of the GDA3 if the representative agent is not disappointment averse. The value of the market risk premium is not estimated, but imposed by the restriction that the market portfolio should be perfectly priced by the model. To make it clear that certain λ values are imposed instead of estimated, we report these values with the superscript i and do not report their standard errors in Table 1 and in subsequent tables throughout the paper.

Panel B presents the results for the GDA3. In the three middle columns, all risk premiums have the expected signs: the market (λ_W) and market downside ($\lambda_{W\mathcal{D}}$) factors have a positive premium, while the downstate factor ($\lambda_{\mathcal{D}}$) has a negative premium. Also, the estimated premiums are statistically significant.[7] For the size/book-to-market and size/investment portfolios however, the downstate premium is positive and the market downside premium is not statistically significant.

Panel C shows the results for the GDA5. Recall that the GDA5 involves two cross-price restrictions. We substitute out the volatility related premiums using these restrictions and estimate the premiums on the rest of the factors. Additionally, the value of $\lambda_{\mathcal{D}}$ is imposed by the restriction that the market portfolio is perfectly priced, similar to the GDA3. In all the columns, the signs are as expected both on the estimated and on the implied premiums: the market (λ_W) and market downside ($\lambda_{W\mathcal{D}}$) factors have a positive premium, while the premiums on the downstate ($\lambda_{\mathcal{D}}$), the volatility (λ_X), and the volatility downside ($\lambda_{X\mathcal{D}}$) factors are negative. The only exception is $\lambda_{\mathcal{D}}$ for the size-investment portfolios. All estimated risk premiums are statistically significant. In the case of the GDA5, the parameter a in the definition of the disappointing event (12) is also estimated. The value of a is less than one in four of the five cases and the typical value is close to 0.5. Recall that an a value less than one means that the market return has a bigger weight in determining disappointing states than changes in market volatility.

Table 2 presents risk premium estimates for the same models when index option and currency portfolios are also used as test assets. Note that if multiple asset classes are included, each asset class is represented with the same number of portfolios, so that they have similar importance in the estimation. Panel A of Table 2 presents the CAPM. The market risk premium, λ_W, is positive for all five sets of portfolios. Panel B corresponds to the GDA3. All risk premiums have the expected signs, and all the estimated risk premiums are statistically significant. Panel C presents result for the GDA5. Similar to the GDA3, all risk premiums

[7]Note again, that the value of $\lambda_{\mathcal{D}}$ is not directly estimated, but is imposed by the restriction that the market portfolio should be perfectly priced.

have the expected signs, and all the estimated risk premiums are statistically significant.[8]

To facilitate model comparison, both Table 1 and Table 2 report the root-mean-squared pricing error (RMSPE) of the models, expressed in basis points (bps) per month, and the ratio of RMSPE to root-mean-squared returns (in brackets after the RMSPE values). The GDA3 provides a better fit than the CAPM for all sets of test assets, and the improvement is considerable in several cases. For example, the RMSPE reduces from 39 to 24 bps in case of the size-momentum stock portfolios, reduces from 44 to 12 bps for the option portfolios, and reduces from 50 to 20 bps when all three asset classes are included in the estimation. The GDA5 provides further improvement compared to GDA3 for all the ten sets of portfolios presented in Table 1 and Table 2.

Figure 1 shows scatter plots of actual versus predicted returns, corresponding to the case when all three asset classes are included in the estimation.[9] Panel A highlights the failure of the CAPM to price our test portfolios. Within each asset class, the actual returns vary considerably, but the CAPM predicts similar returns for all portfolios. Consequently, portfolios within each asset class line up close to a vertical line. The improvement in fit is evident when we move from the CAPM to the GDA3 in Panel B, where the portfolios lie much closer to the 45 degree line. Finally, the portfolios line up almost perfectly along the 45 degree line in Panel C, which corresponds to the GDA5.

In Section A.7 of the Online Appendix, we provide a detailed discussion on why the

[8]To put the magnitudes of the risk premium estimates into perspective, we compare them to the corresponding values implied by the asset pricing model of Section 2, calibrated as in Bonomo et al. (2011). In summary, despite the correct signs, estimates of market downside risk premium, volatility risk premium, and volatility downside risk premium of Tables 1 and 2 are larger than what the calibrated model can actually replicate. We argue that this is due to potential estimation biases that may come from different sources, in particular when portfolios are used as test assets, as discussed in Ang et al. (2016), and Gagliardini et al. (2016). To verify our assertion, we also estimate the factor risk premiums using a large cross-section of individual stocks as test assets. We find that the risk premium estimates obtained with individual stocks are close to the calibration-implied values. A detailed description and discussion of these findings can be found in Section A.8 of the Online Appendix.

[9]In particular, the scatter plots in the top row of Figure 1 correspond to the last column of Table 2. The Online Appendix contains scatter plots similar to the ones in Figure 1 for several other sets of portfolios.

GDA model is successful in pricing the option portfolios of Constantinides et al. (2013). We rely on option greeks to study how the sensitivity of the option price to the underlying price and to volatility varies with option moneyness when disappointment sets in. Portfolios containing OTM calls have the lowest sensitvity to the price of the underlying, conditional on disappointment. They are followed by portfolios with ITM calls, then ITM puts, and portfolios with OTM puts have the highest sensitivity. When considering the sensitivity to volatility conditional on disappointment, the ordering is reversed: portfolios containing OTM puts have the lowest sensitivity and portfolios with OTM calls have the highest. Since market downside risk carries a positive premium and volatility downside risk carries a negative premium, these imply that the GDA model predicts the lowest return for the OTM call portfolios and the highest return for the OTM put portfolios; which is in line with the data.

Daniel and Moskowitz (2016) argue that momentum profits are linked to the option like behavior of the momentum strategy. Clarida et al. (2009) show that currency carry trade strategies resemble the payoff and risk characteristics of FX option strategies. These results, together with our previous discussion on option portfolios, may explain why the GDA model is also successful in pricing the momentum equity and currency portfolios.

3.2.1. Disappointing states

Panel A of Figure 2 plots the market return and the NBER recession periods for our longest sample starting in July 1964 and ending in December 2016. The horizontal line indicates a 3% drop in the market index. According to the simple definition $\mathcal{D}_{At} \equiv \{r_{W,t} < -0.03\}$, disappointing months are those when the market return is below this line. Out of 630 months in the sample, 102 are classified as disappointing, giving a 16.2% unconditional probability of disappointment. There are 90 NBER recession months during this period, out of which 28 are classified as disappointing. This implies a 31.1% probability of disappointment conditional on being in recession, and a 13.7% probability of disappointment conditional on being outside of recession. There is a clear positive relationship between recessions and disappointing states as the conditional probability of disappointment more than doubles in recession periods.

Panel B of Figure 2 shows the value of $r_{Wt} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt}$ with $a = 0.5$ for the same period. We use $a = 0.5$ since most of the estimated values in Table 1 and Table 2 are around this value. The horizontal line is at -0.03, and disappointing states are defined $\mathcal{D}_{Bt} \equiv \left\{r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < -0.03\right\}$. The disappointment definitions in Panel A and Panel B are empirically close to each other. Out of 630 months, 101 are classified as disappointing according to $\mathcal{D}_{Bt}$, giving a 16.0% unconditional probability of disappointment. There are only 10 months in the sample that are disappointing according to $\mathcal{D}_{At}$ but not disappointing according to $\mathcal{D}_{Bt}$; these are highlighted with the diamond markers in Panel B. At the same time, there are 9 months that are disappointing according to $\mathcal{D}_{Bt}$ but not disappointing according to $\mathcal{D}_{At}$; these are highlighted with the round markers in Panel B.

The main reason for $\mathcal{D}_{At}$ and $\mathcal{D}_{Bt}$ being empirically close is that decreasing market return and increasing market volatility tend to coincide empirically; which is also known as the leverage effect. That is, even if increasing market volatility is not explicitly included in the definition, disappointment tend to be accompanied with increasing volatility. In the period from July 1964 to December 2016, the unconditional correlation between r_{Wt} and $\Delta\sigma^2_{Wt}$ is -0.25 in our sample. Their conditional correlation (conditional on being in the disappointing state according to $\mathcal{D}_{At}$) is even stronger, -0.46. Extreme volatility increases also happen in disappointing months when disappointment is defined as $r_{Wt} < -0.03$. Nine of the largest ten $\Delta\sigma^2_{Wt}$ values in our stock sample period are realized in disappointing months (and 16 of the largest 20).

Finally, Panel C of Figure 2 shows quarterly consumption growth throughout the period, and the red circles indicate quarters with two or three disappointing months.[10] Out of the 209 quarters in the sample, there are 20 in which at least two out of three months within the quarter are disappointing. Quarters with multiple disappointing months are associated with a higher probability of declining consumption. There are 16 quarters with negative consumption growth and 7 of them have multiple disappointing months. These values imply that the conditional probability of declining consumption is 35.0% if there are two or more disappointing months in a given quarter, and only 4.8% if there is at most one disappointing month.

3.2.2. Risk premium estimates without restrictions

Table 3 shows risk premium estimates for selected sets of test portfolios when we do not impose the restriction that market should be perfectly priced.[11] Thus, the downstate premium is not imposed, but is estimated as a free parameter in Table 3. Panel A and B correspond to the GDA3 and GDA5 models, respectively. The estimated risk premiums have the expected signs and their magnitudes are similar to those reported for our benchmark specifications in Table 1 and Table 2. The $\lambda_{\mathcal{D}}$ estimate is statistically significant in all but one of the cases. Also note that the model fit is better if we do not impose the restriction on the market portfolio, as the number of free parameters increases.

The GDA5 model can also be estimated without imposing any cross-price restrictions on the risk premiums and assuming that the disappointing event is of the simple form $\mathcal{D}_t = \{r_{W,t} < -0.03\}$. We refer to this specification as the "unrestricted GDA5". The

[10] Aggregate consumption growth is calculated using quarterly data on Personal Consumption Expenditures (PCE) from the U.S. Bureau of Economic Analysis. Aggregate consumption is defined on a per capita basis as services plus non-durables. We use seasonally adjusted series and deflate aggregate consumption by the PCE price index (the base year is 2009). The consumption growth data is available until 2016Q3. The red circles indicate quarters with two or three disappointing months according to $\mathcal{D}_{At}$. Note, however, that exactly the same plot arises if $\mathcal{D}_{Bt}$ is used instead.

[11] In order to save space, results for the other five sets of portfolios are presented in the Online Appendix. Those results lead to very similar conlcusions to the ones presented in Table 3.

unrestricted GDA5 has a couple of advantages compared to our main GDA5 specification. First, it is easier to estimate because there are no cross-price restrictions to be imposed and the definition of disappointment is fixed (no a to be estimated). Second, the GDA3 is nested in the unrestricted GDA5, which facilitates a more direct comparison between the two models. Third, results from the unrestricted GDA5 are also more comparable to previous studies analyzing downside risk, since those studies also define the downstate in terms of the market return only. Panel C of Table 3 provides risk premium estimates for the unrestricted GDA5. All risk premiums are statistically significant and have the expected signs. Magnitudes of the volatility related risk premiums for the unrestricted GDA5 are somewhat higher than in the case of our benchmark GDA5 specification, but the magnitudes of the other premiums are reasonably similar. In terms of model fit, the unrestricted GDA5 delivers lower pricing errors than the other two models in Table 3. Thus, the fourth advantage of the unrestricted GDA5 is that it actually provides a better fit than the other two GDA models.

Despite all its advantages, we do not focus on the unrestricted GDA5 throughout the paper, as it has one major disadvantage compared to the GDA5: it is less related to the theoretical predictions from Section 2.1. Lewellen et al. (2010) suggest that when theory provides predictions for the risk price estimates, these predictions should be taken seriously.

3.2.3. Comparison to alternative models

In this section we compare the fit of the GDA models to alternative models proposed in previous literature. Table 4 presents results corresponding to four alternative models using the same five sets of portfolios as in Table 3. Results for the other sets of portfolios are relegated to the Online Appendix and lead to very similar conclusions to those presented in Table 4.

The model in Panel A, labeled as “VOL”, contains only two priced factors: market return and market volatility. The VOL model can be viewed as a restricted version of the GDA5 that arises if the representative agent is not disappointment averse. The results in Panel A show that volatility risk carries a negative premium. Pricing errors decrease compared to the CAPM, but the size of the improvement varies across different asset classes. The RMSPE barely decreases in case of the stock portfolios, but the improvement is considerable in case of the option portfolios (from 44 to 14 bps) and when all three asset classes are included (from 50 to 26 bps). It is more important for the current paper to compare the VOL and GDA3 models. The GDA3 delivers lower pricing errors than the VOL model for all five sets of portfolios, and the improvement in fit can be considerable as in the case of the size/momentum stock portfolios (from 35 to 24 bps) and when all three asset classes are included (from 26 to 20 bps).

The cross-sectional implications of market downside risk have been previously studied by Ang et al. (2006a) and Lettau et al. (2014). These authors propose slightly different models to incorporate the effect of market downside risk. More importantly, it can be shown that our GDA3 specification nests the models from both of these studies, with different restrictions on the value of $\lambda_{\mathcal{D}}$. Ang et al. (2006a) specify the model for expected returns as

$$\begin{aligned} E\left[R^e_{it}\right] &= \lambda^+ \beta_i^+ + \lambda^- \beta_i^- \ , \qquad \text{with} \\ \beta_i^+ &= \frac{Cov\left(R^e_{it}, r_{Wt} \mid \mathcal{U}_t\right)}{Var\left(r_{Wt} \mid \mathcal{U}_t\right)} \qquad \text{and} \qquad \beta_i^- = \frac{Cov\left(R^e_{it}, r_{Wt} \mid \mathcal{D}_t\right)}{Var\left(r_{Wt} \mid \mathcal{D}_t\right)} \ , \end{aligned} \tag{19}$$

where $\mathcal{U}$ refers to the upside event, which is the complement of the disappointing event $\mathcal{D}$. The model in (19) is equivalent to the GDA3 in (16) with

$$\lambda_W = \lambda^+ + \lambda^- \ , \qquad \lambda_{\mathcal{D}} = 0 \ , \qquad \lambda_{W\mathcal{D}} = \lambda^- \ . \tag{20}$$

That is, the model proposed by Ang et al. (2006a) imposes the restriction $\lambda_{\mathcal{D}} = 0$. On the other hand, Lettau et al. (2014) propose

$$E\left[R_{it}^e\right] = \lambda \beta_i + \lambda^- \left(\beta_i^- - \beta_i\right) \ , \tag{21}$$

where β_i is the CAPM beta and β_i^- is the same downside beta as in (19). The specification in (21) is equivalent to the GDA3 in (16) with

$$\lambda_W = \lambda \ , \qquad \lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}\left(\lambda_W - \lambda_{W\mathcal{D}}\right) \ , \qquad \lambda_{W\mathcal{D}} = \gamma_1 \lambda + \left(1-\gamma_1\right)\lambda^- \ , \tag{22}$$

where

$$\gamma_1 \equiv \frac{Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)} \ , \qquad \gamma_2 \equiv \frac{Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)} \ . \tag{23}$$

That is, the model proposed by Lettau et al. (2014) imposes $\lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}\left(\lambda_W - \lambda_{W\mathcal{D}}\right)$. The derivation of the above results is shown in Appendix B.

Panels B and C of Table 4 present risk premiums for the model of Ang et al. (2006a) and Lettau et al. (2014), respectively. The models are estimated using GMM, imposing the linear restriction on $\lambda_{\mathcal{D}}$ during the estimation for both models.[12] Note that the restriction imposed by the model of Ang et al. (2006a) ($\lambda_{\mathcal{D}} = 0$) is rejected in four out of five cases for the GDA3 model in Table 3 (where we can assess the statistical significance of $\lambda_{\mathcal{D}}$), where $\lambda_{\mathcal{D}}$ is negative and significantly different from zero. Comparing model fit, the GDA3 model is always associated with lower pricing errors than the model of Ang et al. (2006a), and the difference can be substantial, as in case of the option portfolios (12 bps for the GDA3 in

[12]Without the restriction that market portfolio should be priced perfectly, estimating λ^+ and λ^- from (19) using the Fama-MacBeth (1973) procedure and applying the transformations in (20) leads to the same λ values as estimating the GDA3 using GMM with the restriction $\lambda_{\mathcal{D}} = 0$. Similarly, estimating λ and λ^- from (21) using the Fama-MacBeth (1973) procedure and applying the transformations in (22) leads to the same λ values as estimating the GDA3 using GMM with the restriction on $\lambda_{\mathcal{D}}$ from (22). In Panels B and C of Table 4, we also impose the restriction that market portfolio should be correctly priced.

Table 2 and 20 bps for the Ang et al., 2006a model in Table 4) and in the case when all three asset classes are included (20 bps versus 28 bps). The model of Lettau et al. (2014) imposes a different restriction on $\lambda_{\mathcal{D}}$, and as it can be seen in Panel C of Table 4, all the implied $\lambda_{\mathcal{D}}$ values are positive. Since the downstate premium values are typically negative for the GDA3, this restriction is also not in line with the data. The Lettau et al. (2014) model provides a poorer fit than the other two models. The only exception is the size/book-to-market portfolios, where the RMSPE of the Lettau et al. (2014) model is marginally lower than the RMSPE of the Ang et al. (2006a) model (but not lower than that of the GDA3). In general, the models proposed by Ang et al. (2006a) and Lettau et al. (2014) impose restrictions, compared to the GDA3, that are not supported by the data. Panels E and F in Figure 1 show scatter plots of actual versus predicted returns for the Ang et al. (2006a) and Lettau et al. (2014) models. These plots provide a visual evidence that the GDA3 has a better fit than the two nested models.

Panel D of Table 4 corresponds to the four-factor model of Carhart (1997), which is an important benchmark in the literature. The Carhart (1997) model does a good job in pricing the size/book-to-market and size/momentum portfolios. This is not surprising, as the four-factor model is tailor-made to price these stock portfolios correctly. When we consider other asset classes, the Carhart (1997) model is much less successful. When estimating the model using option portfolios, the pricing error is twice as much as that of the GDA5 model, but even more importantly, the estimated risk premiums change considerably compared to the stock portfolios. In other words, the estimated risk premiums are very different, when different asset classes are used. Consequently, the Carhart (1997) model performs badly when estimated using the three asset classes jointly: only the CAPM provides higher RMSPE values. In general, the four-factor model works well for pricing stock portfolios, but it is less successful in pricing portfolios from other asset classes. This is also illustrated in

Panel H of Figure 1. The stock portfolios line up along the 45 degree line, but the portfolios from other asset classes do not.

3.2.4. Variation in the risk premium estimates across test portfolios

Risk premium estimates for all models vary across different sets of test assets, and it is hard to tell whether the variation we observe is substantial or not. To address this concern, we carry out the following exercise: for all the models, we take risk premium estimates from a given set of test portfolios, and calculate out-of-sample RMSPEs on the other sets of portfolios using these risk premiums. In other words, we assess the model fit when the same risk premium estimates are used across different sets of test portfolios. The out-of-sample RMSPE values are reported in Table 5. The shaded column in all three panels indicates which set is used for estimating the risk premiums. In Panel A, the λ-s correspond to the case when 30 stock portfolios consisting of 10 size, 10 book-to-market, and 10 momentum portfolios are used for the estimation. Panel B corresponds to the case when 54 index option portfolios are used for the estimation of the λ-s. In Panel C, λ-s are estimated using 6 size/momentum, 6 option, and 6 currency portfolios.

The picture is very clear when considering sets that do not solely include stock portfolios: the lowest pricing errors are delivered by the GDA models, regardless of which set of portfolios is used for estimation. In the last five columns of Table 5, the lowest RMSPE values in all panels are produced by the GDA models, and the lowest pricing error typically corresponds to the GDA5. When considering stock-only sets in the first five columns of Table 5, the results are more mixed. In Panel A, where the λ-s are estimated using stock portfolios only, the lowest out-of-sample pricing errors are delivered by the Carhart (1997) model. Nevertheless, the second lowest RMSPE typically corresponds to the GDA5. In Panel B, where the λ-s are estimated using option portfolios only, the lowest out-of-sample pricing errors are typically delivered by the GDA models. Note also that the out-of-sample RMSPE values from the Carhart (1997) model are extremely high in this case. Finally, there are no clear tendencies for the stock-only sets in Panel C. Altogether, the results in Table 5 show that the GDA models perform well even if the same risk premiums are used across the different sets of test portfolios.

