Asymmetries and Portfolio Choice

Magnus Dahlquist
Stockholm School of Economics and CEPR

Adam Farago
University of Gothenburg

Roméo Tédongap
ESSEC Business School Paris-Singapore

Abstract

We examine the portfolio choice of an investor with generalized disappointment-aversion preferences who faces log returns described by a normal-exponential model. We derive a three-fund separation strategy: the investor allocates wealth to a risk-free asset, a standard mean-variance efficient fund, and an additional fund reflecting return asymmetries. The optimal portfolio is characterized by the investor's endogenous effective risk aversion and implicit asymmetry aversion. In empirical applications, we find that disappointment aversion is associated with much larger asymmetry aversion than are standard preferences. Our model explains patterns in popular portfolio advice across both risk appetites and investment horizons. (*JEL* G11)

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Return distributions are asymmetric and display fatter tails than does the normal distribution. Correlations between asset returns conditional on downside and upside moves display asymmetric patterns.[1] There is also evidence that investors have asymmetric attitudes toward risk across downward and upward movements. In particular, they place larger weights on losses than on gains when assessing their portfolio risk. We study the joint impact of asymmetries in asset returns and in risk attitudes on investor portfolio choice.

We propose a simple and parsimonious theoretical setup in a static setting, explicitly ruling out any effect that might otherwise arise from dynamic channels. We model asymmetric investor preferences using the generalized disappointment aversion of Gul (1991) and Routledge and Zin (2010), a preference framework in which investors place different weights on downside losses and upside gains. These preferences are consistent with the experimental behavior observed in the Allais (1979) paradox and are supported by further experimental evidence from Choi et al. (2007) and Gill and Prowse (2012). Moreover, this utility specification is axiomatic, firmly grounded in formal decision theory under uncertainty, and power utility arises as a special case in which the degree of disappointment aversion is zero.

To capture return asymmetries, we propose a setup in which log returns on assets are generated by a normal-exponential model. The model assumes that idiosyncratic asset risks follow a multivariate normal distribution, while skewness is generated by a single common factor with an exponential distribution, which assets load differently on. We demonstrate that the model can match key statistical features of the data, such as skewness, coskewness, fat tails, and asymmetric correlations. A special case is one in which log returns are jointly normal, and we refer to this as the case with no return asymmetry.

We derive an analytical solution to the portfolio choice problem which leads to a three-fund separation strategy. The first fund is a risk-free asset, and the second is a standard

[1] Correlations between stocks tend to be greater for downside moves than for upside moves (see, e.g., Ang and Chen 2002; Hong, Tu, and Zhou 2007), and long-term bonds tend to be negatively correlated with stocks conditional on down markets and positively correlated with stocks conditional on up markets (see, e.g., Baele, Bekaert, and Inghelbrecht 2010; Campbell, Sunderam, and Viceira 2013; David and Veronesi 2013).

mean-variance efficient fund. The third fund is an "asymmetry-variance" fund, whose composition is determined by the asymmetry of the risky asset returns. It takes a short position in negatively skewed assets and a long position in positively skewed assets. The weight an investor assigns to each fund primarily depends on her preference parameters. Using the analytical solution, we can characterize the effects of asymmetries in returns and preferences.

If there is no return asymmetry, the asymmetry-variance fund becomes redundant, and the standard two-fund separation applies. The investor's optimal portfolio weight in the single risky fund (the mean-variance fund) is determined by one parameter, effective risk aversion. We contribute to the literature by deriving the formula for the disappointment-averse investor's effective risk aversion and showing that several sets of parameters of the generalized disappointment-aversion preferences lead to the same effective risk aversion.

If returns are asymmetric, the investor also allocates some wealth to the asymmetry-variance fund, and the relative portfolio weight in this fund is determined by her implicit asymmetry aversion. Using a calibrated example involving bonds and stocks as risky assets, we demonstrate that the asymmetry aversion implied by disappointment-aversion preferences can significantly differ from the values implied by the standard power utility. The asymmetry aversion implied by the power utility is low in magnitude, resulting in only a small investment in the asymmetry-variance fund. In this case, return asymmetry only has a marginal effect on optimal portfolios and, consistent with the results of Levy and Markowitz (1979) and Hlawitschka (1994), the investor behaves as if she had mean-variance preferences.

For a disappointment-averse investor, the optimal choice strongly depends on the reference point distinguishing disappointing from nondisappointing outcomes. First, when the reference point equals the certainty equivalent of the investment, a sufficiently disappointment-averse investor invests all wealth in the risk-free asset. In contrast, a disappointment-averse investor whose reference point differs from the certainty equivalent always finds it optimal to hold risky assets. Second, when the reference point is lower than the certainty equivalent, the implicit asymmetry aversion is positive and large in magnitude. Negative skewness is associated with an increased probability of large losses. Therefore, an investor who focuses on avoiding large losses will reduce investment in negatively skewed assets by taking a relatively large long position (compared to a power-utility investor) in the asymmetry-variance fund. This can induce a significant shift from the negatively skewed stocks toward bonds. Third, when the reference point is higher than the investor's certainty equivalent, the implicit asymmetry aversion is negative, which leads to an implicit preference for negative skewness. When the disappointment threshold is high, the investor pays special attention not only to left-tail outcomes but also to central outcomes. For a given mean and variance, negative skewness leads to a fatter left tail as well as to a shift of the mode to the right. With a high threshold, the benefits of having a higher probability mass on small positive returns outweigh the utility costs of the fatter left tail. Therefore, the investor shifts toward negatively skewed assets in her risky portfolio.

Asymmetry of asset returns coupled with generalized disappointment aversion and a disappointment threshold lower than the certainty equivalent can help us understand patterns in popular portfolio advice that are puzzling to standard models. According to the standard two-fund separation theorem, everyone should hold risky assets in the same proportion, and only the relative weights in the risky portfolio and in cash should vary across investors (Tobin 1958). This stands in sharp contrast to the recommendations of financial advisors, across investors with both different risk tolerances and different investment horizons.

First, regarding differences across risk tolerances, Canner, Mankiw, and Weil (1997) document that when dividing a portfolio between cash, bonds, and stocks, financial advisors often recommend that conservative investors should allocate more of their risky portfolio to bonds, while aggressive investors should allocate more to stocks. Canner, Mankiw, and Weil (1997) refer to this as the "asset allocation puzzle". The advisors' recommendation can be rationalized if we think of a conservative investor as someone with a higher degree of disappointment aversion. This conservative investor optimally invests more in the asymmetry-variance fund and, consequently, achieves a higher bond/stock allocation ratio than does an aggressive investor.

Second, advisors often recommend that investors with short investment horizons should favor bonds in their risky portfolios, while long-horizon investors should favor stocks. At short investment horizons, the negative skewness deters disappointment-averse investors from holding stocks. Assuming independent return dynamics, asset returns become less asymmetric as the investment horizon increases. Hence, long-horizon investors hold relatively more stocks than bonds. We thus provide a reason for shifting from stocks to bonds as the horizon decreases that differs from the reason due to the effective mean reversion in prices (Campbell and Viceira 2002, 2005) or nontradable human capital (Jagannathan and Kocherlakota 1996; Cocco, Gomes, and Maenhout 2005).

Our work relates to the literature on the effect of skewness on optimal portfolios. The investor's asymmetry aversion in these studies is usually implied by a Taylor expansion of some standard utility function (Conine and Tamarkin 1981; Jondeau and Rockinger 2006; Guidolin and Timmermann 2008; Martellini and Ziemann 2010; Ghysels, Plazzi, and Valkanov 2014) or set exogenously to an ad hoc value (Mitton and Vorkink 2007; Harvey et al. 2010). Our approach differs. We consider a nonstandard utility specification and study the sign and magnitude of the asymmetry aversion that it implies. Das and Uppal (2004) examine portfolio choice when asset returns exhibit jumps that occur simultaneously; we consider similar return asymmetries, but let the investor explicitly care about the downside risk underlying the return asymmetry. Our work also relates to that of Ang, Bekaert, and Liu (2005), who consider portfolio choice under disappointment aversion and normally distributed asset returns. We extend their analysis in several directions: we consider the generalized disappointment-aversion utility, study the effect of asymmetric return distributions, and derive an analytical solution to the optimal portfolio problem that easily accommodates multiple risky assets.