3.3. Robustness checks

3.3.1. Additional portfolios as test assets

In this section we add corporate bond, sovereign bond, and commodity futures portfolios as test assets. Five corporate bond portfolios, sorted annually on their credit spread, are from Nozawa (2012). Sovereign bond and commodity futures portfolios are from Asness et al. (2013), who create three value and three momentum portfolios in both asset classes. A more detailed description of these portfolios and the data sources can be found in Appendix A. Risk premium estimates for the GDA models are reported in Table 6.[13] The results are robust to the addition of these asset classes. All the risk premiums have the expected signs, and all the estimated risk premiums are statistically significant. In terms of pricing error, the GDA5 delivers lower RMSPE values than any of the alternative models considered in the paper.

We also consider the robustness of our results when the same asset classes are used as in Table 2, but different test portfolios are chosen to represent a given asset class. When stocks and options are jointly considered, we also use the size/operating-profit and size/investment portfolios to represent stocks. When stocks, options, and currencies are jointly considered, we use 10 industry portfolios to represent stocks and also use 6 portfolios provided by Lustig

[13]The corresponding results for alternative models are in the Online Appendix. Also note that stocks are represented by the 6 size/book-to-market portfolios in Table 6. Results with the 6 size/momentum portfolios are in the Online Appendix.

et al. (2011) to represent currencies. The results, presented in the Online Appendix, show that the risk premium estimates are robust to the choice of test portfolios.

3.3.2. Changing the disappointment threshold

The disappointment threshold is set to $b = -0.03$ throughout the paper. As a robustness check, we consider the thresholds $b \in \{0, -0.015, -0.04\}$. The results and a more detailed assessment can be found in the Online Appendix, but we provide a brief summary here for the GDA5. The results remain very similar for the lower threshold ($b = -0.04$). When the threshold is higher ($b = -0.015$ or $b = 0$), the risk premiums, with one exception, also remain similar to those in our benchmark specification. The exception is the downstate premium, $\lambda_{\mathcal{D}}$, which comes closer to zero and can eventually turn into positive as the threshold increases. That is, disappointing events should be sufficiently out in the left tail so that the downstate factor commands a negative premium. In terms of model fit, the lowest RMSPE is typically provided by the models with low disappointment threshold (either $b = -0.03$ or $b = -0.04$).

The results on the model fit have implications on the preference specification in our theoretical model. Recall that in the generalized disappointment aversion framework, parameter κ determines the level of the disappointment threshold relative to the certainty equivalent. Our results that the model fit is better when disappointing events are sufficiently out in the left tail suggest that we should consider $\kappa < 1$ in a representative agent setup.

3.3.3. Alternative measures of market volatility

We also consider how the risk premiums for the GDA5 change if different measures of market volatility are used. Our alternative measures are the option-implied volatility index (VIX), realized volatility calculated from intra-daily market returns, and a model implied volatility calculated using an EGARCH specification. Details on how these alternative measures are calculated and the estimated risk premiums are presented in the Online Appendix. The conclusions are similar to our benchmark case, where monthly volatility is measured as realized volatility of the daily market returns during the month. The signs on the risk premiums are as expected, their magnitudes are similar, and the estimated premiums are statistically significant. The model fit is also similar across the different volatility measures.

4. Conclusion

This paper provides an analysis of downside risks in asset prices. Our empirical tests are motivated by the cross-sectional implications of a dynamic consumption-based general equilibrium model where the representative investor has generalized disappointment aversion preferences and macroeconomic uncertainty is time-varying. We explicitly characterize the factors that are valued by an investor in such setting. Besides the market return and market volatility, three disappointment-related factors are also priced: a downstate factor, a market downside factor, and a volatility downside factor. We also show that in addition to a fall in the market return, downside risk may also be associated with a rise in market volatility. The empirical tests confirm that these factors are priced in the cross-section of various asset classes, including stocks, options, currencies, treasury bonds, corporate bonds, and commodity futures.

The related literature has mainly focused on the time series implications of this general equilibrium setting, discussing the preference parameter values necessary to match empirical regularities in equity returns, risk-free rate, variance premium and options. Estimating these preference parameter values to jointly target both the time series and the cross-section of asset returns constitutes an interesting avenue for future research.

A. Data

Return data on US stock portfolios are from Kenneth French's data library.[14] We use various sets of stock portfolios in our tests. The sample period for the stock portfolios is from July 1964 to December 2016.

Index option returns are from Constantinides et al. (2013).[15] They construct a panel of S&P 500 index option portfolios. The data set contains leverage-adjusted (that is, with a targeted market beta of one) monthly returns of 54 ($2\times3\times9$) option portfolios split across two types (call and put), three targeted time to maturities (30, 60, or 90 days), and 9 targeted moneyness levels (10% ITM, 7.5% ITM, 5% ITM, 2.5% ITM, ATM, 2.5% OTM, 5% OTM, 7.5% OTM, and 10% OTM). The option data are available from April 1986 to January 2012. In estimations when we use only 24 ($2\times3\times4$) option portfolios, we use only a subset of the portfolios corresponding to two types (call and put), three maturities (30, 60, or 90 days), and 4 moneyness levels (5% ITM, ATM, 5% OTM, and 10% OTM). In cases when we use only 6 (2×3) option portfolios, these contain short maturity (30 days) options split across two types (call and put) and three moneyness levels (ATM, 5% OTM, and 10% OTM).

Currency returns are from Lettau et al. (2014), who use monthly data on 53 currencies to create six portfolios by sorting them in ascending order of their respective interest rates.[16] The sixth (highest interest rate) portfolio is split into two baskets, 6A and 6B, and portfolio 6B has currencies with annualized inflation at least 10% higher than US inflation in the same month. We follow Lettau et al. (2014) and use the 6A portfolio to obtain our results. Currency returns are available from January 1974 to March 2010.

Corporate bond portfolios, sorted annually on their credit spread, are from Nozawa (2012). Five portfolios are obtained by equally weighting the ten portfolios in the benchmark

[14] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[15] Data are from Alexi Savov's website at http://pages.stern.nyu.edu/asavov/alexisavov/Alexi_Savov.html
[16] Data are from Michael Weber's website at http://faculty.chicagobooth.edu/michael.weber

analysis of Nozawa (2012) into five baskets.[17] The corporate bond returns are available from October 1975 to March 2010.

Sovereign bond portfolios are from Asness et al. (2013) who sort government bond indexes into three portfolios based on value and three portfolios based on momentum, separately. We use all six portfolios in our estimation.[18] The portfolio returns are available from January 1983 to December 2016.

Commodity futures portfolios are also from Asness et al. (2013) who sort commodity futures into three portfolios based on value and three portfolios based on momentum, separately. We use all six portfolios in our estimation. The portfolio returns are available from January 1972 to December 2016.

In cases when multiple asset classes are used at the same time, the sample period is always the longest possible period for which all asset classes have data available.

[17]Data are from Michael Weber's website, from the replication data set connected to Lettau et al. (2014).

[18]An updated and extended version of the portfolios used by Asness et al. (2013) is available from the AQR website at https://www.aqr.com/library/data-sets

B. The GDA3 and nested models

To calculate betas in the GDA3 model, the following regression is estimated:

$$R_{it}^e = \alpha_i + \beta_{iW} r_{Wt} + \beta_{i\mathcal{D}} I(\mathcal{D}_t) + \beta_{iW\mathcal{D}} r_{Wt} I(\mathcal{D}_t) + \varepsilon_{it} \tag{B.1}$$

The mechanics of the OLS implies $E[\varepsilon_{it}] = E[\varepsilon_{it} r_{Wt}] = E[\varepsilon_{it} I(\mathcal{D}_t)] = E[\varepsilon_{it} r_{Wt} I(\mathcal{D}_t)] = 0$, where ε_{it} denotes residuals from the estimation. Then, with the estimated α_i and β_i-s,

$$E[R_{it}^e] = \alpha_i + \beta_{iW} E[r_{Wt}] + \beta_{i\mathcal{D}} \pi + \beta_{iW\mathcal{D}} E[r_{Wt} \mid \mathcal{D}_t] \pi \tag{B.2}$$

$$E[R_{it}^e r_{Wt}] = \alpha_i E[r_{Wt}] + \beta_{iW} E\left[r_{Wt}^2\right] + \beta_{i\mathcal{D}} E[r_{Wt} \mid \mathcal{D}_t] \pi + \beta_{iW\mathcal{D}} E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] \pi \tag{B.3}$$

$$E[R_{it}^e \mid \mathcal{D}_t] = (\alpha_i + \beta_{i\mathcal{D}}) + (\beta_{iW} + \beta_{iW\mathcal{D}}) E[r_{Wt} \mid \mathcal{D}_t] \tag{B.4}$$

$$E[R_{it}^e r_{Wt} \mid \mathcal{D}_t] = (\alpha_i + \beta_{i\mathcal{D}}) E[r_{Wt} \mid \mathcal{D}_t] + (\beta_{iW} + \beta_{iW\mathcal{D}}) E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] \ , \tag{B.5}$$

where $\pi \equiv E[I(\mathcal{D}_t)]$ is the unconditional probability of disappointment. Also note that the occurrence of the upside event, the complement of the disappointing event, can be written as $I(\mathcal{U}_t) = 1 - I(\mathcal{D}_t)$, hence (B.1) can be rewritten as

$$\begin{aligned} R_{it}^e &= \alpha_i + \beta_{iW} r_{Wt} + \beta_{iW\mathcal{D}} r_{Wt} \cdot [1 - I(\mathcal{U}_t)] + \beta_{i\mathcal{D}} [1 - I(\mathcal{U}_t)] + \varepsilon_{it} \\ &= (\alpha_i + \beta_{i\mathcal{D}}) + (\beta_{iW} + \beta_{iW\mathcal{D}}) r_{Wt} - \beta_{iW\mathcal{D}} r_{Wt} \cdot I(\mathcal{U}_t) - \beta_{i\mathcal{D}} I(\mathcal{U}_t) + \varepsilon_{it} \ . \end{aligned} \tag{B.6}$$

Again, the mechanics of the OLS, namely $E[\varepsilon_{it} I(\mathcal{U}_t)] = E[\varepsilon_{it} r_{Wt} I(\mathcal{U}_t)] = 0$, gives us

$$E[R_{it}^e | \mathcal{U}_t] = \alpha_i + \beta_{iW} E[r_{Wt} | \mathcal{U}_t] \tag{B.7}$$

$$E[R_{it}^e r_{Wt} | \mathcal{U}_t] = \alpha_i E[r_{Wt} | \mathcal{U}_t] + \beta_{iW} E\left[r_{Wt}^2 | \mathcal{U}_t\right] \ . \tag{B.8}$$

Using (B.4) and (B.5), it can be shown that the market downside beta is

$$\beta_i^- \equiv \frac{Cov\left(R_{it}^e, r_{Wt}|\mathcal{D}_t\right)}{Var\left(r_{Wt}|\mathcal{D}_t\right)} = \frac{E\left[R_{it}^e r_{Wt}|\mathcal{D}_t\right] - E\left[R_{it}^e|\mathcal{D}_t\right] E\left[r_{Wt}|\mathcal{D}_t\right]}{Var\left(r_{Wt}|\mathcal{D}_t\right)} = \beta_{iW} + \beta_{iW\mathcal{D}} \tag{B.9}$$

Using (B.7) and (B.8), the upside beta is

$$\beta_i^+ \equiv \frac{Cov\left(R_{it}^e, r_{Wt}|\mathcal{U}_t\right)}{Var\left(r_{Wt}|\mathcal{U}_t\right)} = \frac{E\left[R_{it}^e r_{Wt}|\mathcal{U}_t\right] - E\left[R_{it}^e|\mathcal{U}_t\right] E\left[r_{Wt}|\mathcal{U}_t\right]}{Var\left(r_{Wt}|\mathcal{U}_t\right)} = \beta_{iW} \tag{B.10}$$

Finally, using (B.2) and (B.3) it can be shown that

$$Cov\left(R_{it}^e, r_{Wt}\right) = \beta_{iW} Var\left(r_{Wt}\right) + \beta_{iW\mathcal{D}} Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right) + \beta_{i\mathcal{D}} Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right) . \tag{B.11}$$

Hence, the CAPM beta is

$$\beta_i \equiv \frac{Cov\left(R_{it}^e, r_{Wt}\right)}{Var\left(r_{Wt}\right)} = \beta_{iW} + \beta_{iW\mathcal{D}} \underbrace{\frac{Cov\left(r_{Wt} I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)}}_{\equiv\gamma_1} + \beta_{i\mathcal{D}} \underbrace{\frac{Cov\left(I\left(\mathcal{D}_t\right), r_{Wt}\right)}{Var\left(r_{Wt}\right)}}_{\equiv\gamma_2} \tag{B.12}$$

Using (B.9) and (B.10), the model in (19) can be written as

$$E\left[R_{it}^e\right] = \lambda^+ \beta_i^+ + \lambda^- \beta_i^- = \left(\lambda^+ + \lambda^-\right) \beta_{iW} + \lambda^- \beta_{iW\mathcal{D}} . \tag{B.13}$$

Using (B.9) and (B.12) the model in (21) can be written as

$$\begin{aligned} E\left[R_i^e\right] &= \lambda \beta_i + \lambda^- \left(\beta_i^- - \beta_i\right) \\ &= \lambda \beta_{iW} + \left(\gamma_1 \lambda + \left(1 - \gamma_1\right) \lambda^-\right) \beta_{iW\mathcal{D}} + \gamma_2 \left(\lambda - \lambda^-\right) \beta_{i\mathcal{D}} . \end{aligned} \tag{B.14}$$

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Table 1: Risk premiums for the CAPM and GDA models using stock portfolios

	25 S×BM	25 S×Mom	10 S,B,M	25 S×OP	25 S×INV
A. CAPM					
λ_W	0.0050^{i}	0.0050^{i}	0.0050^{i}	0.0050^{i}	0.0050^{i}
RMSPE	30.6 [0.40]	39.4 [0.52]	24.9 [0.39]	23.9 [0.33]	29.0 [0.38]
B. GDA3					
λ_W	0.0065***	0.0070***	0.0065***	0.0066***	0.0064***
	(0.0016)	(0.0008)	(0.0008)	(0.0017)	(0.0013)
$\lambda_{\mathcal{D}}$	0.0367^{i}	-0.2449^{i}	-0.2022^{i}	-0.0917^{i}	0.1895^{i}
$\lambda_{W\mathcal{D}}$	0.0125	0.0256***	0.0197***	0.0173*	0.0060
	(0.0091)	(0.0070)	(0.0061)	(0.0103)	(0.0062)
RMSPE	25.7 [0.34]	23.6 [0.31]	19.8 [0.31]	18.8 [0.26]	22.6 [0.30]
C. GDA5					
λ_W	0.0078***	0.0068***	0.0064***	0.0069***	0.0072***
	(0.0015)	(0.0008)	(0.0007)	(0.0017)	(0.0009)
$\lambda_{\mathcal{D}}$	-0.1825^{i}	-0.1673^{i}	-0.1717^{i}	-0.0778^{i}	0.1363^{i}
$\lambda_{W\mathcal{D}}$	0.0261**	0.0202**	0.0171***	0.0181*	0.0135*
	(0.0114)	(0.0094)	(0.0063)	(0.0105)	(0.0070)
λ_X	-0.0021^{i}	-0.0018^{i}	-0.0012^{i}	-0.0024^{i}	-0.0035^{i}
$\lambda_{X\mathcal{D}}$	-0.0036^{i}	-0.0023^{i}	-0.0017^{i}	-0.0026^{i}	-0.0040^{i}
a	0.8462	0.4692	0.5335	0.3275	1.2827
	(0.5485)	(1.0453)	(0.7009)	(0.5046)	(1.1586)
RMSPE	21.4 [0.28]	20.7 [0.27]	18.7 [0.29]	17.5 [0.24]	16.9 [0.22]

The table shows risk premium estimates for the CAPM and GDA models using five different sets of US stock portfolios as test assets: (i) 25 (5×5) portfolios formed on size and book-to-market, (ii) 25 (5×5) portfolios formed on size and momentum, (iii) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum portfolios, (iv) 25 (5×5) portfolios formed on size and operating profitability, and (v) 25 (5×5) portfolios formed on size and investment. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table 2: Risk premiums for the CAPM and GDA models using further asset classes

Stocks		25 S×BM	25 S×Mom	6 S×BM	6 S×Mom
Options	54	24	24	6	6
Currencies				6	6
A. CAPM					
λ_W	0.0053^i	0.0053^i	0.0053^i	0.0051^i	0.0051^i
RMSPE	43.8 [0.71]	39.8 [0.57]	42.4 [0.60]	49.4 [0.70]	50.3 [0.71]
B. GDA3					
λ_W	0.0068***	0.0067***	0.0067***	0.0066***	0.0064***
	(0.0006)	(0.0004)	(0.0004)	(0.0005)	(0.0005)
$\lambda_{\mathcal{D}}$	-0.1672^i	-0.1442^i	-0.1863^i	-0.2294^i	-0.2884^i
$\lambda_{W\mathcal{D}}$	0.0179***	0.0166***	0.0178***	0.0205***	0.0210***
	(0.0059)	(0.0056)	(0.0041)	(0.0053)	(0.0044)
RMSPE	11.9 [0.19]	21.5 [0.31]	21.1 [0.30]	20.5 [0.29]	19.8 [0.28]
C. GDA5					
λ_W	0.0070***	0.0070***	0.0067***	0.0069***	0.0065***
	(0.0008)	(0.0009)	(0.0005)	(0.0007)	(0.0008)
$\lambda_{\mathcal{D}}$	-0.2927^i	-0.2250^i	-0.1974^i	-0.3753^i	-0.3418^i
$\lambda_{W\mathcal{D}}$	0.0228***	0.0201***	0.0179***	0.0265***	0.0222***
	(0.0035)	(0.0057)	(0.0047)	(0.0060)	(0.0066)
λ_X	-0.0006^i	-0.0007^i	-0.0012^i	-0.0001^i	-0.0002^i
$\lambda_{X\mathcal{D}}$	-0.0017^i	-0.0020^i	-0.0016^i	-0.0014^i	-0.0007^i
a	0.5820	0.7026	0.3816	0.5259	0.3170
	(0.6764)	(0.8943)	(0.9693)	(0.4416)	(0.5694)
RMSPE	10.0 [0.16]	19.1 [0.27]	18.8 [0.27]	18.6 [0.26]	17.3 [0.24]

The table shows risk premium estimates for the CAPM and GDA models using various sets of test assets: (i) 54 index option portfolios from Constantinides et al. (2013); (ii) 25 (5×5) size/book-to-market and 24 index option portfolios; (iii) 25 (5×5) size/momentum and 24 index option portfolios; (iv) 6 size/book-to-market, 6 option, and 6 currency (from Lettau et al. 2014) portfolios; and (v) 6 size/momentum, 6 option, and 6 currency portfolios. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table 3: Risk premiums when the perfect market pricing restriction is not imposed

Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. GDA3					
λ_W	0.0069***	0.0078***	0.0056*	0.0070**	0.0070**
	(0.0022)	(0.0021)	(0.0032)	(0.0032)	(0.0034)
$\lambda_{\mathcal{D}}$	-0.0473	-0.3519***	-0.2438***	-0.2200**	-0.2783***
	(0.0896)	(0.1247)	(0.0606)	(0.0940)	(0.1015)
$\lambda_{W\mathcal{D}}$	0.0103	0.0272***	0.0185***	0.0204***	0.0210***
	(0.0067)	(0.0067)	(0.0045)	(0.0047)	(0.0044)
RMSPE	24.8 [0.33]	21.8 [0.29]	11.6 [0.19]	20.3 [0.29]	19.4 [0.27]
B. GDA5					
λ_W	0.0081***	0.0080***	0.0059*	0.0079**	0.0078***
	(0.0023)	(0.0020)	(0.0031)	(0.0031)	(0.0030)
$\lambda_{\mathcal{D}}$	-0.2785*	-0.3071***	-0.3384***	-0.3741***	-0.3397**
	(0.1518)	(0.1061)	(0.0632)	(0.1256)	(0.1374)
$\lambda_{W\mathcal{D}}$	0.0262***	0.0238***	0.0229***	0.0273***	0.0241***
	(0.0093)	(0.0065)	(0.0035)	(0.0056)	(0.0058)
λ_X	-0.0012^i	-0.0011^i	-0.0007^i	0.0001^i	-0.0005^i
$\lambda_{X\mathcal{D}}$	-0.0027^i	-0.0016^i	-0.0012^i	-0.0016^i	-0.0013^i
a	0.8483**	0.4193	0.4008*	0.6212**	0.4324
	(0.3863)	(0.3943)	(0.2398)	(0.2573)	(0.3674)
RMSPE	20.6 [0.27]	16.4 [0.22]	9.7 [0.16]	17.8 [0.25]	16.1 [0.23]
C. Unrestricted GDA5					
λ_W	0.0089***	0.0082***	0.0086***	0.0093***	0.0080***
	(0.0023)	(0.0021)	(0.0028)	(0.0032)	(0.0029)
$\lambda_{\mathcal{D}}$	-0.6088***	-0.2242**	-0.3744***	-0.3245***	-0.1945*
	(0.1640)	(0.1060)	(0.0588)	(0.1195)	(0.1165)
$\lambda_{W\mathcal{D}}$	0.0355***	0.0205***	0.0312***	0.0279***	0.0157**
	(0.0094)	(0.0061)	(0.0062)	(0.0065)	(0.0074)
λ_X	-0.0050***	-0.0025***	-0.0022***	-0.0032	-0.0035*
	(0.0011)	(0.0008)	(0.0006)	(0.0020)	(0.0020)
$\lambda_{X\mathcal{D}}$	-0.0063***	-0.0031***	-0.0045***	-0.0053**	-0.0043**
	(0.0013)	(0.0008)	(0.0012)	(0.0026)	(0.0019)
RMSPE	18.7 [0.25]	15.4 [0.20]	9.3 [0.15]	11.1 [0.16]	13.1 [0.18]

The table shows risk premium estimates for GDA models using various sets of test portfolios (the same sets of portfolios as in Table 4) *without* imposing the restriction that the market portfolio is perfectly priced. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by cross-price restrictions for the GDA5. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table 4: Risk premiums for alternative models

Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. VOL					
λ_W	0.0054***	0.0054***	0.0058***	0.0057***	0.0057***
	(0.0005)	(0.0005)	(0.0001)	(0.0002)	(0.0002)
λ_X	-0.0023^i	-0.0024^i	-0.0030^i	-0.0035^i	-0.0036^i
RMSPE	25.9 [0.34]	35.4 [0.46]	14.2 [0.23]	26.2 [0.37]	26.8 [0.38]
B. Ang et al. (2006a)					
λ_W	0.0066***	0.0070***	0.0070***	0.0072***	0.0072***
	(0.0017)	(0.0008)	(0.0005)	(0.0006)	(0.0005)
$\lambda_{\mathcal{D}}$	0^i	0^i	0^i	0^i	0^i
$\lambda_{W\mathcal{D}}$	0.0140^i	0.0169^i	0.0138^i	0.0175^i	0.0171^i
RMSPE	25.8 [0.34]	28.3 [0.37]	19.5 [0.31]	25.9 [0.36]	28.4 [0.40]
C. Lettau et al. (2014)					
λ_W	0.0065***	0.0066***	0.0069***	0.0073***	0.0071***
	(0.0016)	(0.0009)	(0.0006)	(0.0006)	(0.0005)
$\lambda_{\mathcal{D}}$	0.0557^i	0.0607^i	0.0531^i	0.0896^i	0.0835^i
$\lambda_{W\mathcal{D}}$	0.0116^i	0.0122^i	0.0112^i	0.0146^i	0.0140^i
RMSPE	25.7 [0.34]	31.0 [0.41]	24.0 [0.39]	30.4 [0.43]	33.3 [0.47]
D. Carhart (1997)					
λ_W	0.0054***	0.0052***	0.0059***	0.0054***	0.0053***
	(0.0002)	(0.0000)	(0.0004)	(0.0004)	(0.0001)
λ_{SMB}	0.0026^i	0.0020^i	0.0117^i	0.0023^i	0.0023^i
λ_{HML}	0.0047***	0.0076**	0.0451	0.0041	0.0102
	(0.0014)	(0.0034)	(0.0372)	(0.0026)	(0.0106)
λ_{WML}	0.0254	0.0073***	0.0018	0.0168	0.0064
	(0.0203)	(0.0021)	(0.0181)	(0.0335)	(0.0039)
RMSPE	11.2 [0.15]	13.9 [0.18]	19.4 [0.31]	45.6 [0.64]	44.4 [0.63]

The table shows risk premium estimates for different models using various sets of test assets: (i) 25 (5×5) size/book-to-market portfolios; (ii) 25 (5×5) size/momentum portfolios; (iii) 54 index option portfolios from Constantinides et al. (2013); (iv) 6 size/book-to-market, 6 option, and 6 currency (from Lettau et al. 2014) portfolios; and (v) 6 size/momentum, 6 option, and 6 currency portfolios. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by the restriction in (20) for the model in Panel B and the restriction in (22) for the model in Panel C). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table 5: Fit of models when the same risk premiums are used

Stocks	S×BM	S×Mom	S,B,M	S×OP	S×INV		S×BM	S×Mom	S×BM	S×Mom
Options						✓	✓	✓	✓	✓
Currencies									✓	✓
A. Stocks only										
CAPM	30.6	39.4	24.9	23.9	29.0	43.8	40.5	42.8	49.7	50.5
GDA3	27.2	25.1	19.8	19.4	25.8	**<u>12.9</u>**	**22.7**	**22.4**	**<u>20.8</u>**	**<u>21.1</u>**
GDA5	**25.2**	**21.8**	**18.7**	**18.4**	24.3	**<u>12.9</u>**	**<u>20.8</u>**	**<u>20.8</u>**	**23.0**	**22.7**
Ang et al.	25.8	28.7	21.9	19.1	23.4	22.3	25.3	27.1	27.8	29.9
Lettau et al.	26.0	31.7	23.0	19.4	23.4	26.7	28.1	29.7	35.0	36.8
VOL	26.1	35.5	23.4	19.1	**23.0**	20.9	25.8	28.3	33.1	34.0
Carhart	**<u>14.0</u>**	**<u>17.7</u>**	**<u>9.7</u>**	**<u>13.9</u>**	**<u>15.0</u>**	39.7	34.3	34.1	46.1	45.8
B. Options only										
CAPM	28.7	38.6	24.1	21.8	26.9	43.8	39.8	42.4	48.9	49.9
GDA3	25.7	**24.9**	**<u>18.6</u>**	**<u>17.9</u>**	23.9	**11.9**	**21.7**	**<u>21.3</u>**	**21.3**	**21.6**
GDA5	**<u>24.6</u>**	**<u>21.7</u>**	**18.8**	**18.1**	24.7	**<u>10.0</u>**	**<u>20.2</u>**	**21.8**	**<u>19.0</u>**	**<u>19.0</u>**
Ang et al.	**25.2**	29.3	21.6	18.9	**22.7**	19.5	24.4	26.0	27.5	29.5
Lettau et al.	25.3	31.7	23.0	19.4	**<u>22.4</u>**	24.0	26.7	28.8	31.9	34.1
VOL	26.6	36.2	24.0	20.6	23.0	14.2	24.4	26.5	26.8	27.8
Carhart	233.4	155.5	157.7	154.6	169.0	19.4	180.9	126.0	149.2	91.8
C. Stocks, options, and currencies										
CAPM	29.9	39.1	24.6	23.1	28.2	43.7	40.2	42.6	49.4	50.3
GDA3	28.1	25.5	**<u>19.4</u>**	19.9	26.9	**14.9**	**24.3**	**23.0**	**20.9**	**19.8**
GDA5	30.3	**25.0**	**20.3**	22.1	29.5	**<u>13.1</u>**	**<u>22.3</u>**	**<u>22.0</u>**	**<u>19.2</u>**	**<u>17.3</u>**
Ang et al.	**<u>26.0</u>**	28.6	21.9	**19.8**	23.6	24.2	26.8	29.1	25.9	28.4
Lettau et al.	**26.1**	32.1	23.9	20.7	**23.3**	28.8	29.3	32.1	30.5	33.3
VOL	27.6	36.5	24.6	21.9	23.9	16.6	26.4	27.8	26.2	26.8
Carhart	32.1	**<u>17.0</u>**	24.7	**<u>17.6</u>**	**<u>16.2</u>**	36.4	41.4	33.2	49.4	44.4

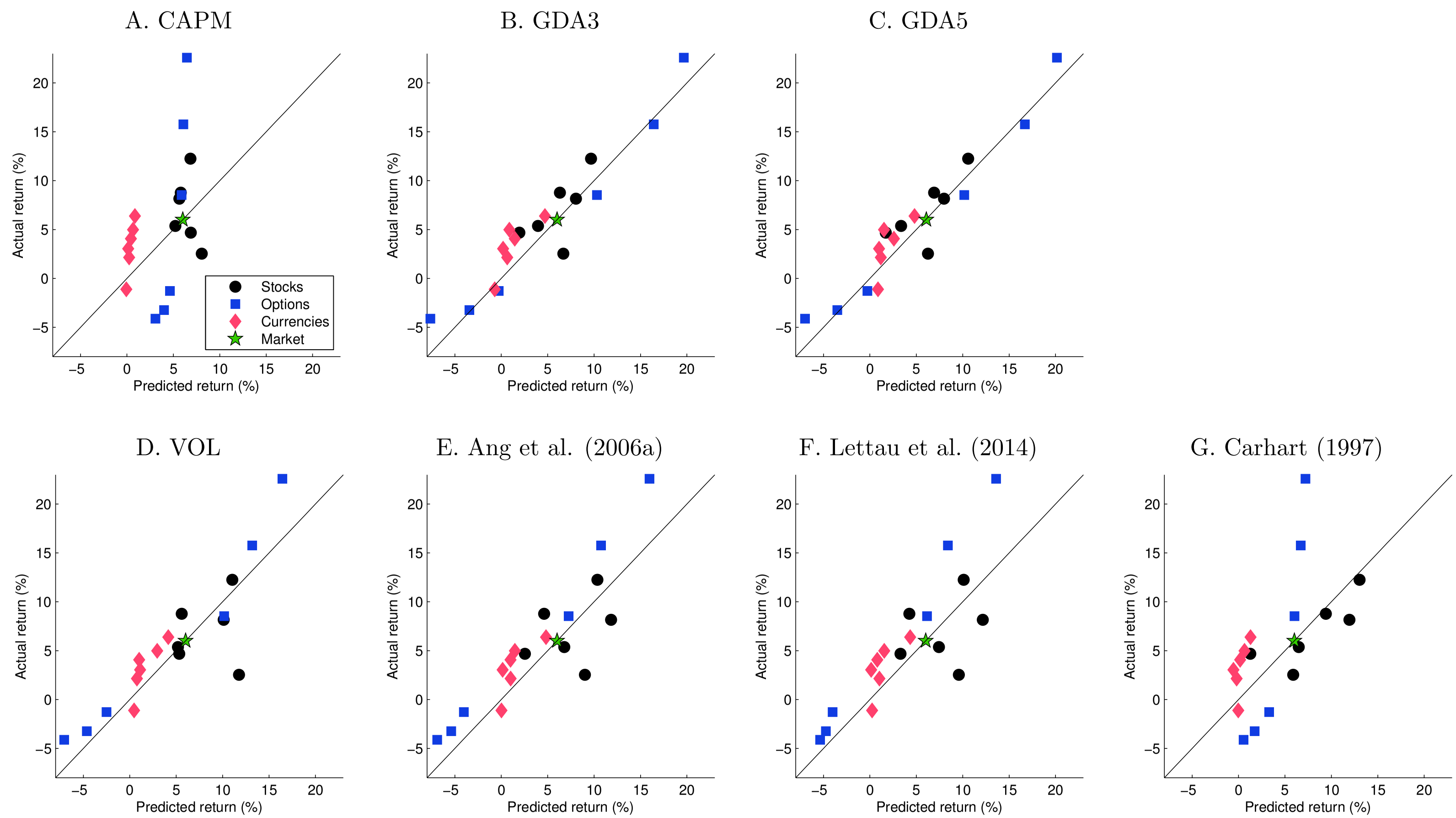
The table shows the root mean squared pricing error (reported in basis points per month) of different models on different sets of portfolios when the same risk premiums are used across all sets. The shaded column in all three panels indicates which set is used for estimating the risk premiums. In Panel A, 10 size, 10 book-to-market, and 10 momentum portfolios are used to estimate the λ-s (third column of Table 1). In Panel B, 54 index option portfolios are used to estimate the λ-s (first column of Table 2 and third column of Table 4). In Panel C, 6 size/momentum, 6 option, and 6 currency portfolios are used to estimate the λ-s (last column of Table 2 and Table 4). The test portfolios in the first five columns are the same as in Table 1, while the test portfolios in the last five columns are the same as in Table 2. Within each column the lowest RMSPE value is boldfaced and underlined, while the second lowest RMSPE value is boldfaced.

Table 6: Risk premiums for the GDA models using additional asset classes

Stocks	6 S×BM	6 S×BM	6 S×BM	6 S×BM
Options	6	6	6	6
Currencies	6	6	6	6
Corp. bonds	5			5
Sov. bonds		6		6
Commodities			6	6
A. GDA3				
λ_W	0.0069***	0.0060***	0.0068***	0.0066***
	(0.0004)	(0.0004)	(0.0007)	(0.0006)
$\lambda_{\mathcal{D}}$	-0.1546^i	-0.3238^i	-0.1615^i	-0.1472^i
$\lambda_{W\mathcal{D}}$	0.0201***	0.0193***	0.0198***	0.0178***
	(0.0055)	(0.0051)	(0.0053)	(0.0052)
RMSPE	21.5 [0.34]	26.2 [0.42]	23.2 [0.35]	28.1 [0.50]
B. GDA5				
λ_W	0.0067***	0.0063***	0.0066***	0.0066***
	(0.0007)	(0.0006)	(0.0006)	(0.0005)
$\lambda_{\mathcal{D}}$	-0.2592^i	-0.3980^i	-0.2009^i	-0.1997^i
$\lambda_{W\mathcal{D}}$	0.0213***	0.0231***	0.0193***	0.0191***
	(0.0051)	(0.0052)	(0.0040)	(0.0040)
λ_X	-0.0010^i	-0.0002^i	-0.0014^i	-0.0015^i
$\lambda_{X\mathcal{D}}$	-0.0015^i	-0.0005^i	-0.0017^i	-0.0018^i
a	0.3170	0.3115	0.2545	0.3171
	(0.6596)	(0.3642)	(0.6878)	(0.5433)
RMSPE	20.0 [0.31]	23.5 [0.37]	23.1 [0.35]	26.9 [0.47]

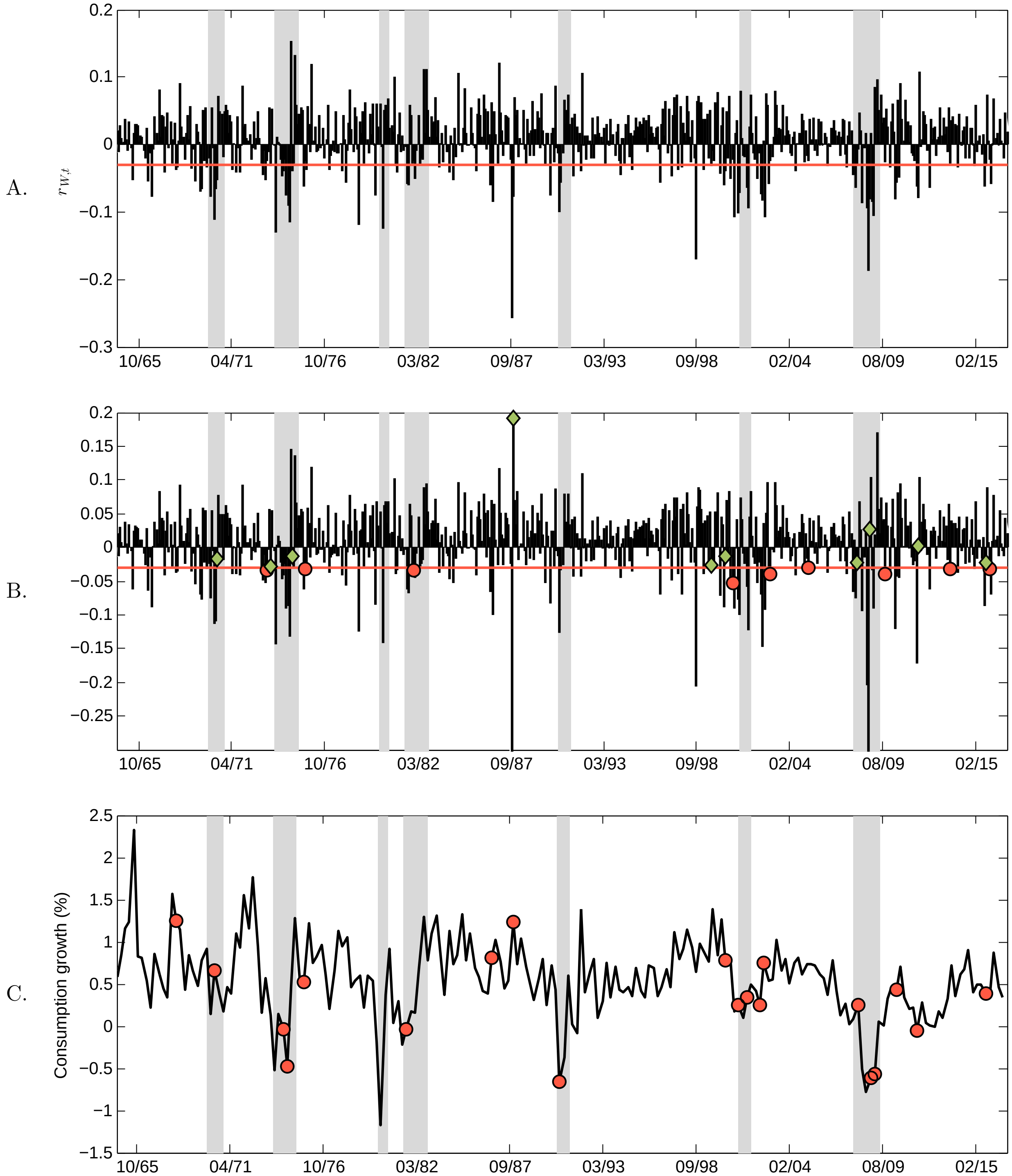
The table shows risk premium estimates for the GDA models when we add corporate bond, sovereign bond, and commodity futures portfolios to our benchmark set of test assets. The benchmark set of test assets consists of 6 stock portfolios (size/book-to-market), 6 option portfolios, and 6 currency portfolios. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Figure 1: Actual versus predicted returns



The scatter plots show the realized average excess returns for stock portfolios, option portfolios, currency portfolios, and the market return (see the legend in Panel A) against the predicted average excess returns from models reported in the last column of Table 2 (CAPM, GDA3, and GDA5) and Table 4 (VOL, Ang et al. (2006a), Lettau et al. (2014), and Carhart (1997)).

Figure 2: Disappointing states



The sample period in all the panels is from July 1964 to December 2016, and the shaded intervals correspond to NBER recession periods. Panel A shows the monthly market return ($r_{W,t}$) with the horizontal red line indicating the -3% level. Panel B shows the value of $r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt}$. The diamond markers in Panel B indicate months that are disappointing according to $\mathcal{D}_{At} \equiv \{r_{W,t} < -0.03\}$ but not disappointing according to $\mathcal{D}_{Bt} \equiv \left\{r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < -0.03\right\}$. The round markers in Panel B indicate months that are disappointing according to $\mathcal{D}_{Bt}$ but not disappointing according to $\mathcal{D}_{At}$. Panel C shows quarterly consumption growth and the round markers indicate quarters with two or three disappointing months.