Disappointment aversion is not the only preference framework in which investors are more sensitive to outcomes below a certain reference point. Kőszegi and Rabin (2006, 2007) introduce a model of reference-dependent preferences in which the agent is loss averse around a stochastic reference point. We show that by replacing their stochastic reference point with the constant but endogenous reference point of disappointment aversion, the general setup of Kőszegi and Rabin (2006, 2007) nests generalized disappointment aversion as a special case. To the best of our knowledge, this relationship between the two preference frameworks is new to the literature. The cumulative prospect theory of Tversky and Kahneman (1992) features loss aversion around an exogenously given reference point as one of its building blocks. We show that if the effect of loss aversion is isolated using the so-called kinked power utility (in which case we provide an analytical solution to the portfolio choice problem), it leads to optimal portfolios similar to those stemming from disappointment aversion.

We focus on the portfolio choice implications of skewness; another strand of the literature focuses on its asset pricing implications (see, e.g., Kraus and Litzenberger 1976; Harvey and Siddique 2000; Dittmar 2002; Langlois 2013). In the Online Appendix, we derive the asset pricing implications of our dual-asymmetry setting and show that they are similar to those of Simaan (1993). We find a negative relationship between asset skewness and expected return, which is consistent with Barberis and Huang (2008), Mitton and Vorkink (2007), and Boyer, Mitton, and Vorkink (2010) regarding the overpricing of positively skewed securities. Empirical tests of these implications using a large cross-section of assets would constitute an interesting avenue for future research.

1 Theoretical Setup

An investor with generalized disappointment-aversion utility, as in Routledge and Zin (2010), can allocate wealth between N risky securities ($i = 1, 2, \dots, N$) and a risk-free asset ($i = f$). Similar to Ang and Bekaert (2002), Das and Uppal (2004), Ang, Bekaert, and Liu (2005), and Guidolin and Timmermann (2008), we consider a finite-horizon setup with utility defined over terminal wealth. Our model of asset returns is set in discrete time.

1.1 Investor attitude toward risk

Generalized disappointment-aversion (GDA) preferences capture the idea that investors care differently about downside losses than upside gains. The investor's objective is to maximize the utility of the certainty equivalent of terminal wealth, W. Following Routledge and Zin (2010), the certainty equivalent of terminal wealth, $\mathcal{R}(W)$, is implicitly defined by

$$\theta U(\mathcal{R}(W)) = E[U(W)] - \ell E[(U(\kappa\mathcal{R}(W)) - U(W)) I(W < \kappa\mathcal{R}(W))] \ , \tag{1}$$

where $I(\cdot)$ is an indicator function that equals 1 if the condition is met and 0 otherwise, and

$$U(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0 \ \text{ and } \gamma \neq 1 \ , \\ \ln x & \text{if } \gamma = 1 \ . \end{cases} \tag{2}$$

The parameter $\gamma > 0$ measures the investor's risk aversion, $\ell \geq 0$ is the investor's degree of disappointment aversion, and $\kappa > 0$ is the percentage of her certainty equivalent below which outcomes are considered disappointing. Parameter θ is defined as

$$\theta \equiv \begin{cases} 1 & \text{if } \kappa \leq 1 \\ 1 - \ell(\kappa^{1-\gamma} - 1) & \text{if } \kappa > 1 \end{cases} \tag{3}$$

and ensures that the certainty equivalent is appropriately scaled even when $\kappa > 1$, so that the certainty equivalent of a constant value, x, equals itself (i.e., $\mathcal{R}(x) = x$).

If the investor's degree of disappointment aversion is zero ($\ell = 0$), the definition of the certainty equivalent from (1) simplifies to

$$U(\mathcal{R}(W)) = E[U(W)] \ . \tag{4}$$

In this case, the investor has expected utility (EU) preferences with power utility. Through-out the paper, we refer to such an investor as the EU investor. When $\ell > 0$, outcomes lower than $\kappa\mathcal{R}(W)$ receive an extra weight and lower the investor's certainty equivalent relative to EU. As the objective is to maximize the certainty equivalent, a disappointment-averse investor would like to avoid outcomes below $\kappa\mathcal{R}(W)$. The penalty for disappointing outcomes increases with ℓ, so this parameter modulates the importance of disappointment versus satisfaction and can be interpreted as the degree of disappointment aversion.

Parameter κ sets the threshold for disappointing outcomes relative to the certainty equivalent. The special case of $\kappa = 1$ corresponds to the original disappointment-aversion (DA) preferences of Gul (1991). If $\kappa < 1$, the random future value is considered disappointing if it lies sufficiently below today's certainty equivalent; if $\kappa > 1$, the random future value must be sufficiently far above the certainty equivalent to be considered not disappointing. Previous literature on disappointment aversion is predominantly concerned with the $\kappa < 1$ case. Routledge and Zin (2010) briefly discuss the $\kappa > 1$ possibility, but otherwise the literature has ignored this setting. The setting in which the reference point is lower than the certainty equivalent is arguably more relevant for understanding real-life investor behavior, though it might be of general interest to study the portfolio choice implications of a high disappointment threshold. We demonstrate that different values of κ lead to diverse investor behavior. We refer to an investor for whom $\kappa = 1$ as a DA investor and to an investor for whom $\kappa \neq 1$ as a GDA investor.

Terminal wealth may be written as

$$W = W_0 R_W \ , \tag{5}$$

where W_0 is the initial wealth and R_W is the gross return on the investor's portfolio over the investment horizon. Due to the homogeneity of utility function (2),

$$\mathcal{R}(W) = W_0 \mathcal{R}(R_W) \ . \tag{6}$$

Ultimately, the investor's objective is simply to maximize the certainty equivalent of the portfolio gross return, $\mathcal{R}(R_W)$, given by

$$\theta U(\mathcal{R}) = E[U(R_W)] - \ell E[(U(\kappa\mathcal{R}) - U(R_W)) I(R_W < \kappa\mathcal{R})] \ , \tag{7}$$

in which we have used the short-hand notation $\mathcal{R}$ for $\mathcal{R}(R_W)$. Maximizing the certainty equivalent leads to the same solution as does maximizing its logarithm, $\eta \equiv \ln \mathcal{R}$. We show in Appendix A that the investor's log certainty equivalent is implicitly given by

$$\eta = \begin{cases} \frac{1}{1-\gamma} \ln E[\exp((1-\gamma) r_W)] \\ \qquad - \frac{1}{1-\gamma} \ln\left(\theta + \ell\kappa^{1-\gamma}(1 - E[\exp((\gamma-1) p_W)])\right) & \text{if } \gamma > 0 \text{ and } \gamma \neq 1 \\ E[r_W] - \ell E[p_W] + \ell \max(\ln\kappa, 0) & \text{if } \gamma = 1 \end{cases} , \tag{8}$$

where

$$p_W \equiv \max(\ln\kappa + \eta - r_W, 0) \tag{9}$$

corresponds to the payoff of a European put option on the portfolio's log return, r_W, with a strike equal to $\ln\kappa + \eta$, the investor's endogenous threshold of disappointment.

The intuition for (8) is most straightforward when $\gamma = 1$. The investor's log certainty equivalent is a sum of two components: the first is the log certainty equivalent of the EU investor, and the second is a downside risk penalty for achieving a portfolio return below the endogenous disappointment threshold. The downside risk is valued as a European put option on the portfolio return with a strike equal to the disappointment threshold. If the portfolio return at the end of the investment period is below the disappointment threshold, the option matures in the money, reducing the utility of the investor. The total cost of downside risk is the expected payoff of this put option, $E[p_W]$, times the degree of disappointment aversion, ℓ. The parameter ℓ also may be interpreted as the marginal cost of downside risk, as a one-basis-point increase in $E[p_W]$ translates into an ℓ-basis-point decrease in the investor's certainty equivalent. When $\gamma \neq 1$, the intuition remains the same. The first component of (8) is the log certainty equivalent of the EU investor. The second component is the downside risk penalty, which is nonpositive by definition and a decreasing function of the put option's payoff.

1.2 Model of asset returns

We propose a simple extension to the multivariate normal distribution to capture the asymmetry of asset returns. Specifically, we assume that log returns on N risky assets are described by the model

$$r_t = \mu - \sigma \circ \delta + (\sigma \circ \delta)\, \varepsilon_{0,t} + \left(\sigma \circ \sqrt{\iota - \delta \circ \delta}\right) \circ \varepsilon_t \ , \tag{10}$$

where μ, σ, and δ are N-dimensional vectors, ι is a vector of ones, and $\circ$ denotes the Schur product (element-wise product) of vectors. The scalar $\varepsilon_{0,t}$ is a common shock across all assets that follows an exponential distribution with a rate parameter equal to one.[2] The N-dimensional vector, ε_t, represents asset-specific shocks and has a multivariate normal distribution, independent of $\varepsilon_{0,t}$, with standard normal marginal densities and correlation matrix Ψ. Parameters μ, σ, Ψ, and δ together describe the return-generating model. If $\delta = 0$, then r_t follows a multivariate normal distribution with mean μ, standard deviation vector σ, and correlation matrix Ψ. Hence, our setup conveniently nests the case in which asset returns are jointly lognormal. In our extended model, N additional parameters in δ are needed compared with the multivariate normal distribution; these additional parameters describe the asymmetry of returns. Note that both Barberis and Huang (2008) and Mitton