Online Appendix to

"Downside Risks and the Cross-Section of Asset Returns"

February 28, 2017

This appendix contains additional details that are omitted from the main text for brevity.

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A.1 Derivation of the cross-sectional implications

In this section we outline the derivation of the cross-sectional implications of the GDA model and derive the sign restrictions on the risk prices.

A.1.1 Substituting out consumption

The logarithm of $M_{t-1,t}$ (denoted as $m_{t-1,t}$) and the disappointing event $\mathcal{D}_t$ are

$$m_{t-1,t} = \ln\delta - \gamma\Delta c_t - \left(\gamma - \frac{1}{\psi}\right)\Delta z_{Vt} \quad \text{and} \quad \mathcal{D}_t = \{\Delta c_t + \Delta z_{Vt} < \ln\kappa\} \ , \tag{A.1}$$

where

$$\Delta c_t \equiv \ln\left(\frac{C_t}{C_{t-1}}\right) = \ln C_t - \ln C_{t-1} \quad \text{and} \quad \Delta z_{Vt} \equiv \ln\left(\frac{V_t}{C_t}\right) - \ln\left(\frac{\mathcal{R}_{t-1}(V_t)}{C_{t-1}}\right) \tag{A.2}$$

represent the change in the log consumption level (or consumption growth) and the change in the log welfare valuation ratio (or welfare valuation ratio growth), respectively.

Following Epstein and Zin (1989), Hansen et al. (2007) and Routledge and Zin (2010) the log return on wealth is related to consumption growth and the welfare valuation ratio growth through

$$r_{Wt} = -\ln\delta + \Delta c_t + \left(1 - \frac{1}{\psi}\right)\Delta z_{Vt}. \tag{A.3}$$

Substituting out consumption growth using the above relationship, the equations in (A.1) can be rewritten as

$$m_{t-1,t} = (1-\gamma)\ln\delta - \gamma r_{Wt} - \left(\frac{\gamma - 1}{\psi}\right)\Delta z_{Vt} \quad \text{and} \quad \mathcal{D}_t = \{r_{Wt} + (1/\psi)\,\Delta z_{Vt} < \ln(\kappa/\delta)\} \, . \tag{A.4}$$

Note that the market return r_{Wt} is not directly observed by the econometrician. The return to a stock market index is sometimes used to proxy for this return as in Epstein and Zin

(1991). The welfare valuation ratios,

$$z_{Vt} \equiv \ln\left(V_t/C_t\right) \quad \text{and} \quad z_{\mathcal{R}t} \equiv \ln\left(\mathcal{R}_t\left(V_{t+1}\right)/C_t\right) \ , \tag{A.5}$$

are also unobservable. Following Hansen et al. (2008) and Bonomo et al. (2011), we can exploit the dynamics of aggregate consumption growth and the utility recursion, in addition to the definition of the certainty equivalent to solve for the unobserved welfare valuation ratios.

From equation (A.3) it follows that stochastic volatility of aggregate consumption growth is a sufficient condition for stochastic volatility of the market return. In that case, market volatility measures time-varying macroeconomic uncertainty. In all what follows, this additional assumption is coupled with our assumption on investors' preferences. More specifically, assume for example that the logarithm of consumption follows a heteroscedastic random walk as in Bonomo et al. (2011) were the stochastic volatility of consumption growth is an AR(1) process that can be well-approximated in population by a two-state Markov chain. Then it can be shown that the welfare valuation ratios satisfy

$$z_{Vt} = \varphi_{V0} + \varphi_{V\sigma}\sigma^2_{Wt} \ \text{ and } \ z_{\mathcal{R}t} = \varphi_{\mathcal{R}0} + \varphi_{\mathcal{R}\sigma}\sigma^2_{Wt} \tag{A.6}$$

were $\sigma^2_{Wt} \equiv Var_t\left[r_{Wt+1}\right]$ is the conditional variance of the market return, and were the drift coefficients φ_{V0} and $\varphi_{\mathcal{R}0}$ and the loadings $\varphi_{V\sigma}$ and $\varphi_{\mathcal{R}\sigma}$ depend on investor's preference parameters and on parameters of the consumption growth dynamics. In this case, $m_{t-1,t}$ and the disappointing event in equation (A.4) may be written as

$$\begin{aligned} m_{t-1,t} &= (1-\gamma)\ln\delta^* - \gamma r_{Wt} - \left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\Delta\sigma^2_{Wt} \\ \mathcal{D}_t &= \left\{r_{Wt} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma^2_{Wt} < \ln\left(\kappa/\delta^*\right)\right\}, \end{aligned} \tag{A.7}$$

where

$$\Delta\sigma_{Wt}^2 \equiv \sigma_{Wt}^2 - \frac{\varphi_{\mathcal{R}\sigma}}{\varphi_{V\sigma}}\sigma_{Wt-1}^2 \quad \text{and} \quad \ln\delta^* = \ln\delta + \frac{1}{\psi}\left(\varphi_{V0} - \varphi_{\mathcal{R}0}\right).$$

Our definitions and notations for Δz_{Vt} and $\Delta\sigma_{Wt}^2$ presume that $z_{\mathcal{R}t} \approx z_{Vt}$, meaning that $\varphi_{\mathcal{R}\sigma} \approx \varphi_{V\sigma}$. This shows that changes in the welfare valuation ratio can empirically be proxied by changes in a stock market volatility index, where volatility can be estimated by a generalized autoregressive conditional heteroscedasticity (GARCH) model, can be computed from high-frequency index returns (realized volatility), or can be measured by the option-implied volatility (VIX). Disappointment may occur due to a fall in the market return. It may also occur following a rise in market volatility. This means that the coefficient $\varphi_{V\sigma}$ in the definition of disappointment in (A.7) is negative. In fact, when macroeconomic uncertainty rises, everything else being equal, the investor is pessimistic about the future. She then assigns a low valuation to the continuation value and is willing to accept with certainty a lower welfare to avoid the risk in future consumption. Therefore, the ratio of welfare valuation to current consumption falls. We take as given that $\varphi_{V\sigma} < 0$ and $\varphi_{\mathcal{R}\sigma} \approx \varphi_{V\sigma}$, and we show in our calibration assessment in Section A.8 of this Online Appendix that this important theoretical implication of the model holds for a broad range of reasonable values of preference parameters.

Finally, the disappointing event in equation (A.7) may also be expressed as

$$\mathcal{D}_t = \left\{r_{Wt} - a\left(\sigma_W/\sigma_X\right)\Delta\sigma_{Wt}^2 < b\right\}\ , \tag{A.8}$$

with

$$a \equiv -\left(1/\psi\right)\varphi_{V\sigma}\left(\sigma_X/\sigma_W\right) \quad \text{and} \quad b \equiv \ln\left(\kappa/\delta^*\right), \tag{A.9}$$

where $\sigma_W = Std\left[r_{Wt}\right]$ and $\sigma_X = Std\left[\Delta\sigma_{Wt}^2\right]$ are the respective unconditional volatilities of the market return and changes in market volatility. Note that $\varphi_{V\sigma} < 0$ implies $a > 0$.

A.1.2 Cross-sectional implications of GDA preferences

For every asset i, optimal consumption and portfolio choice by the representative investor induces a restriction on the simple excess return R_{it}^e that is implied by the Euler condition:

$$E_{t-1}\left[M_{t-1,t}^{GDA}R_{it}^e\right] = 0 \ , \tag{A.10}$$

where $R_{it}^e = R_{it} - R_{ft}$ denotes the excess return, R_{it} is the simple gross return of asset i, and R_{ft} denotes the risk-free simple gross return. Using the definition of $M_{t-1,t}^{GDA}$, equation (A.10) can be written as

$$\begin{aligned} E_{t-1}\left[M_{t-1,t}\left(\frac{1+\ell I\left(\mathcal{D}_t\right)}{1+\kappa^{1-\gamma}\ell E_{t-1}\left[I\left(\mathcal{D}_t\right)\right]}\right)R_{it}^e\right] &= 0 \\ E_{t-1}\left[M_{t-1,t}\left(1+\ell I\left(\mathcal{D}_t\right)\right)R_{it}^e\right] &= 0 \ . \end{aligned} \tag{A.11}$$

By the law of iterated expectations, the above expression also holds unconditionally:

$$E\left[M_{t-1,t}\left(1+\ell I\left(\mathcal{D}_t\right)\right)R_{it}^e\right] = 0 \ . \tag{A.12}$$

Dividing both sides by $E\left[M_{t-1,t}\right]$, we get

$$E\left[H_{t-1,t}\left(1+\ell I\left(\mathcal{D}_t\right)\right)R_{it}^e\right] = 0 \ , \tag{A.13}$$

where $H_{t-1,t}$ denotes the risk-adjustment density defined by

$$H_{t-1,t} \equiv \frac{M_{t-1,t}}{E\left[M_{t-1,t}\right]} \approx 1 + m_{t-1,t} - E\left[m_{t-1,t}\right]. \tag{A.14}$$

The log-linear approximation of the nonlinear risk-adjustment density $H_{t-1,t}$ as shown in equation (A.14) is common in the asset pricing literature (see for example Yogo, 2006).

After some algebraic manipulation, (A.13) may be written as

$$E\left[R_{it}^{e}\right]=\frac{1}{1+\ell\pi^{\mathbb{H}}}\left[Cov\left(R_{it}^{e},-H_{t-1,t}\right)+\ell Cov\left(R_{it}^{e},-H_{t-1,t}I\left(\mathcal{D}_{t}\right)\right)\right] \tag{A.15}$$

where $E^{\mathbb{H}}[\cdot]$ denotes the expectation under the risk-adjustment density $H_{t-1,t}$ and $\pi^{\mathbb{H}} \equiv E^{\mathbb{H}}[I(\mathcal{D}_t)]$ is the risk-adjusted disappointment probability. Equation (A.15) shows that an asset premium is the sum of two covariances. The first covariance $Cov(R_{it}^{e}, -H_{t-1,t})$ is the compensation for regular risks, while the second covariance $Cov(R_{it}^{e}, -H_{t-1,t}I(\mathcal{D}_t))$ reveals compensation for downside risks conditional upon disappointment.

Using the approximation (A.14) in the pricing relation (A.15), we obtain the cross-sectional linear factor model from the main text:

$$E\left[R_{it}^{e}\right]=p_{W}\sigma_{iW}+p_{\mathcal{D}}\sigma_{i\mathcal{D}}+p_{W\mathcal{D}}\sigma_{iW\mathcal{D}}+p_{X}\sigma_{iX}+p_{X\mathcal{D}}\sigma_{iX\mathcal{D}}\ , \tag{A.16}$$

where the risk prices are given by:

$$\begin{aligned}
p_{W} &= \frac{1}{1+\ell\pi^{\mathbb{H}}}\gamma \\
p_{\mathcal{D}} &= -\frac{1}{1+\ell\pi^{\mathbb{H}}}\ell\left(1+\gamma\mu_{W}+\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\mu_{X}\right) \\
p_{W\mathcal{D}} &= \frac{1}{1+\ell\pi^{\mathbb{H}}}\ell\gamma \\
p_{X} &= \frac{1}{1+\ell\pi^{\mathbb{H}}}\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma} \\
p_{X\mathcal{D}} &= \frac{1}{1+\ell\pi^{\mathbb{H}}}\ell\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\ ,
\end{aligned} \tag{A.17}$$

and where $\mu_W \equiv E[r_{Wt}]$ and $\mu_X \equiv E[\Delta\sigma_{Wt}^2]$ are the means of the market return and changes in market volatility, respectively.

Let us consider the signs of these risk prices. The consumption-based asset pricing literature generally agrees on $\gamma > 1$, which implies $p_W > 0$. Thus, investors require a premium for a security that has positive covariance with the market return. Maintaining the assumption that $\gamma > 1$, it follows from equation (A.17) that $p_X \neq 0$ if and only if $\psi < \infty$. Thus, compensation for covariance with changes in market volatility is due to imperfect intertemporal substitution. The representative investor's risk aversion $\gamma > 1$ and imperfect intertemporal substitution $\psi < \infty$ together imply that $p_{X,t} < 0$. The next observation is that $p_{\mathcal{D}} \neq 0$ if and only if $\ell \neq 0$, regardless of the values of γ and ψ. Compensation for covariance with the downstate factor $I(\mathcal{D}_t)$ is exclusively due to disappointment aversion. Since $\ell \geq 0$, the associated risk price is negative, $p_{\mathcal{D},t} < 0$. Next, $p_{W\mathcal{D}} \neq 0$ if and only if both $\gamma \neq 0$ and $\ell \neq 0$. Both risk aversion and disappointment aversion are needed to explain the required compensation for covariance with the market downside factor. Risk aversion $\gamma > 1$ and disappointment aversion $\ell > 0$ together imply that $p_{W\mathcal{D}} > 0$. Finally, $p_{X\mathcal{D}} \neq 0$ if and only if $\gamma \neq 1$, $\ell \neq 0$, and $\psi \neq \infty$ are all satisfied. Thus, risk aversion, disappointment aversion, and imperfect intertemporal substitution of the representative investor are all needed to explain the required compensation for covariance with the volatility downside factor. Recall that we take $\varphi_{V\sigma} < 0$ as given , so $\gamma > 1$, $\ell > 0$, and $\psi < \infty$ together imply that $p_{X\mathcal{D}} < 0$.

There are two cross-price restrictions that are implied by the risk prices in (A.17). First, it can be easily seen that

$$\frac{p_{W\mathcal{D}}}{p_W} = \frac{p_{X\mathcal{D}}}{p_X} \ . \tag{A.18}$$

Second, using the equations for $p_{W\mathcal{D}}$ and $p_{X\mathcal{D}}$, and the definition of a in (A.9), we can write

$$p_{X\mathcal{D}} = -a\frac{\sigma_W}{\sigma_X}\frac{\gamma-1}{\gamma}p_{W\mathcal{D}} \ . \tag{A.19}$$

If we further assume that the risk aversion, γ, of the representative investor is high enough, then $\frac{\gamma-1}{\gamma} \approx 1$, and (A.19) simplifies to

$$p_{X\mathcal{D}} = -a\frac{\sigma_W}{\sigma_X}p_{W\mathcal{D}} \ . \tag{A.20}$$

When estimating the GDA5 model in the paper, we use the assumption $\frac{\gamma-1}{\gamma} = 1$. We also considered $\frac{\gamma-1}{\gamma}=0.75$ (which corresponds to $\gamma=3$), and the (unreported) empirical results are similar to those in the main text.

A.2 Additional restriction that the market is perfectly priced

When the test asset is the market return ($i=W$), the GDA5 model can be written as

$$E\left[R^e_{Wt}\right]=\lambda_W\beta_{WW}+\lambda_{\mathcal{D}}\beta_{W\mathcal{D}}+\lambda_{W\mathcal{D}}\beta_{WW\mathcal{D}}+\lambda_X\beta_{WX}+\lambda_{X\mathcal{D}}\beta_{WX\mathcal{D}}\ , \tag{A.21}$$

where the betas are calculated from the regression

$$R^e_{Wt}=\alpha_W+\beta_{WW}r_{Wt}+\beta_{W\mathcal{D}}I\left(\mathcal{D}_t\right)+\beta_{WW\mathcal{D}}r_{Wt}I\left(\mathcal{D}_t\right)+\beta_{WX}\Delta\sigma^2_{Wt}+\beta_{WX\mathcal{D}}\Delta\sigma^2_{Wt}I\left(\mathcal{D}_t\right)+\varepsilon_{Wt} \tag{A.22}$$

Since the return to be explained (the simple excess return on the market, R^e_{Wt}) and the market factor (the log-return on the market, r_{Wt}) are not exactly the same, non of the betas from the above regression will be zero. Hence, for (A.21) to hold, we can impose the following restriction on the downstate premium:

$$\lambda_{\mathcal{D}}=\frac{E\left[R^e_{Wt}\right]}{\beta_{W\mathcal{D}}}-\lambda_W\frac{\beta_{WW}}{\beta_{W\mathcal{D}}}-\lambda_{W\mathcal{D}}\frac{\beta_{WW\mathcal{D}}}{\beta_{W\mathcal{D}}}-\lambda_X\frac{\beta_{WX}}{\beta_{W\mathcal{D}}}-\lambda_{X\mathcal{D}}\frac{\beta_{WX\mathcal{D}}}{\beta_{W\mathcal{D}}}\ . \tag{A.23}$$

A similar restriction can be derived if we do not pick the downstate premium, but another one instead (e.g., λ_W or $\lambda_{W\mathcal{D}}$). Also, it is straightforward to derive a similar restriction for the GDA3 model. When requiring the market to be perfectly priced, we impose the linear restriction in (A.23) on the downstate premium.

If the market factor is the simple excess return on the market, then (A.22) becomes

$$R^e_{Wt}=\alpha'_W+\beta'_{WW}R^e_{Wt}+\beta'_{W\mathcal{D}}I\left(\mathcal{D}_t\right)+\beta'_{WW\mathcal{D}}R^e_{Wt}I\left(\mathcal{D}_t\right)+\beta'_{WX}\Delta\sigma^2_{Wt}+\beta'_{WX\mathcal{D}}\Delta\sigma^2_{Wt}I\left(\mathcal{D}_t\right)+\varepsilon_{Wt}\ . \tag{A.24}$$

It is easy to see that in this case $\beta'_{WW} = 1$ and $\alpha'_W = \beta'_{W\mathcal{D}} = \beta'_{WW\mathcal{D}} = \beta'_{WX} = \beta'_{WX\mathcal{D}} = 0$. Hence, (A.21) becomes

$$E\left[R^e_{Wt}\right] = \lambda_W \ . \tag{A.25}$$

That is, imposing the restriction that the market is priced correctly is equivalent to setting the market premium equal to the expected excess return on the market. Table A.1 shows the risk premium estimates for the GDA models with the restriction that the market return is correctly priced when R^e_{Wt} is used as the market factor.

A.3 Further risk premium estimates

This section provides risk premium estimates from various specifications that are left out from the main text for brevity.

Table 3 of the main text reports risk premium estimates for the GDA3, GDA5, and unrestricted GDA5 models without imposing the restriction that the market portfolio is perfectly priced using five selected sets of portfolios. Results for the other five sets of portfolios from the benchmark analysis are presented in Table A.2.

Table 4 of the main text reports risk premium estimates for alternative models using five selected sets of portfolios from our benchmark analysis. Results for the other five sets of portfolios from the benchmark analysis are presented in Table A.3.

Table 6 of the main text shows risk premium estimates for the GDA models when corporate bonds, sovereign bonds, and commodities are added to the set of test assets. Corresponding results for the alternative models considered in the paper are presented in Table A.4.

We also consider the robustness of our results when different test portfolios (compared to the main text) are chosen to represent a given asset class. The sources of the return data are described in Appendix A of the main text. There are two additional sets of portfolios used here: 10 US stock portfolios sorted by industry (10 Ind) from Kenneth French's website and six currency portfolios from Lustig et al. (2011). Lustig et al. (2011) use 35 currencies to create six portfolios by sorting them based on their respective interest rates. The sample period of the original paper is from November 1983 to December 2009, but the authors provide an updated version of the return data on their website.* We use data up to December 2013. The risk premium estimates for the GDA3 and GDA5 models are presented in Table A.5. Conclusions regarding the signs, magnitudes, and statistical significances of the risk premiums are very similar to those obtained in the main text for the benchmark test portfolios.

A.4 Different disappointment thresholds

For our main results the disappointment threshold is set to $b = -0.03$. Table A.6 and Table A.7 present risk premium estimates for the GDA models using the values $b \in \{0, -0.015, -0.04\}$. In the following discussion, we focus our attention to the results corresponding to the GDA5 in Table A.7.

When $b = -0.04$, the disappointment threshold becomes lower. The disappointment probability with $\mathcal{D}_t = \{r_{Wt} < -0.04\}$ and using the period between 1964 and 2013 is 12.3%, which is very close to the 16.3% obtained in our benchmark scenario with $b = -0.03$. Consequently, the results remain similar: all the estimated risk premiums in Panel C of Table A.7 are statistically significant and have the expected signs (the single exception is $\lambda_{\mathcal{D}}$ for the size/book-to-market portfolios, which is not statistically significant, but has the expected sign). The magnitudes of the premiums are similar to the benchmark scenario. In terms of model fit, the $b = -0.04$ specification provides lower RMSPE for the size/book-to-market and the option portfolios, but the $b = -0.03$ specification provides lower pricing errors for the other three portfolios.

As the threshold becomes higher, disappointment is triggered more easily. The disappointment probability with $\mathcal{D}_t = \{r_{Wt} < b\}$ is 26.7% for $b = -0.015$, and 38.5% for $b = 0$.

*Return data on the currency portfolios of Lustig et al. (2011) are obtained from Adrien Verdelhan's website at http://web.mit.edu/adrienv/www/Data.html

Risk premium estimates for the GDA5 with these thresholds are reported in Panel A and B of Table A.7, respectively. The estimated risk premiums, with the exception of $\lambda_{\mathcal{D}}$, have the expected sign and the estimates are statistically significant. As the disappointment threshold increases, the premium on the downstate factor becomes insignificant. In some cases it becomes positive and statistically significant. That is, disappointing events should be sufficiently out in the left tail so that the downstate factor is priced in the cross-section. In terms of model fit, the lowest RMSPE is provided by the models with low disappointment threshold (either $b = -0.03$ or $b = -0.04$) for all five sets of portfolios reported in Table A.7.

A.5 Different measures of market volatility

In this section we explore how the estimates for the GDA5 model change if different measures of market volatility are considered. In the main text, monthly volatility is measured as the realized volatility of the daily market returns during the month:

$$\sigma_{Wt}^2 = \sum_{\tau=1}^{N_t} \left(r_{Wt,\tau} - \mu_{Wt}\right)^2 \ , \tag{A.26}$$

where $r_{Wt,\tau}$ is the daily market return on the τ-th trading day of month t, μ_{Wt} is the mean of the daily market returns in month t, and N_t is the number of trading days in month t.

The alternative measures considered here are the option-implied volatility index (VIX), realized volatility calculated from intra-daily market returns, and a model implied volatility calculated using an EGARCH specification. The option-implied monthly volatility is calculated as

$$\sigma_{Wt}^{2,VIX} = \frac{1}{N_t} \sum_{\tau=1}^{N_t} \left(\frac{VIX_{t,\tau}}{100 \cdot \sqrt{12}} \right)^2 \ , \tag{A.27}$$

where $VIX_{t,\tau}$ is the value of the VIX index on the τ-th trading day of month t. The daily value of the VIX index is obtained from CBOE through the WRDS service. Monthly realized volatility from intra-daily market returns is calculated as

$$\sigma_{Wt}^{2,RV} = \sum_{\tau=1}^{N_t} \sum_{j=1}^{N_\tau} r_{Wt,\tau,j}^2 \ , \tag{A.28}$$

where $r_{Wt,\tau,j}$ denotes the 10-minute log return series on the τ-th trading day of month t and N_τ is the number intra-daily returns within a trading day. We use intra-daily return series of the S&P 500. The data comes from Olsen Financial Technologies. Finally, in the model based approach, we fit a model with conditional heteroskedasticity to the daily log market return series $r_{W\tau}$. We consider the EGARCH(1,1,1) by Nelson (1991),

$$\begin{aligned} r_{W\tau} &= \mu + \sigma_{W\tau}\varepsilon_\tau \ , \quad \text{with} \ \ \varepsilon_\tau \overset{\text{iid}}{\sim} \mathcal{N}(0,1) \\ \ln\left(\sigma_{W\tau}^2\right) &= \omega + \nu\left(|\varepsilon_\tau| - \sqrt{2/\pi}\right) + \theta\varepsilon_\tau + \phi \ln\left(\sigma_{W\tau-1}^2\right) \end{aligned} \tag{A.29}$$

Then the model-implied monthly volatility is calculated as

$$\sigma_{Wt}^{2,EGARCH} = \sum_{\tau=1}^{N_t} \hat{\sigma}_{Wt,\tau}^2 \ , \tag{A.30}$$

where $\hat{\sigma}_{Wt,\tau}^2$ is the estimated daily variance on the τ-th trading day of month t. Change in monthly volatility for all of the above measures is calculated as

$$\Delta\sigma_{Wt}^2 = \sigma_{Wt}^2 - \sigma_{Wt-1}^2 \ . \tag{A.31}$$

Note that the measures are available for different time periods. The VIX data is available starting from 1986 and our intra-daily return data covers only the period from February 1986 to September 2010. The model implied volatility is available for the entire sample period. We use the longest possible sample for each specification.

Risk premium estimates are presented in Table A.8. The results are similar across different volatility measures. The signs on the risk premiums are as expected and, apart from a few cases, the estimated risk premiums are statistically significant. It is hard to compare the model fit across volatility measures, since the panels in Table A.8 correspond to different sample periods. However, the RMSPE to root-mean-squared-returns ratios (reported in brackets) are similar across the different measures.

A.6 Additional scatter plots of results in the main text

Figure A.1 to Figure A.4 show scatter plots of actual versus predicted returns corresponding to three different sets of portfolios and seven asset pricing models. The models are the same as in Figure 1 of the main text, and the portfolios are also from the main text (for detailed description see Appendix A of the main text):

- 6 (3×2) size/book-to-market, 6 option, and 6 currency portfolios in Figure A.1,
- 25 (5×5) size/book-to-market portfolios in Figure A.2,
- 25 (5×5) size/momentum portfolios in Figure A.3, and
- 54 option portfolios from Constantinides et al. (2013) in Figure A.4.

A.7 Option sensitivities to the GDA factors

In the emprical analysis we use the index option portfolios of Constantinides et al. (2013), who create leverage-adjusted (to have a target CAPM beta of one) portfolios of S&P 500 index options sorted on moneyness. To achieve a target CAPM beta of one, they approximate the elasticity of the options with respect to the market index with the elasticity implied by the Black and Scholes (1973) model:

$$\vartheta_W \equiv \left.\frac{\partial \pi}{\partial S}\right|_{S=S_0} \frac{S_0}{\pi_0} \ , \tag{A.32}$$

where S_0 is the current price of the underlying, π_0 is the current price of the option, and the partial derivative is calculated from the Black-Scholes formula. Then they create a hypothetical portfolio that invests ϑ_W^{-1} dollars in the option and $1 - \vartheta_W^{-1}$ dollars in the risk-free rate. In our empirical analysis, we use these hypothetical portfolios. Panel A of Figure A.5 shows the ϑ_W values of options with different moneyness (K/S_0) levels.[†] Note that the elasticity is $\vartheta_W > 1$ for call options and $\vartheta_W < -1$ for put options. Therefore, a leverage-adjusted call option portfolio consists of a long position in a fraction of a call and some investment in the risk-free rate, while a leverage-adjusted put portfolio consists of a short position in a fraction of a put and more than 100% investment in the risk-free rate.

To assess the options' sensitivity to the market downside factor, we calculate a measure inspired by the ϑ_W of Constantinides et al. (2013): the sensitivity to changes in the price of the underlying after a 5% drop in the price of the underlying. That is, we calculate

$$\vartheta_{W\mathcal{D}} \equiv \left.\frac{\partial \pi}{\partial S}\right|_{S=0.95S_0} \frac{S_0}{\pi_0} . \tag{A.33}$$

Since the index option portfolios we analyze invest ϑ_W^{-1} fraction into the option, the sensitivity of these portfolios to the market downside factor is $\vartheta_W^{-1}\vartheta_{W\mathcal{D}}$. This value is shown in Panel B of Figure A.5 for different moneyness levels. OTM put options have the largest sensitivity, followed by ITM puts, then ITM calls, and finally OTM calls. For comparison, we show various betas of the option portfolios in Table A.9. The market downside beta, $\beta_{iW}^{-} = \frac{Cov\left(R_{it}^e, r_{Wt}|\mathcal{D}_t\right)}{Var(r_{Wt}|\mathcal{D}_t)}$, measures the portfolio's sensitivity to the market, given disappointment. Note that since $\vartheta_{W\mathcal{D}}$ is only an approximation based on the Black-Scholes formula, we do not expect the $\vartheta_W^{-1}\vartheta_{W\mathcal{D}}$ and β_{iW}^{-} values to exactly coincide. However, it is clear that the ordering of the β_{iW}^{-} values in Table A.9 is the same as that of the $\vartheta_W^{-1}\vartheta_{W\mathcal{D}}$ values in Panel B of Figure A.5.

[†]We use $S_0 = 10$, $T = 1/12$ (one month maturity), 30% annual volatility for the underlying, and a risk-free rate of zero when creating the plots in Figure A.5. The general conclusions do not hinge on these particular parameter values.

To assess the options' sensitivity to volatility, we calculate

$$\vartheta_X \equiv \left.\frac{\partial \pi}{\partial \sigma}\right|_{S=S_0} \frac{S_0}{\pi_0} \ , \tag{A.34}$$

where σ denotes the volatility of the underlying. Again, the sensitivity of the option portfolios can be calculated as $\vartheta_W^{-1}\vartheta_X$. This value is shown in Panel C of Figure A.5. OTM put options have the lowest sensitivity, followed by ITM puts, then ITM calls, and finally OTM calls have the highest sensitivity. This is in line with the ordering of the volatility betas in Table A.9, measured as $\beta_{iX} = \frac{Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}\right)}{Var\left(\Delta\sigma^2_{Wt}\right)}$.

Finally, to assess the sensitivity of these portfolios to the volatility downside factor, we calculate the sensitivity to changes in the volatility after the price of the underlying drops by 5%:

$$\vartheta_{X\mathcal{D}} \equiv \left.\frac{\partial \pi}{\partial \sigma}\right|_{S=0.95S_0} \frac{S_0}{\pi_0} \tag{A.35}$$

The $\vartheta_W^{-1}\vartheta_{X\mathcal{D}}$ values are shown in Panel D of Figure A.5. The sensitivities have the same ordering as in Panel C, which is in line with the ordering of the volatility downside betas in Table A.9, measured as $\beta^-_{iX} = \frac{Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}|\mathcal{D}_t\right)}{Var\left(\Delta\sigma^2_{Wt}|\mathcal{D}_t\right)}$.

A.8 Calibration assessment and estimation with individual stocks

In this section, we further strengthen our main empirical results by showing that they reflect a rational economic model where agents care about the level and volatility of consumption, and are aware of downside risk in consumption growth. In other words, in this section, we rationalize, in the context of a consumption-based reduced-form general equilibrium setting, the empirical evidence on cross-sectional asset pricing by GDA factors as presented and discussed in the main text.

We analyze the factor risk premiums, λ_f with $f \in \{W, X, \mathcal{D}, W\mathcal{D}, X\mathcal{D}\}$, generated by a GDA endowment economy, reasonably calibrated to match the risk-free rate and the aggregate stock market behavior. In setting up the calibration, we closely follow Bonomo et al. (2011). They study an asset pricing model with generalized disappointment aversion and long-run volatility risk and show that it produces first and second moments of price-dividend ratios and asset returns as well as return predictability patterns in line with the data. Using the same endowment dynamics, we focus on the cross-sectional implications by studying the model-implied disappointment probability and factor risk premiums.

We assume that consumption and equity dividend growth are conditionally normal, unpredictable, and their conditional variances fluctuate according to a two-state Markov chain:

$$\begin{aligned} \Delta c_t &= \mu + \sqrt{\omega_c\left(s_{t-1}\right)}\varepsilon_{ct} \\ \Delta d_t &= \mu + \nu_d \sqrt{\omega_c\left(s_{t-1}\right)}\varepsilon_{dt} \ , \end{aligned} \tag{A.36}$$

where Δc_t is the aggregate consumption growth, Δd_t is the equity dividend growth, s_{t-1} indicates the state of the world, and ε_{ct} and ε_{dt} follow a bivariate IID standard normal process with mean zero and correlation ρ. The two states of the economy naturally correspond to a low (L) and a high (H) volatility state.

The endowment dynamics is calibrated at the monthly frequency to match the sample mean, volatility, and first-order autocorrelation of the real annual US consumption growth and stock market dividend growth from 1930 to 2012. These moments remain stable if the data are updated until more recently. Panel A of Table A.10 shows the parameters of the calibrated endowment process. The state transition probabilities are $p_{LL} = 0.9989$ and $p_{HH} = 0.9961$, and the corresponding long-run probabilities are 78.9% and 21.1% for the low and high volatility states, respectively. We set the preference parameters similar to the benchmark calibration of Bonomo et al. (2011). The values are presented in Panel B of Table A.10. For the GDA3 model, we simply set $\psi = \infty$, everything else being equal.

The first set of results in Panel C shows that our calibration matches well the first and second moments of consumption and dividend growth in the data. The model-implied annualized (time-averaged) mean, volatility, and first-order autocorrelation of consumption growth are respectively 1.80%, 2.07%, and 0.25, and are consistent with the observed annual values of 1.84%, 2.20%, and 0.48, respectively. The mean, volatility, and first-order autocorrelation of dividend growth are respectively 1.80%, 13.29%, and 0.25, and the observed annual values are 1.05%, 13.02%, and 0.11, respectively.

Given these endowment dynamics, we solve for welfare valuation ratios in closed form, which we combine with consumption growth to derive the endogenous market return and market variance processes. We refer the reader to Bonomo et al. (2011) for formal derivations. The second set of results in Panel C of Table A.10 shows that the model generates moments of asset prices that are consistent with empirical evidence. The level of the risk-free rate, 0.46% for GDA3 and 0.76% for GDA5, is close to the actual value of 0.57%. The equity premium, 8.06% for GDA3 and 6.61% for GDA5, is slightly larger than the actual value of 5.50%, but remains comparable to other sample values estimated in the literature, for example 7.25% in Bonomo et al. (2011). The equity volatility generated by the model, 17.65% for GDA3 and 16.84% for GDA5, is also comparable to the actual value of 20.25%.

As mentioned earlier, the main purpose of this calibration is to study the model implications for the disappointing event and the GDA factor risk premiums. The model-implied disappointment probability and factor risk premiums are reported in Panel D of Table A.10. The unconditional model-implied monthly disappointment probability is 17.43% for the GDA3 model and 16.06% for the GDA5 model. These numbers are closely related to their corresponding empirical values of 16.3% and 16.0% respectively, as discussed in Section 3.2.1. of the main article. Let us focus now on the monthly model-implied factor risk premiums in Panel D of Table A.10. The market risk premium is equal to $\lambda_W = 0.0065$ for the GDA3 model, and $\lambda_W = 0.0042$ for the GDA5, while the volatility risk premium is $\lambda_X = 0$ for the GDA3, and $\lambda_X = -1.38 \times 10^{-6}$ in the GDA5 model. The market downside risk premium is $\lambda_{W\mathcal{D}} = 0.0038$ for the GDA3 model, and $\lambda_{W\mathcal{D}} = 0.0023$ for the GDA5, while the volatility downside risk premium is $\lambda_{X\mathcal{D}} = 0$ for the GDA3, and $\lambda_{X\mathcal{D}} = -1.16 \times 10^{-6}$ for the GDA5.

Finally, the downside risk premium is $\lambda_{\mathcal{D}} = -0.3494$ for the GDA3, and $\lambda_{\mathcal{D}} = -0.3010$ for the GDA5.

The λ values from the calibration are to be compared to their data counterparts estimated in the empirical section of the main text. Our benchmark for comparison are factor risk premium estimates when all three asset classes (stocks, index options, and currencies) are included in the estimation. The results are reported in the last two columns of Table 2 in the main text. The model-implied values of the market risk premium and the downstate risk premium compare favorably to their data counterparts as they lie within one or two standard errors around their estimated data counterparts. The remaining model-implied factor risk premiums are much lower in magnitude than the empirical estimates. However, the estimated values must be considered with care due to at least two main sources of bias. First, as discussed in Section 3 of the main article, the estimation uses an empirical proxy of the true market return with potentially very different properties, especially moments and dynamics. Second, our estimation in the main article uses standard sets of few portfolios as test assets. Ang et al. (2016), and Gagliardini et al. (2016) discuss cross-sectional tests using a large cross-section of individual stocks versus fewer portfolios. They prove theoretically and observe empirically that using portfolios may destroy important information necessary for obtaining efficient estimates of the cross-sectional risk premiums, and those risk premium estimates obtained from a large cross-section of individual stocks can substantially depart from risk premium estimates on standard sets of portfolios. Their main point is that individual stocks provide a much larger dispersion in betas, an important prior to cross-sectional tests. To illustrate the effect of the second point, we carry out an empirical exercise in the following subsection, where we use individual stocks to estimate factor risk premiums in the GDA models.

A.8.1 Risk premium estimates using individual stocks

We follow the methodology used by Ang et al. (2006). In particular, we use the two-stage cross-sectional regression method of Fama and MacBeth (1973). In the first stage, we use short-window regressions to estimate the stocks' sensitivities (betas) to the factors. For every month $t \geq 12$ in the sample, we use twelve months of daily data from month $t-11$ to month t to run a time-series regression for each stock i that has return data over the given period. For example, in case of the GDA5, we run the regression

$$R^e_{i,\tau} = \alpha_{i,t} + \beta_{iW,t} r_{W,\tau} + \beta_{iW\mathcal{D},t} r_{W,\tau} I\left(\mathcal{D}_\tau\right) + \beta_{i\mathcal{D},t} I\left(\mathcal{D}_\tau\right) + \beta_{iX,t} \Delta\sigma^2_{W,\tau} + \beta_{iX\mathcal{D},t} \Delta\sigma^2_{W,\tau} I\left(\mathcal{D}_\tau\right) + \varepsilon^i_\tau \tag{A.37}$$

where τ refers to daily observations over the one-year period and t refers to the current month. The second stage of the Fama-Macbeth procedure corresponds to estimating the cross-sectional regressions

$$R^e_{i,t+1} = \beta_{iW,t}\lambda_{W,t} + \beta_{iW\mathcal{D},t}\lambda_{W\mathcal{D},t} + \beta_{i\mathcal{D},t}\lambda_{\mathcal{D},t} + \beta_{iX,t}\lambda_{X,t} + \beta_{iX\mathcal{D},t}\lambda_{X\mathcal{D},t} + \eta^i_t, \tag{A.38}$$

where the dependent variable is the excess return for stock i in month $t+1$. That is the betas, calculated using data from months $t-11$ to t, are related to stock returns in the following moth $(t+1)$. These two steps are repeated for all months in the sample. The unconditional factor risk premiums are obtained by averaging the lambdas over the sample period, i.e., $\hat{\lambda}_f = \hat{E}\left[\lambda_{f,t}\right]$ for factor f. Since this approach uses overlapping information when calculating the betas, we calculate standard errors using the Newey and West (1987) estimator (with 12 lags).

We use all common stocks traded on the NYSE, AMEX and NASDAQ markets (the data comes from CRSP). The sample period is from July, 1963 to December, 2013. To measure daily market volatility used in the first stage regressions, we fit an exponential GARCH to the time series of daily market returns. Note that our unreported analysis shows that the risk premium estimates are robust to using alternatives ways to measure market volatility, including the options-implied volatility index (VIX), realized volatility from intra-daily market returns, or the volatility implied by different GARCH specifications. The disappointing event in the first-stage regressions is defined as $\mathcal{D}_\tau = \left\{ r_{W,\tau} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{W,\tau} < q_{0.16} \right\}$. Note that the disappointment threshold, $q_{0.16}$, is set in each one-year period for the first-stage regressions so that the disappointment probability (i.e., the percentage of disappointing days) is 16%. We apply this definition to match the 16% unconditional probability of disappointment from the empirical section of the main text. Also note that results are robust to varying the probability of disappointment between 15% and 20%.

Table A.11 shows the risk premium estimates for the GDA3 and several GDA5 models. We use $a = 0$ for the GDA3 and $a \in \{0, 0.5, 1\}$ for the GDA5. All the estimated risk premiums are statistically significant and have the expected signs. Moreover, for all risk factors, the estimated values are comparable in magnitude to the calibration-implied factor risk premiums in Panel D of Table A.10.