[2] That is, $\varepsilon_{0,t} \sim \exp(1)$. We also have considered an alternative model of asset returns, known as the extended skew-normal distribution, in which the common shock has a truncated normal distribution. The normal-exponential model in (10) has several advantages. First, the formulas for the return moments are simpler. Second, the extended skew-normal model needs one additional parameter. Third, we can derive the exact distribution of multiperiod returns for the normal-exponential model, while we have to use approximated distributions if we work with the skew-normal model. Moreover, Adcock and Shutes (2012) show that the normal-exponential model is a certain limiting case of the extended skew-normal distribution, and the two models lead to very similar results in empirical applications.

and Vorkink (2007) consider a case in which only one of the N risky assets is skewed, the others having symmetric returns. The model in (10) nests this scenario by setting $\delta_1 = ... = \delta_{N-1} = 0$ and letting only δ_N differ from zero.

The log return on asset i may be written as

$$r_{i,t} = \mu_i - \sigma_i\delta_i + (\sigma_i\delta_i)\,\varepsilon_{0,t} + \left(\sigma_i\sqrt{1-\delta_i^2}\right)\varepsilon_{i,t} \ . \tag{11}$$

Parameter δ_i, belonging to the interval $(-1, 1)$, determines the sensitivity of the asset return to the exponentially distributed common shock $\varepsilon_{0,t}$. The exponential distribution is suitable for characterizing the occurrence of extreme events, such as large and infrequent losses. For example, the waiting time until the next event in a Poisson process has an exponential distribution. The Poisson process is often used to characterize the occurrence of jumps in continuous-time models (see, e.g., Merton 1976; Bates 1996; Broadie, Chernov, and Johannes 2007). Assets with large negative sensitivities to $\varepsilon_{0,t}$ are subject to large, but infrequent, negative returns, while assets with large positive sensitivities are subject to large, but infrequent, positive returns. Model (10) assumes that the occurrence of such extreme movements is simultaneous across assets, so it may be interpretable as a systematic event. In this sense, our discrete-time return dynamics share the properties of the continuous-time dynamics considered by Das and Uppal (2004).

It is straightforward to show that the mean, variance, skewness, and excess kurtosis of $r_{i,t}$ are given by

$$E(r_{i,t}) = \mu_i \ , \qquad Var(r_{i,t}) = \sigma_i^2 \ , \qquad Skew(r_{i,t}) = 2\delta_i^3 \ , \qquad Xkurt(r_{i,t}) = 6\delta_i^4 \ . \tag{12}$$

The correlation and coskewness of the returns of asset i and asset j are

$$\begin{aligned} Corr(r_{i,t}, r_{j,t}) &= \psi_{ij}\sqrt{1-\delta_i^2}\sqrt{1-\delta_j^2} + \delta_i\delta_j \ , \\ Coskew(r_{i,t}, r_{j,t}) &\equiv \frac{E\left[(r_{i,t} - E(r_{i,t}))^2 (r_{j,t} - E(r_{j,t}))\right]}{Var(r_{i,t})\sqrt{Var(r_{j,t})}} = 2\delta_i^2\delta_j \ . \end{aligned} \tag{13}$$

The formulas in (12) and (13) illustrate how the vector δ characterizes the nonnormality of returns, as it leads to nonzero skewness, coskewness, and excess kurtosis. The parameters of the distribution can be estimated by the generalized method of moments (GMM) using the moments given in (12) and (13). The investment horizon is assumed to be one period for the main part of the paper, but we also consider longer investment horizons in Section 3.2.

The asymmetry of asset returns is attributed to a common source of risk in the normal-exponential model (10). Boyer, Mitton, and Vorkink (2010) argue that idiosyncratic skewness is also important in explaining cross-sectional differences in asset returns. In the Online Appendix we discuss a simple extension to the normal-exponential model that accounts for the assets' idiosyncratic skewness. We also demonstrate that the main conclusions regarding optimal portfolios do not much change when the extended model is considered.