A.8.2 Sensitivity of the calibration results

We also conduct a sensitivity analysis of our calibration results. We study how the quantities of interest vary as preference parameters change within reasonable ranges. We set the regular risk aversion parameter γ and GDA threshold parameter κ to their base case values ($\gamma = 2.5$ and $\kappa = \delta = 0.998$) and vary the disappointment aversion parameter $\ell \in [1, 4]$ and the elasticity of intertemporal substitution $\psi \in \{0.75, 1, 1.5, \infty\}$. Results are shown in Figures A.6 and A.7. Panels A and B of Figure A.6 show that the model-implied annualized mean and volatility of the risk-free rate belong to a reasonable range of values used in the asset pricing literature. The same goes for the mean and volatility of the equity excess return in Panels G and H.

Panels C and D of Figure A.6 show that the welfare valuation ratios loads negatively on market volatility, consistent with the economic intuition that asset values and, consequently, investor's wealth and welfare fall in periods of high uncertainty in financial markets. The model-implied loadings of the welfare valuation ratios onto market volatility are $\varphi_{V\sigma}$ and $\varphi_{\mathcal{R}\sigma}$ are very close, as the ratio of loadings $\varphi_{\mathcal{R}\sigma}/\varphi_{V\sigma}$ is close to one. Thus, panels C and D confirm that $\varphi_{\mathcal{R}\sigma} < 0$ and $\varphi_{\mathcal{R}\sigma} \approx \varphi_{V\sigma}$ hold for reasonable preference parameter values.

Figure A.7 shows the sensitivity of the factor risk premiums. Again, the lower magnitudes of model-implied premiums compared to their estimated data counterparts may directly result from the fact that our empirical proxy of the market return, the return on a stock market index may have different time series properties than the true (but unobservable) market return, besides other sources of estimation bias such as the use of standard sets of fewer portfolios rather than a large cross-section of individual stocks. Factor risk premiums in Figure A.7 are order of magnitude comparable to estimates based on individual stocks reported in Table A.11. The signs of the risk premiums are, however, all consistent with economic intuition and our estimation results in the main text. Finally, Panel F of Figure A.7 shows the disappointment probability when we vary the disappointment aversion parameter ℓ.

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Table A.1: Risk premiums when the market is priced correctly and R^e_W is used

Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. GDA3					
λ_W	0.0050^i	0.0050^i	0.0052^i	0.0050^i	0.0050^i
$\lambda_{\mathcal{D}}$	0.0726	-0.2790	-0.1596	-0.1217	-0.2274*
	(0.1606)	(0.3145)	(0.3270)	(0.1499)	(0.1300)
$\lambda_{W\mathcal{D}}$	0.0096	0.0245***	0.0173*	0.0182***	0.0203***
	(0.0102)	(0.0079)	(0.0098)	(0.0060)	(0.0044)
RMSPE	27.4 [0.36]	22.2 [0.29]	12.3 [0.20]	22.4 [0.32]	22.2 [0.31]
B. GDA5					
λ_W	0.0050^i	0.0050^i	0.0052^i	0.0050^i	0.0050^i
$\lambda_{\mathcal{D}}$	-0.3276***	-0.2206*	-0.2351	-0.3039**	-0.2344**
	(0.1261)	(0.1209)	(0.1697)	(0.1303)	(0.1123)
$\lambda_{W\mathcal{D}}$	0.0256**	0.0198**	0.0196***	0.0234***	0.0192***
	(0.0129)	(0.0085)	(0.0052)	(0.0052)	(0.0041)
λ_X	-0.0011^i	-0.0013^i	-0.0014^i	-0.0012^i	-0.0013^i
$\lambda_{X\mathcal{D}}$	-0.0020^i	-0.0018^i	-0.0018^i	-0.0018^i	-0.0013^i
a	0.5012	0.4361	0.3826	0.3691	0.1154
	(0.5193)	(1.1508)	(0.8836)	(0.5489)	(0.6698)
RMSPE	24.0 [0.32]	19.8 [0.26]	11.7 [0.19]	22.1 [0.31]	20.8 [0.29]

The table shows risk premium estimates for the GDA models using various sets of test portfolios (in columns; the same sets of portfolios as in Table 4 of the main text). The simple excess return on the market (R^e_W) is used as the market factor as opposed to our benchmark specification, where the log market return (r_W) is used. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.2: Risk premiums when the perfect market pricing restriction is not imposed

Stocks	10 S,B,M	25 S×OP	25 S×INV	25 S×BM	25 S×Mom
Options				24	24
A. GDA3					
λ_W	0.0072***	0.0069***	0.0067***	0.0067**	0.0068**
	(0.0021)	(0.0022)	(0.0021)	(0.0031)	(0.0032)
$\lambda_{\mathcal{D}}$	-0.3075***	-0.2068**	0.0935	-0.1460*	-0.1847*
	(0.0984)	(0.0832)	(0.0784)	(0.0831)	(0.0955)
$\lambda_{W\mathcal{D}}$	0.0210***	0.0152*	0.0060	0.0167***	0.0178***
	(0.0062)	(0.0085)	(0.0064)	(0.0042)	(0.0040)
RMSPE	17.4 [0.27]	17.4 [0.24]	21.9 [0.29]	21.5 [0.31]	21.1 [0.30]
B. GDA5					
λ_W	0.0073***	0.0072***	0.0076***	0.0074**	0.0076***
	(0.0019)	(0.0022)	(0.0022)	(0.0032)	(0.0028)
$\lambda_{\mathcal{D}}$	-0.2662**	-0.1826**	-0.0427	-0.2266***	-0.1835
	(0.1134)	(0.0916)	(0.0966)	(0.0862)	(0.1183)
$\lambda_{W\mathcal{D}}$	0.0192***	0.0187**	0.0181**	0.0202***	0.0180***
	(0.0062)	(0.0094)	(0.0089)	(0.0051)	(0.0053)
λ_X	-0.0006^i	-0.0014^i	-0.0026^i	-0.0007^i	-0.0011^i
$\lambda_{X\mathcal{D}}$	-0.0012^i	-0.0019^i	-0.0033^i	-0.0020^i	-0.0019^i
a	0.5885	0.5336	0.7776	0.7026	0.6178
	(0.4240)	(0.5340)	(0.6250)	(0.4563)	(0.4908)
RMSPE	16.3 [0.26]	16.5 [0.23]	19.7 [0.26]	18.7 [0.27]	17.8 [0.25]
C. Unrestricted GDA5					
λ_W	0.0077***	0.0078***	0.0103***	0.0076**	0.0070**
	(0.0020)	(0.0021)	(0.0023)	(0.0032)	(0.0030)
$\lambda_{\mathcal{D}}$	-0.2697***	-0.3868***	-0.0376	-0.1883**	-0.1162
	(0.0929)	(0.1186)	(0.0892)	(0.0937)	(0.0964)
$\lambda_{W\mathcal{D}}$	0.0215***	0.0274***	0.0323***	0.0188***	0.0124**
	(0.0057)	(0.0096)	(0.0088)	(0.0064)	(0.0056)
λ_X	-0.0008	-0.0045***	-0.0055***	-0.0029***	-0.0031***
	(0.0008)	(0.0013)	(0.0009)	(0.0011)	(0.0011)
$\lambda_{X\mathcal{D}}$	-0.0013	-0.0054***	-0.0073***	-0.0040***	-0.0036***
	(0.0008)	(0.0016)	(0.0011)	(0.0015)	(0.0010)
RMSPE	16.2 [0.25]	13.6 [0.19]	17.0 [0.22]	16.6 [0.24]	16.5 [0.23]

The table shows risk premium estimates for GDA models using various sets of test portfolios *without* imposing the restriction that the market portfolio is perfectly priced. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by cross-price restrictions for the GDA5. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.3: Risk premiums for alternative models

	10 S,B,M	25 S×OP	25 S×INV	25 S×BM	25 S×Mom
Stocks	10 S,B,M	25 S×OP	25 S×INV	25 S×BM	25 S×Mom
Options				24	24
A. VOL					
λ_W	0.0053***	0.0054***	0.0055***	0.0057***	0.0058***
	(0.0005)	(0.0005)	(0.0005)	(0.0002)	(0.0001)
λ_X	-0.0019^i	-0.0021^i	-0.0025^i	-0.0026^i	-0.0028^i
RMSPE	23.4 [0.37]	19.1 [0.27]	22.6 [0.30]	24.1 [0.35]	26.4 [0.38]
B. Ang et al. (2006)					
λ_W	0.0066***	0.0065***	0.0065***	0.0069***	0.0069***
	(0.0007)	(0.0019)	(0.0015)	(0.0005)	(0.0004)
$\lambda_{\mathcal{D}}$	0^i	0^i	0^i	0^i	0^i
$\lambda_{W\mathcal{D}}$	0.0142^i	0.0132^i	0.0137^i	0.0135^i	0.0132^i
RMSPE	21.9 [0.34]	19.0 [0.26]	23.4 [0.31]	24.3 [0.35]	26.0 [0.37]
C. Lettau et al. (2014)					
λ_W	0.0062***	0.0063***	0.0066***	0.0068***	0.0068***
	(0.0009)	(0.0018)	(0.0015)	(0.0005)	(0.0004)
$\lambda_{\mathcal{D}}$	0.0360^i	0.0457^i	0.0577^i	0.0519^i	0.0480^i
$\lambda_{W\mathcal{D}}$	0.0095^i	0.0105^i	0.0118^i	0.0111^i	0.0107^i
RMSPE	23.0 [0.36]	19.3 [0.27]	23.0 [0.30]	26.7 [0.39]	28.7 [0.41]
D. Carhart (1997)					
λ_W	0.0051***	0.0054***	0.0053***	0.0058***	0.0055***
	(0.0000)	(0.0002)	(0.0001)	(0.0004)	(0.0001)
λ_{SMB}	0.0020^i	0.0014^i	0.0016^i	0.0021^i	0.0026^i
λ_{HML}	0.0033	0.0080**	0.0075***	0.0045**	0.0061
	(0.0025)	(0.0033)	(0.0018)	(0.0023)	(0.0070)
λ_{WML}	0.0062***	0.0194	0.0151	0.0289	0.0067*
	(0.0023)	(0.0171)	(0.0117)	(0.0330)	(0.0039)
RMSPE	9.7 [0.15]	10.8 [0.15]	9.5 [0.13]	32.1 [0.46]	32.4 [0.46]

The table shows risk premium estimates for different models using various sets of test portfolios. The details of the test portfolios are provided in Appendix A of the main paper. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by restrictions that are discussed in detail in the main text for the models in Panel B and Panel C). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.4: Risk premiums with additional asset classes

Stocks	6 S×BM	6 S×BM	6 S×BM	6 S×BM
Options	6	6	6	6
Currencies	6	6	6	6
Corp. bonds	5			5
Sov. bonds		6		6
Commodities			6	6
A. CAPM				
λ_W	0.0051^i	0.0051^i	0.0051^i	0.0051^i
RMSPE	45.5 [0.71]	44.5 [0.71]	48.2 [0.73]	42.4 [0.75]
B. Ang et al. (2006)				
λ_W	0.0073***	0.0068***	0.0069***	0.0066***
	(0.0005)	(0.0005)	(0.0007)	(0.0006)
$\lambda_{\mathcal{D}}$	0^i	0^i	0^i	0^i
$\lambda_{W\mathcal{D}}$	0.0177^i	0.0140^i	0.0146^i	0.0128^i
RMSPE	24.1 [0.38]	33.2 [0.53]	28.5 [0.43]	30.9 [0.55]
C. Lettau et al. (2014)				
λ_W	0.0073***	0.0066***	0.0066***	0.0064***
	(0.0005)	(0.0005)	(0.0007)	(0.0006)
$\lambda_{\mathcal{D}}$	0.0901^i	0.0560^i	0.0549^i	0.0429^i
$\lambda_{W\mathcal{D}}$	0.0146^i	0.0112^i	0.0111^i	0.0099^i
RMSPE	27.9 [0.44]	35.8 [0.57]	32.5 [0.49]	32.9 [0.58]
D. VOL				
λ_W	0.0057***	0.0057***	0.0057***	0.0056***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
λ_X	-0.0035^i	-0.0034^i	-0.0034^i	-0.0033^i
RMSPE	26.0 [0.41]	26.9 [0.43]	26.4 [0.40]	26.6 [0.47]
E. Carhart (1997)				
λ_W	0.0054***	0.0055***	0.0054***	0.0054***
	(0.0002)	(0.0004)	(0.0002)	(0.0001)
λ_{SMB}	0.0025^i	0.0020^i	0.0024^i	0.0022^i
λ_{HML}	0.0041	0.0045*	0.0044	0.0047
	(0.0030)	(0.0027)	(0.0032)	(0.0033)
λ_{WML}	0.0158	0.0238	0.0148	0.0168
	(0.0159)	(0.0295)	(0.0167)	(0.0121)
RMSPE	41.4 [0.65]	40.7 [0.65]	44.1 [0.66]	38.3 [0.68]

The table shows risk premium estimates for the GDA models when we add corporate bond, sovereign bond, and commodity futures portfolios to our benchmark set of test assets. The benchmark set of test assets consists of 6 stock portfolios (size/book-to-market), 6 option portfolios, and 6 currency portfolios. The premiums are estimated using GMM. Standard errors are in parenthesis. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.5: Risk premiums for additional test portfolios

Stocks	10 S,B,M	25 S×OP	25 S×INV	10 Ind	6 S×BM	6 S×Mom	10 Ind	6 S×Mom	10 Ind
Options	24	24	24	6	6	6	6	6	6
Currencies				6 Lettau et al.	6 Lustig et al.	6 Lustig et al.	6 Lustig et al.	6 Lettau et al.	6 Lettau et al.
Corp. bonds								5	5
Sov. bonds								6	6
Commodities								6	6
A. GDA3									
λ_W	0.0067***	0.0068***	0.0068***	0.0069***	0.0070***	0.0068***	0.0073***	0.0066***	0.0066***
	(0.0004)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
$\lambda_{\mathcal{D}}$	-0.1681^i	-0.1577^i	-0.1444^i	-0.1130^i	-0.2359^i	-0.2806^i	-0.1229^i	-0.1642^i	-0.1009^i
$\lambda_{W\mathcal{D}}$	0.0172***	0.0171***	0.0168***	0.0187***	0.0212***	0.0215***	0.0199***	0.0182***	0.0161***
	(0.0040)	(0.0056)	(0.0059)	(0.0045)	(0.0054)	(0.0043)	(0.0045)	(0.0044)	(0.0048)
RMSPE	17.5 [0.27]	20.1 [0.29]	21.2 [0.30]	24.9 [0.37]	20.6 [0.29]	20.3 [0.28]	24.8 [0.36]	28.4 [0.50]	29.0 [0.51]
B. GDA5									
λ_W	0.0067***	0.0069***	0.0069***	0.0068***	0.0069***	0.0068***	0.0071***	0.0066***	0.0066***
	(0.0006)	(0.0007)	(0.0011)	(0.0004)	(0.0006)	(0.0008)	(0.0004)	(0.0005)	(0.0005)
$\lambda_{\mathcal{D}}$	-0.1688^i	-0.1471^i	-0.1666^i	-0.1786^i	-0.2772^i	-0.3145^i	-0.1681^i	-0.2101^i	-0.1549^i
$\lambda_{W\mathcal{D}}$	0.0169***	0.0169***	0.0174**	0.0199***	0.0215***	0.0217***	0.0195***	0.0193***	0.0175***
	(0.0042)	(0.0062)	(0.0082)	(0.0040)	(0.0044)	(0.0052)	(0.0038)	(0.0035)	(0.0043)
λ_X	-0.0014^i	-0.0014^i	-0.0012^i	-0.0018^i	-0.0009^i	-0.0006^i	-0.0019^i	-0.0014^i	-0.0016^i
$\lambda_{X\mathcal{D}}$	-0.0017^i	-0.0021^i	-0.0020^i	-0.0021^i	-0.0013^i	-0.0009^i	-0.0022^i	-0.0017^i	-0.0019^i
a	0.4008	0.6178	0.6188	0.3454	0.3004	0.2532	0.3153	0.3171	0.3375
	(1.1999)	(1.3008)	(1.5671)	(0.5851)	(0.5550)	(0.5721)	(0.4546)	(0.6538)	(0.4698)
RMSPE	16.8 [0.26]	18.7 [0.27]	20.2 [0.29]	22.2 [0.33]	19.0 [0.27]	18.1 [0.25]	22.2 [0.32]	26.7 [0.47]	27.0 [0.48]

The table shows risk premium estimates for the GDA models using different sets of portfolios. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.6: Risk premiums for the GDA3 with alternative disappointment thresholds

	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. $b = 0$					
λ_W	0.0063***	0.0065***	0.0069***	0.0073***	0.0071***
	(0.0006)	(0.0006)	(0.0005)	(0.0006)	(0.0005)
$\lambda_{\mathcal{D}}$	-0.0046^i	-0.1026^i	0.0345^i	0.2780^i	0.1661^i
$\lambda_{W\mathcal{D}}$	0.0144***	0.0177***	0.0148***	0.0170***	0.0166***
	(0.0047)	(0.0041)	(0.0037)	(0.0040)	(0.0033)
RMSPE	25.8 [0.34]	23.2 [0.30]	12.6 [0.20]	17.6 [0.25]	22.6 [0.32]
B. $b = -0.015$					
λ_W	0.0054***	0.0068***	0.0069***	0.0072***	0.0069***
	(0.0007)	(0.0004)	(0.0005)	(0.0005)	(0.0006)
$\lambda_{\mathcal{D}}$	-0.2547^i	-0.1276^i	-0.0899^i	0.1276^i	-0.0202^i
$\lambda_{W\mathcal{D}}$	0.0104	0.0205***	0.0156***	0.0164***	0.0168***
	(0.0093)	(0.0048)	(0.0052)	(0.0043)	(0.0032)
RMSPE	25.4 [0.34]	23.5 [0.31]	12.6 [0.20]	21.8 [0.31]	23.8 [0.33]
C. $b = -0.04$					
λ_W	0.0066***	0.0071***	0.0069***	0.0070***	0.0069***
	(0.0016)	(0.0007)	(0.0005)	(0.0006)	(0.0004)
$\lambda_{\mathcal{D}}$	0.1962^i	-0.2206^i	-0.2370^i	-0.1697^i	-0.2096^i
$\lambda_{W\mathcal{D}}$	0.0051	0.0256***	0.0217*	0.0218***	0.0229***
	(0.0135)	(0.0077)	(0.0114)	(0.0057)	(0.0066)
RMSPE	20.9 [0.28]	25.3 [0.33]	11.6 [0.19]	22.6 [0.32]	24.1 [0.34]

The table shows risk premium estimates for the GDA3 model when the disappointing event is defined as $\mathcal{D}_t = \{r_{W,t} < b\}$. The value of b varies across panels. The test portfolios are the same as in Table 4 of the main text. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.7: Risk premiums for the GDA5 with alternative disappointment thresholds

Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. $b = 0$					
λ_W	0.0075***	0.0064***	0.0068***	0.0067***	0.0069***
	(0.0015)	(0.0004)	(0.0006)	(0.0008)	(0.0005)
$\lambda_{\mathcal{D}}$	0.2182^i	-0.1639^i	-0.0214^i	0.2496^i	0.1304^i
$\lambda_{W\mathcal{D}}$	0.0206*	0.0155***	0.0139***	0.0121**	0.0147*
	(0.0106)	(0.0050)	(0.0034)	(0.0051)	(0.0077)
λ_X	-0.0028^i	-0.0009^i	-0.0017^i	-0.0031^i	-0.0024^i
$\lambda_{X\mathcal{D}}$	-0.0033^i	-0.0013^i	-0.0018^i	-0.0029^i	-0.0026^i
a	0.4590	0.3673	0.2338	0.6317	0.2973
	(0.7554)	(0.5533)	(1.1975)	(0.8769)	(1.2784)
RMSPE	22.5 [0.30]	19.2 [0.25]	12.9 [0.21]	20.5 [0.29]	23.2 [0.33]
B. $b = -0.015$					
λ_W	0.0076***	0.0069***	0.0069***	0.0071***	0.0069***
	(0.0015)	(0.0007)	(0.0007)	(0.0006)	(0.0004)
$\lambda_{\mathcal{D}}$	-0.1470^i	-0.0870^i	-0.1235^i	0.1086^i	0.0519^i
$\lambda_{W\mathcal{D}}$	0.0235*	0.0186**	0.0156***	0.0153**	0.0145***
	(0.0139)	(0.0087)	(0.0037)	(0.0071)	(0.0031)
λ_X	-0.0017^i	-0.0019^i	-0.0015^i	-0.0027^i	-0.0023^i
$\lambda_{X\mathcal{D}}$	-0.0030^i	-0.0022^i	-0.0019^i	-0.0028^i	-0.0027^i
a	0.6799	0.3451	0.4072	0.1714	0.4094
	(0.4869)	(1.0155)	(0.9192)	(0.6534)	(1.7059)
RMSPE	22.1 [0.29]	20.7 [0.27]	13.0 [0.21]	21.6 [0.30]	24.5 [0.34]
C. $b = -0.04$					
λ_W	0.0068***	0.0071***	0.0069***	0.0067***	0.0066***
	(0.0017)	(0.0006)	(0.0009)	(0.0006)	(0.0007)
$\lambda_{\mathcal{D}}$	0.0388^i	-0.2720^i	-0.2940^i	-0.2000^i	-0.2915^i
$\lambda_{W\mathcal{D}}$	0.0131	0.0272***	0.0231***	0.0210***	0.0237***
	(0.0199)	(0.0080)	(0.0031)	(0.0058)	(0.0079)
λ_X	-0.0029^i	-0.0020^i	-0.0006^i	-0.0017^i	-0.0009^i
$\lambda_{X\mathcal{D}}$	-0.0029^i	-0.0022^i	-0.0015^i	-0.0017^i	-0.0014^i
a	0.1288	0.2422	0.5741	0.1025	0.3860
	(0.2882)	(0.7454)	(0.7205)	(1.0316)	(0.7472)
RMSPE	23.9 [0.32]	21.1 [0.28]	9.2 [0.15]	21.1 [0.30]	21.0 [0.30]

The table shows risk premium estimates for the GDA5 model when the disappointing event is defined as $\mathcal{D}_t = \left\{ r_{W,t} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{W,t} < b \right\}$. The value of b varies across panels. The test portfolios are the same as in Table 4 of the main text. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are not estimated, but are imposed. RMSPE is the root-mean-squared pricing error of the model in basis points (bps) per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.8: Risk premiums for the GDA5 using alternative volatility measures

	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Stocks	25 S×BM	25 S×Mom		6 S×BM	6 S×Mom
Options			54	6	6
Currencies				6	6
A. Option implied volatility (VIX)					
λ_W	0.0081***	0.0079***	0.0065***	0.0065***	0.0065***
	(0.0009)	(0.0008)	(0.0014)	(0.0011)	(0.0009)
$\lambda_{\mathcal{D}}$	-0.1071^i	-0.1310^i	-0.3226^i	-0.2698^i	-0.2470^i
$\lambda_{W\mathcal{D}}$	0.0152*	0.0160**	0.0209***	0.0207*	0.0201**
	(0.0083)	(0.0076)	(0.0063)	(0.0111)	(0.0089)
λ_X	-0.0010^i	-0.0012^i	-0.0005^i	-0.0008^i	-0.0009^i
$\lambda_{X\mathcal{D}}$	-0.0016^i	-0.0014^i	-0.0008^i	-0.0010^i	-0.0010^i
a	1.2625	0.4006	0.4546	0.2778	0.1595
	(2.0932)	(1.4324)	(0.8337)	(1.2991)	(1.2682)
RMSPE	23.8 [0.30]	23.6 [0.29]	12.6 [0.20]	21.4 [0.30]	20.3 [0.29]
B. Realized volatlity (intra-daily)					
λ_W	0.0058***	0.0065***	0.0063***	0.0067***	0.0064***
	(0.0012)	(0.0007)	(0.0010)	(0.0004)	(0.0007)
$\lambda_{\mathcal{D}}$	0.1079^i	-0.1728^i	-0.2112^i	-0.1944^i	-0.2820^i
$\lambda_{W\mathcal{D}}$	0.0037	0.0170*	0.0177***	0.0194***	0.0211***
	(0.0109)	(0.0099)	(0.0034)	(0.0040)	(0.0053)
λ_X	-0.0011^i	-0.0008^i	-0.0006^i	-0.0008^i	-0.0006^i
$\lambda_{X\mathcal{D}}$	-0.0010^i	-0.0009^i	-0.0008^i	-0.0011^i	-0.0008^i
a	0.9802	0.2201	0.4378	0.4429	0.3235
	(2.2929)	(0.6148)	(1.8549)	(1.1562)	(1.1562)
RMSPE	23.7 [0.34]	25.0 [0.36]	12.4 [0.21]	19.6 [0.28]	18.1 [0.25]
C. Model implied volatility (EGARCH)					
λ_W	0.0069***	0.0070***	0.0068***	0.0066***	0.0065***
	(0.0017)	(0.0009)	(0.0022)	(0.0004)	(0.0008)
$\lambda_{\mathcal{D}}$	-0.0060^i	-0.1715^i	-0.2701^i	-0.1978^i	-0.2736^i
$\lambda_{W\mathcal{D}}$	0.0155	0.0214***	0.0206*	0.0196***	0.0212***
	(0.0132)	(0.0076)	(0.0109)	(0.0048)	(0.0049)
λ_X	-0.0010^i	-0.0008^i	-0.0004^i	-0.0004^i	-0.0002^i
$\lambda_{X\mathcal{D}}$	-0.0012^i	-0.0010^i	-0.0006^i	-0.0006^i	-0.0004^i
a	0.8603	0.4252	0.4020	0.2069	0.1105
	(0.8198)	(0.7559)	(2.4298)	(1.3322)	(0.8260)
RMSPE	20.6 [0.27]	19.7 [0.26]	11.4 [0.18]	20.5 [0.29]	19.3 [0.27]

The table shows risk premium estimates for the GDA5 model when market volatility is measured in different ways (in panels). The test portfolios are the same as in Table 3 of the main text. The premiums are estimated using GMM. Standard errors are in parenthesis. Values with the superscript i are not estimated, but are imposed. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Table A.9: Betas of the option portfolios

	Return	Betas			
	$E[R^e_{it}]$	β_{iW}	β^-_{iW}	β_{iX}	β^-_{iX}
Call, 5% OTM	-3.45	0.64	0.26	0.33	0.71
Call, ATM	-1.32	0.75	0.36	-0.25	0.35
Call, 5% ITM	1.13	0.80	0.45	-0.78	-0.03
Put, 5% ITM	5.78	0.92	0.76	-2.64	-1.60
Put, ATM	8.99	0.97	0.88	-3.35	-2.13
Put, 5% OTM	16.02	1.01	0.99	-4.13	-2.76

The table presents retunrs and betas of various index option portfolios. The first column presents the annual average excess return of the portfolios. The rest of the table reports the market beta $\beta_{iW} = \frac{Cov(R^e_{it}, r_{Wt})}{Var(r_{Wt})}$, the market downside beta $\beta^-_{iW} = \frac{Cov(R^e_{it}, r_{Wt}|\mathcal{D}_t)}{Var(r_{Wt}|\mathcal{D}_t)}$, the volatility beta $\beta_{iX} = \frac{Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}\right)}{Var\left(\Delta\sigma^2_{Wt}\right)}$, and the volatility downside beta $\beta^-_{iX} = \frac{Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}|\mathcal{D}_t\right)}{Var\left(\Delta\sigma^2_{Wt}|\mathcal{D}_t\right)}$ of the portfolios. The disappointing event is $\mathcal{D}_t = \{r_{Wt} < -0.03\}$.

Table A.10: Model calibration

A. Endowment parameters	B. Preference parameters
$\mu = 0.15\%$, $\sqrt{\omega_c(L)} = 0.46\%$, $\sqrt{\omega_c(H)} = 1.32\%$, $\nu_d = 6.42$, $\rho = 0.3$, $p_{HH} = 0.9961$, $p_{LL} = 0.9989$	$\delta = 0.998$, $\gamma = 2.5$, $\ell = 2.33$, $\kappa = 0.998$

C. Endowment and asset pricing moments

	Sample	GDA3	GDA5
$E[\Delta c_t]$ (%)	1.84	1.80	1.80
$\sigma[\Delta c_t]$ (%)	2.20	2.07	2.07
$AC1(\Delta c_t)$	0.48	0.25	0.25
$E[\Delta d_t]$ (%)	1.05	1.80	1.80
$\sigma[\Delta d_t]$ (%)	13.02	13.29	13.29
$AC1(\Delta d_t)$	0.11	0.25	0.25
$Corr(\Delta c_t, \Delta d_t)$	0.52	0.30	0.30
$E[pd]$ (%)	3.33	2.72	2.89
$\sigma[pd]$ (%)	0.44	0.20	0.11
$E[r_f]$ (%)	0.57	0.46	0.76
$\sigma[r_f]$ (%)	3.77	0.15	1.55
$E[r - r_f]$ (%)	5.50	8.06	6.61
$\sigma[r - r_f]$ (%)	20.25	17.65	16.84

D. Downside event and factor risk premiums

	GDA3	GDA5
ψ	∞	1.5
a	0.00	1.38
b (%)	0.00	-0.10
$Prob(\mathcal{D})$ (%)	17.43	16.09
λ_W	0.0065	0.0042
$\lambda_{\mathcal{D}}$	-0.3494	-0.3010
$\lambda_{W\mathcal{D}}$	0.0038	0.0023
λ_X		-1.38E-6
$\lambda_{X\mathcal{D}}$		-1.16E-6

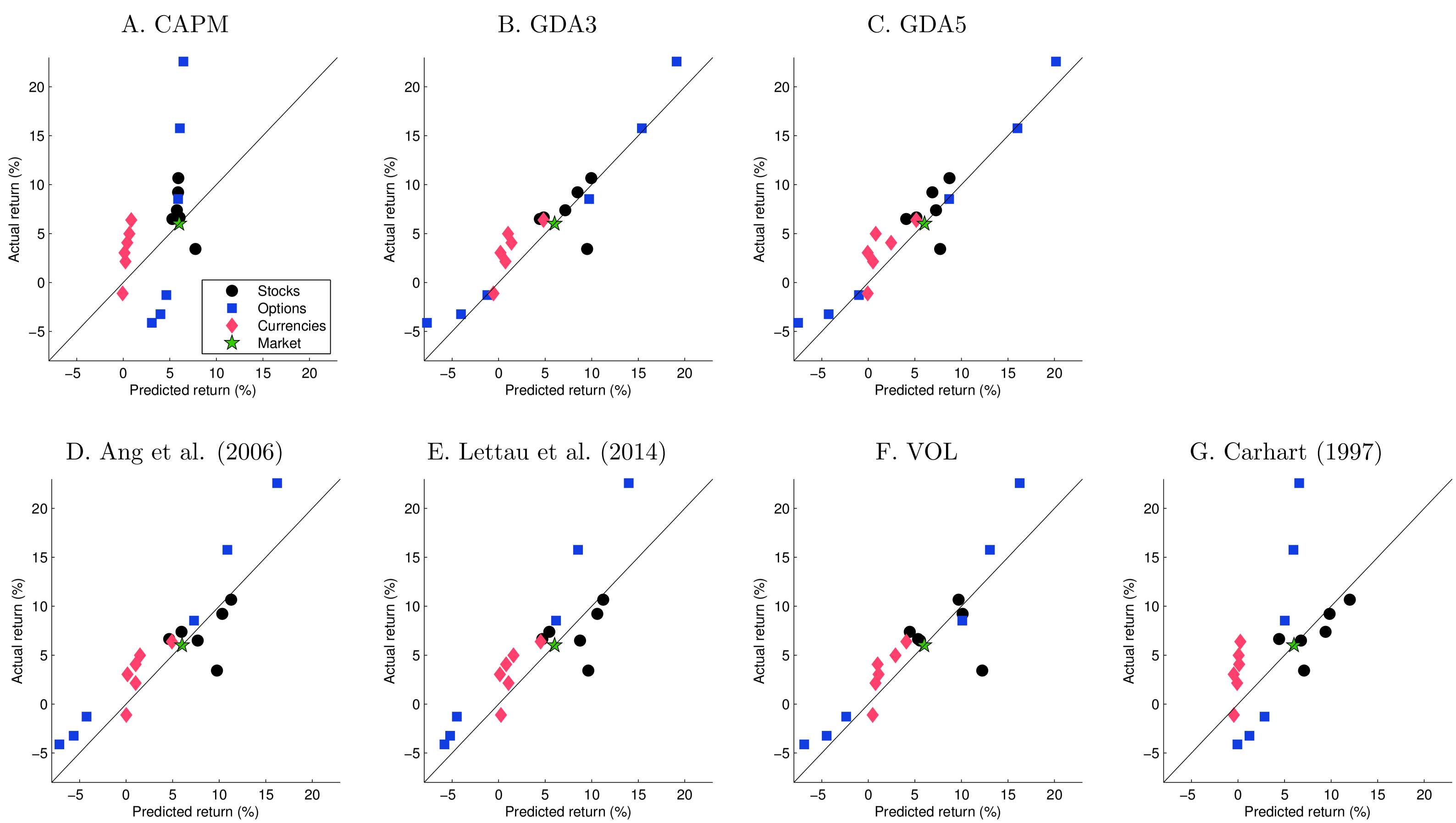
The top panels of the table present the parameter values used for the calibration assesment. Panel A shows the parameters of the endowment dynamics from (A.36), while Panel B presents the values of the preference parameters. Panel C presents the model implied mean (E), standard deviation (σ), and first order autocorrelation ($AC1$) of consumption growth (Δc_t) and dividend growth (Δd_t), and the first and second moments of the log price-dividend ratio (pd), log risk-free rate (r_f), and excess log equity return ($r - r_f$). The first column presents annualized data counterparts over the period from January 1930 to December 2012. Finally, Panel D shows the characteristics of the downside event (parameters a and b from equation (A.4) and the unconditional disappointment probability) and the factor risk premiums (λ-s), as implied by the GDA model.

Table A.11: Risk premium estimates using individual stocks

	GDA3 $a=0$	GDA5 $a=0$	GDA5 $a=0.5$	GDA5 $a=1$
λ_W	0.0054**	0.0051**	0.0051**	0.0052**
	(0.0024)	(0.0023)	(0.0023)	(0.0023)
$\lambda_{\mathcal{D}}$	-0.2112**	-0.1906**	-0.3249***	-0.3561***
	(0.1017)	(0.0957)	(0.1228)	(0.1240)
$\lambda_{W\mathcal{D}}$	0.0045***	0.0041**	0.0044***	0.0040***
	(0.0017)	(0.0016)	(0.0016)	(0.0013)
λ_X		-1.03e-5***	-1.03e-5***	-1.01e-5***
		(3.65e-6)	(3.76e-6)	(3.84e-6)
$\lambda_{X\mathcal{D}}$		-3.15e-6***	-6.57e-6***	-8.31e-6***
		(9.38e-7)	(1.93e-6)	(2.47e-6)

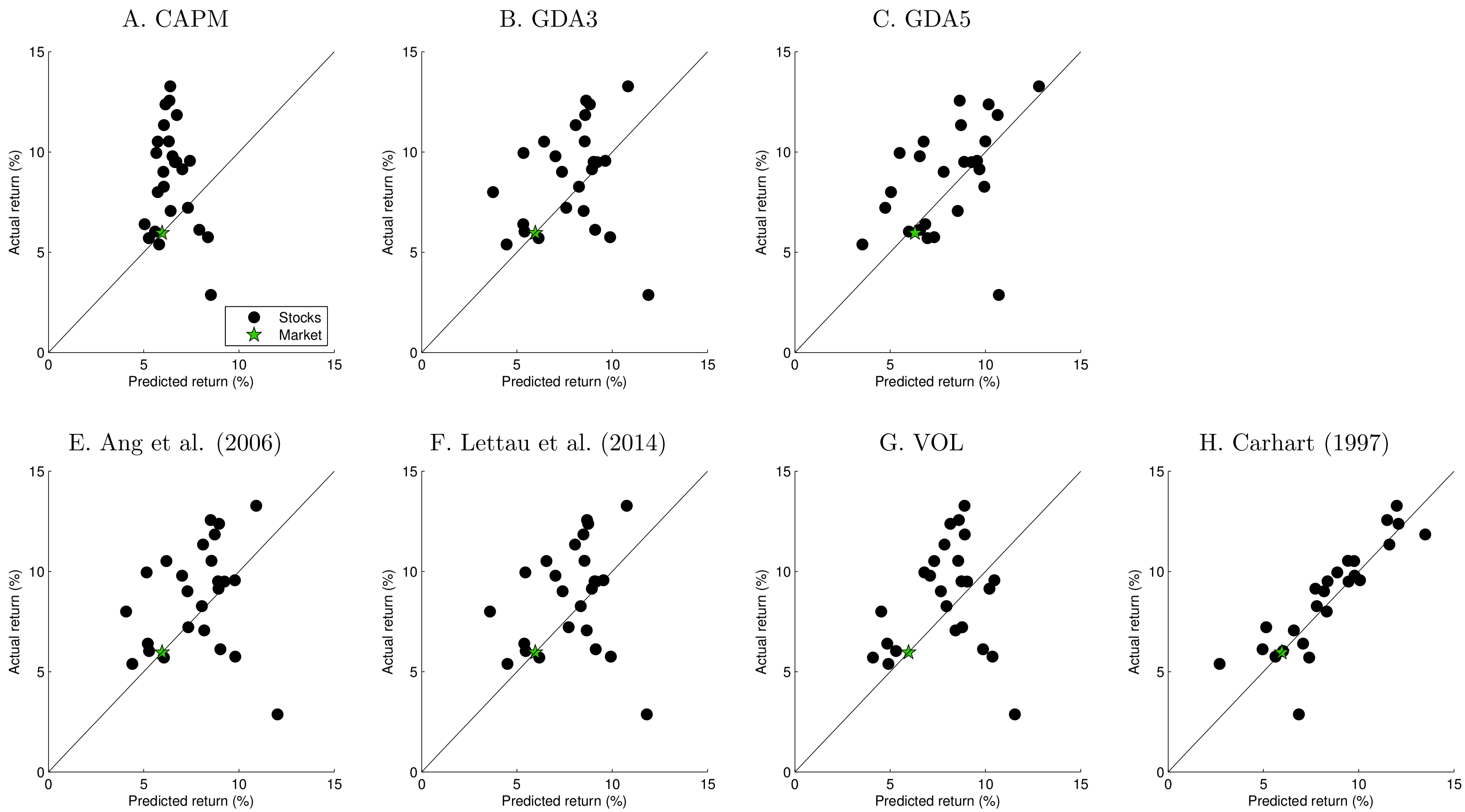
The Table presents results of Fama-MacBeth regressions. For each month $t \geq 12$ the β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the next month $(t+1)$. The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The sample period is from July, 1963 to December, 2013.

Figure A.1: Actual versus predicted returns for stock (size/book-to-market), option, and currency portfolios



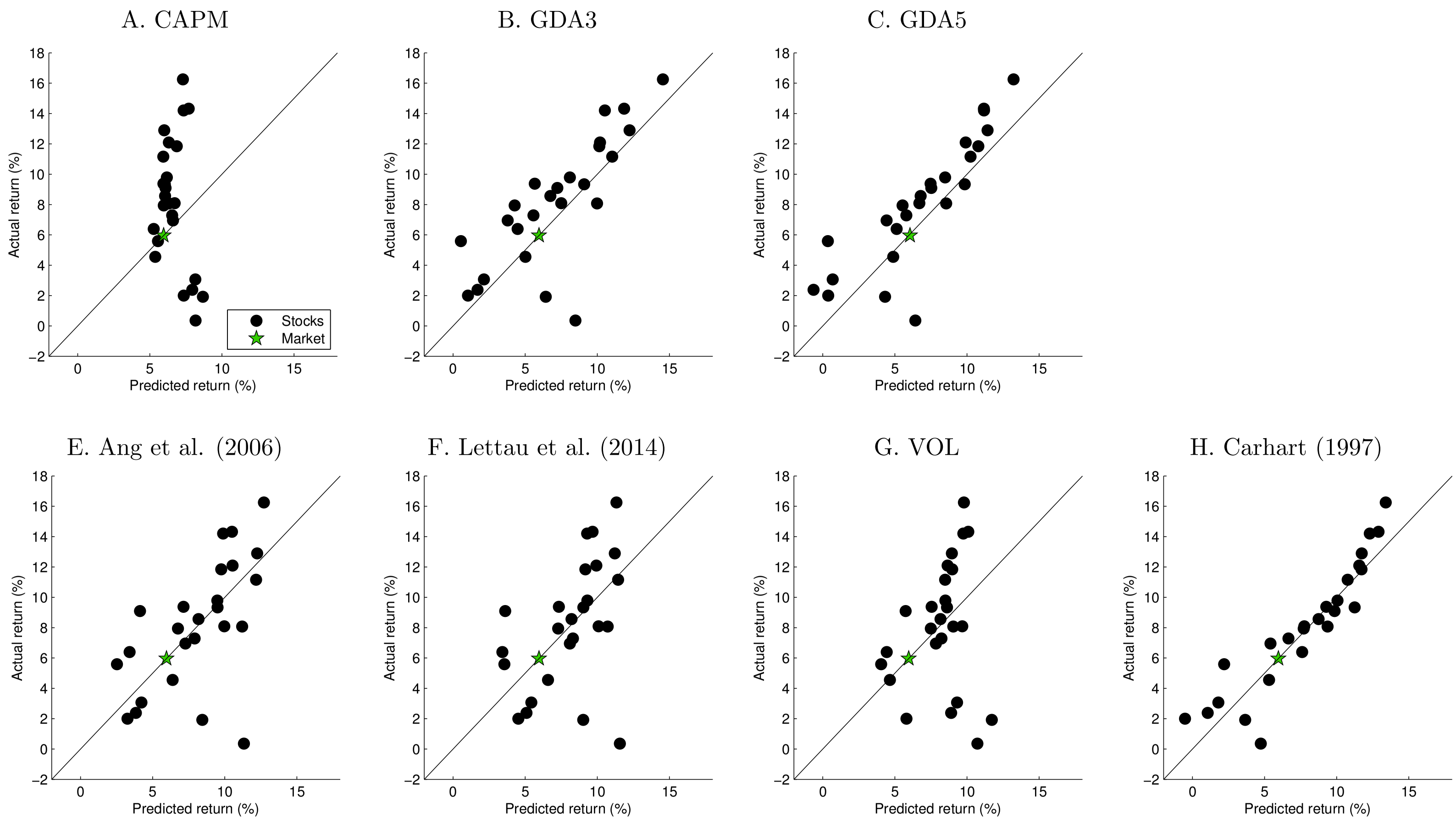
The figure shows the realized average excess returns for 6 (3×2) size/book-to-market, 6 option, 6 currency, and the market portfolio (see the legend in Panel A) against the predicted average excess returns from various models. The sample period is April 1986 – March 2010.

Figure A.2: Actual versus predicted returns for size/book-to-market portfolios



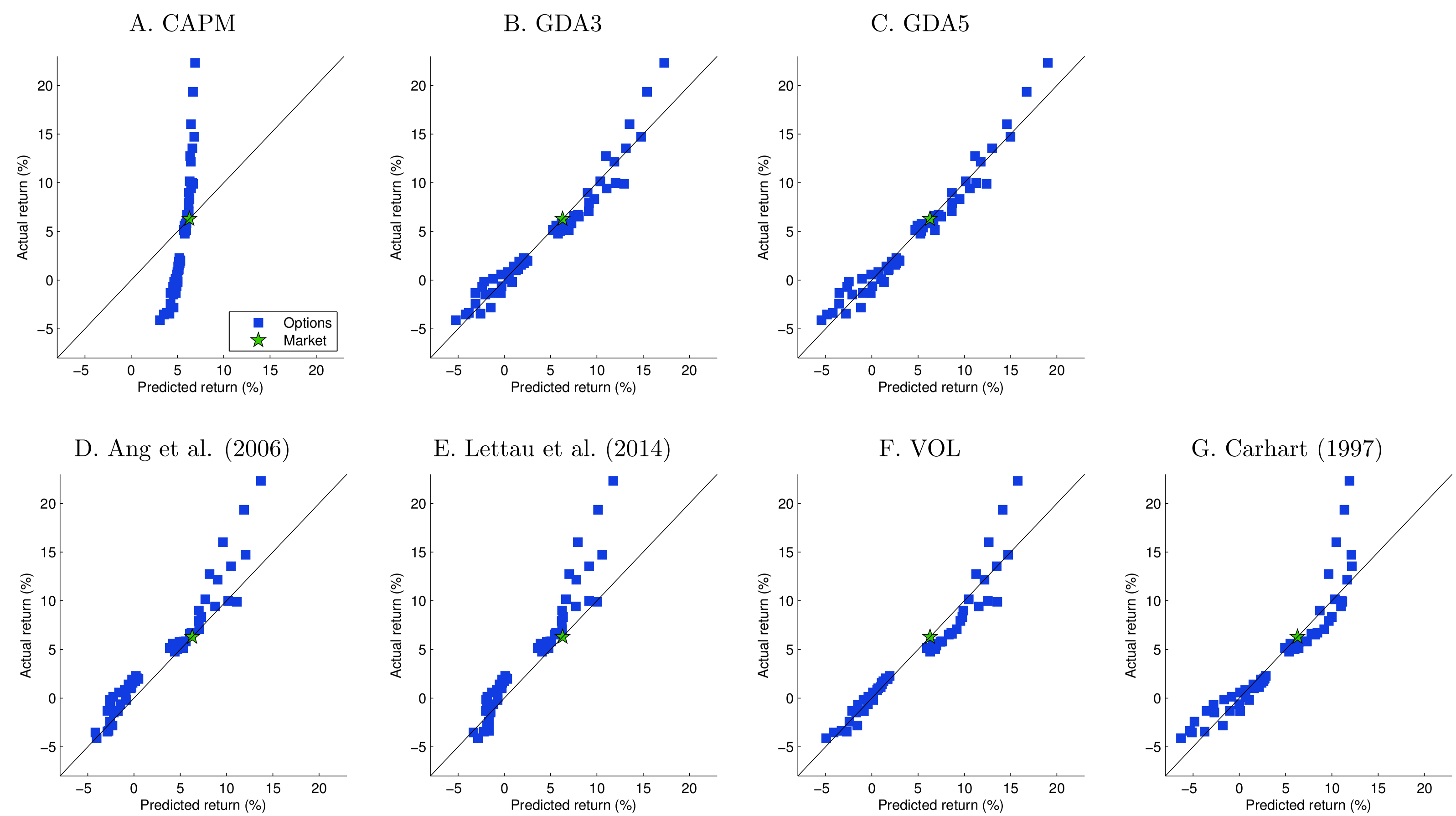
The figure shows the realized average excess returns for the 25 (5×5) size/book-to-market portfolios and the market portfolio (see the legend in Panel A) against the predicted average excess returns from various models. The sample period is July 1964 – December 2016.

Figure A.3: Actual versus predicted returns for size/momentum portfolios



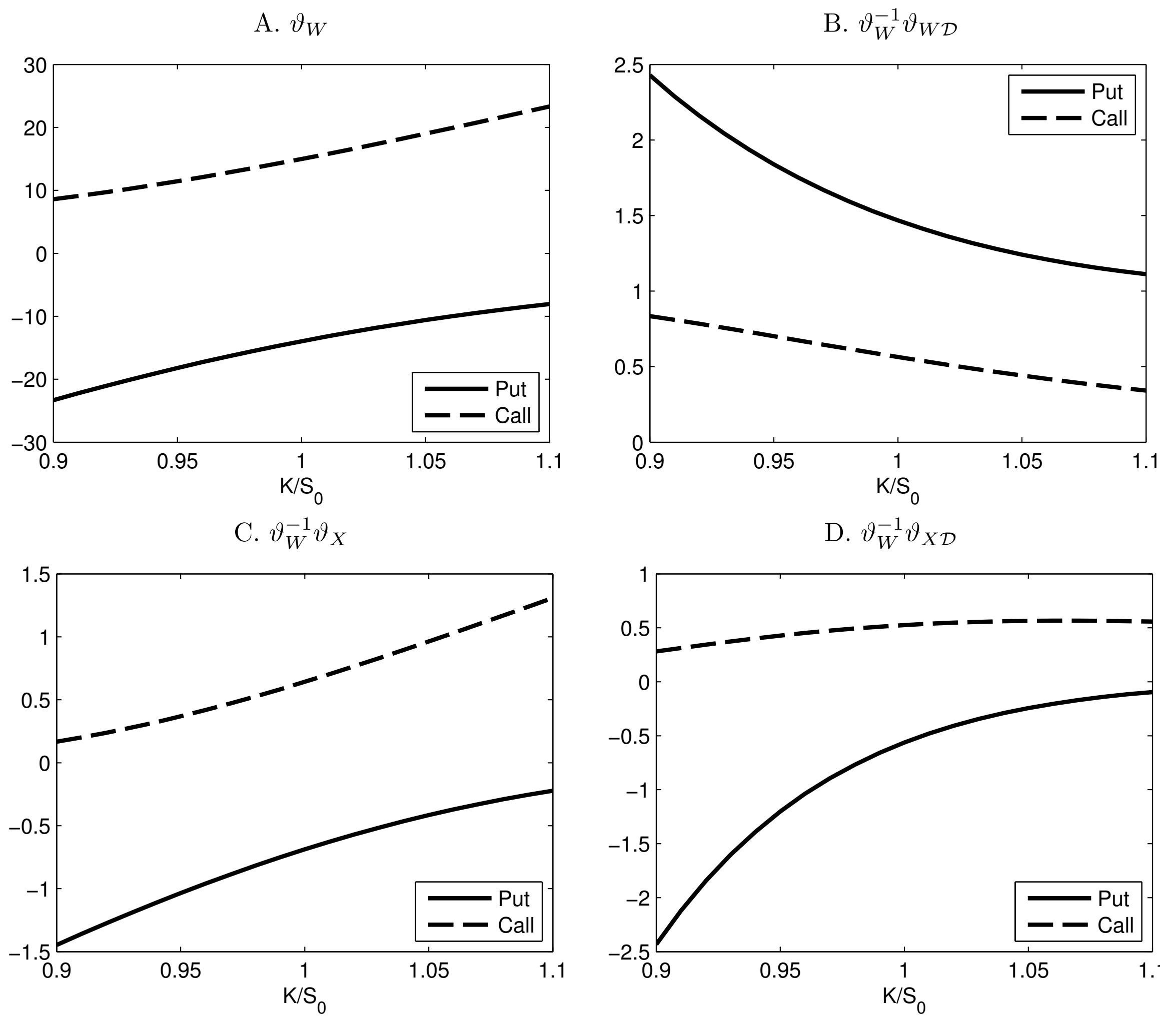
The figure shows the realized average excess returns for the 25 (5×5) size/momentum portfolios and the market portfolio (see the legend in Panel A) against the predicted average excess returns from various models. The sample period is July 1964 – December 2016.

Figure A.4: Actual versus predicted returns for option portfolios



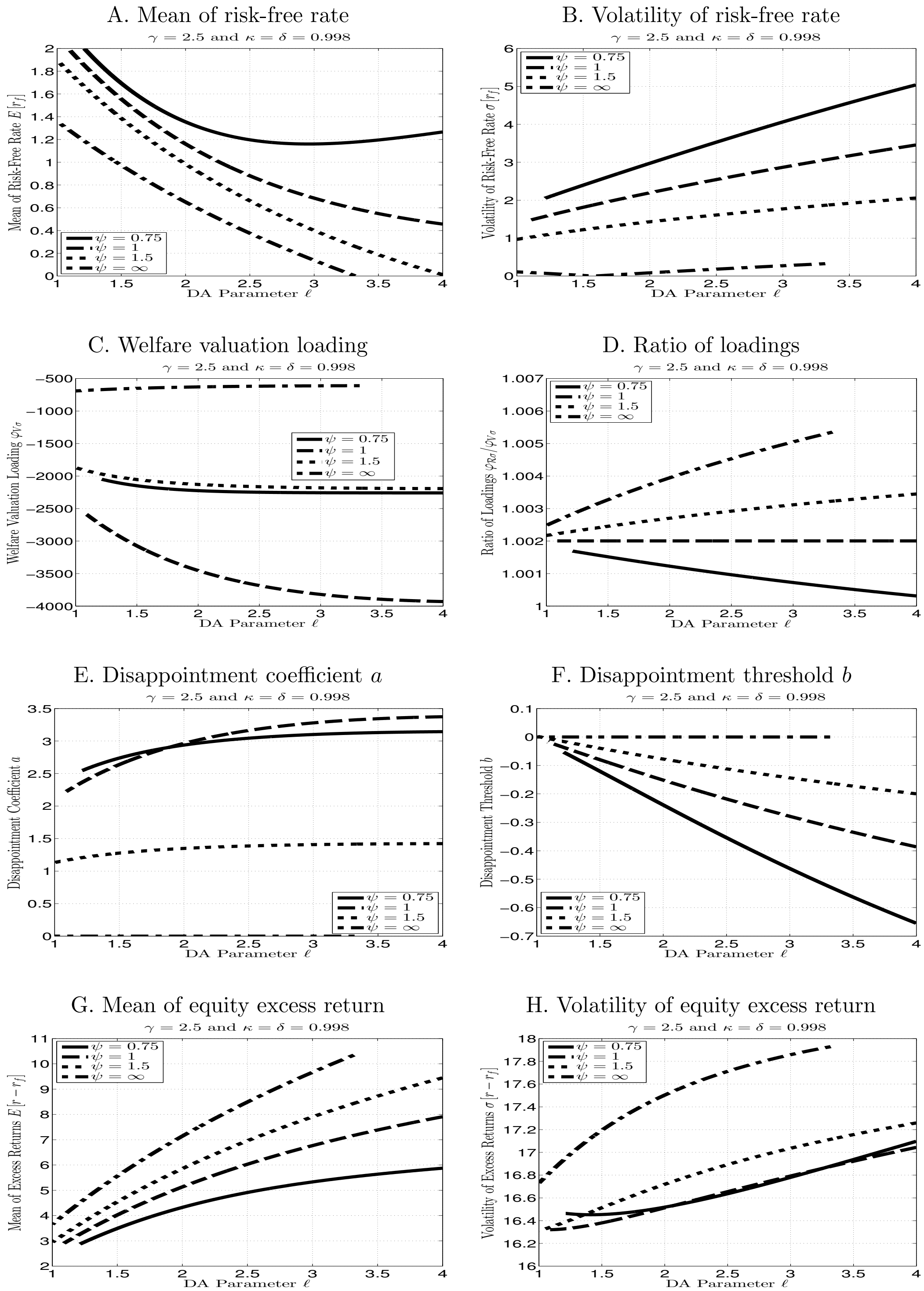
The figure shows the realized average excess returns for the 54 option portfolios of Constantinides et al. (2013) and the market return (see the legend in Panel A) against the predicted average excess returns from various models. The sample period is April 1986 – January 2012.

Figure A.5: Sensitivities of the option portfolios



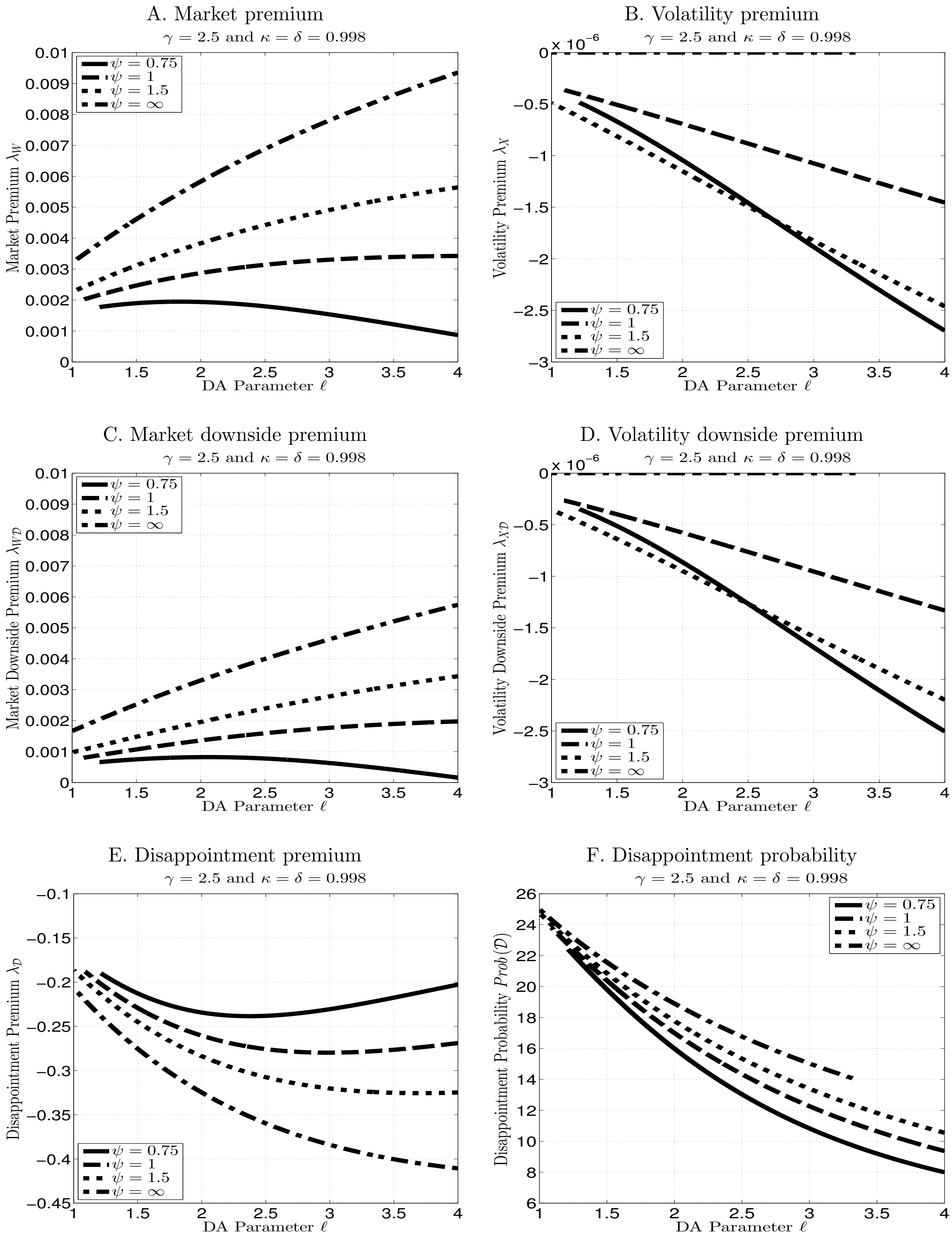
The figure shows option sensitivtes, implied by the Black-Scholes formula, of options with different moneyness (K/S_0) levels. The sensitivities are defined by the equations from (A.32) to (A.35). The parameter values used are $S_0 = 10$, $T = 1/12$ (one month maturity), 30% annual volatility for the underlying, and a risk-free rate of zero. The strike price, K, varies along the horizontal axis of each graph.

Figure A.6: Asset Prices Sensitivity to Disappointment Aversion



The figure displays model-implied annualized mean and volatility of the risk-free rate in Panels A and B, loadings of the welfare valuation ratios onto market volatility and their ratio in Panels C and D, and coefficients that determine the disappointing region in Panels E and F. The equity premium and the equity volatility are finally shown in Panels G and H. All quantities are plotted against the degree of disappointment aversion ℓ, and for different values of the elasticity of intertemporal substitution ψ.

Figure A.7: Factor Risk Premia Sensitivity to Disappointment Aversion



The figure displays model-implied factor risk premiums in Panels A to E, and the disappointment probability in Panel F. All quantities are plotted against the degree of disappointment aversion ℓ, and for different values of the elasticity of intertemporal substitution ψ.