1.3 Optimal portfolio

The second-order Taylor approximation à la Campbell and Viceira (2002) of the portfolio log return is

$$r_{W,t} \approx r_f + w^\top \left(r_t - r_f \iota + \frac{1}{2}\sigma^2 \right) - \frac{1}{2} w^\top \Sigma w \ , \tag{14}$$

where r_f is the risk-free rate, w is the vector of portfolio weights for risky assets, ι is a vector of ones, and σ^2 is the diagonal of the variance-covariance matrix Σ.[3] If individual asset returns are characterized by the return-generating model (10), then using the above approximation, the portfolio log return is also characterized by the normal-exponential model

$$r_{W,t} = \mu_W - \sigma_W \delta_W + (\sigma_W \delta_W)\, \varepsilon_{0,t} + \left(\sigma_W \sqrt{1-\delta_W^2} \right) \varepsilon_{W,t} \ , \tag{15}$$

[3]In the Online Appendix, we find that the approximation works well in our calibration exercise. An alternative is to consider a third-order approximation instead of (14). However, the order of approximation affects only the mean of the portfolio log return, that is, μ_W from Equation (16), but not the higher moments. We find that (14) approximates the true mean of the portfolio log return very well and that the third-order alternative does not lead to a considerable improvement.

with

$$\mu_W = r_f + w^\top \left(\mu - r_f\iota + \frac{1}{2}\sigma^2\right) - \frac{1}{2} w^\top \Sigma w \ , \qquad \sigma_W^2 = w^\top \Sigma w \ , \qquad \delta_W = \frac{w^\top (\sigma \circ \delta)}{\sigma_W} \ , \quad (16)$$

and where $\varepsilon_{W,t}$ is a standard normal shock independent of $\varepsilon_{0,t}$. Given our setup, the following proposition describes the optimal portfolio.

Proposition 1.1. The investor's optimal asset allocation may be written as

$$w = \frac{1}{\tilde{\gamma}} \left(w^{\mathbf{MV}} + \tilde{\chi} w^{\mathbf{AV}} \right) \ , \quad (17)$$

where

$$w^{\mathbf{MV}} \equiv \Sigma^{-1} \left(\mu - r_f\iota + \frac{1}{2}\sigma^2\right) \quad \text{and} \quad w^{\mathbf{AV}} \equiv \Sigma^{-1} (\sigma \circ \delta) \ . \quad (18)$$

Coefficients $\tilde{\gamma}$ and $\tilde{\chi}$ depend on the optimal portfolio weight vector, w (i.e., they are endogenously determined). Analytical expressions for $\tilde{\gamma}$ and $\tilde{\chi}$ are given in Appendix B.

Proof. See Appendix B.

Our setup leads to a three-fund separation strategy similar to that of Simaan (1993). The investor allocates her wealth to two risky funds and invests the remainder of her wealth in the risk-free asset. We call the first risky fund, $w^{\mathbf{MV}}$, the "mean-variance" fund because it is the solution to the mean-variance optimal portfolio problem. Note that the same fund appears in Campbell and Viceira (2002), in the solution to the lognormal model with power utility. We call the second risky fund, $w^{\mathbf{AV}}$, the "asymmetry-variance" fund because its composition depends on the asymmetry vector, δ, and the variance-covariance matrix of the risky asset returns. It is the solution to an asymmetry-variance optimal portfolio problem similar to the mean-variance one.

The weights that the investor assigns to the risky funds are determined by $\tilde{\gamma}$ and $\tilde{\chi}$. These coefficients depend not only on the preference parameters (i.e., γ, ℓ, and κ) but also on the optimal asset allocation, w, itself and the certainty equivalent, η. That is, the coefficients $\tilde{\gamma}$ and $\tilde{\chi}$ and the certainty equivalent η are all endogenous to the model. To solve for these values and for the optimal allocation, w, Equations (8) and (17) must be solved simultaneously.

Given the endogenous values of $\tilde{\gamma}$ and $\tilde{\chi}$, the optimal allocation in (17) also can be achieved by solving the following mean-variance-asymmetry investment problem:

$$\max_{w} \quad \mu_W - r_f - \frac{\tilde{\gamma} - 1}{2}\sigma_W^2 + \tilde{\chi}\sigma_W\delta_W \ , \tag{19}$$

where μ_W and σ_W^2 are the mean and variance, respectively, of the portfolio log return given in Equation (16), while δ_W describes its asymmetry. Therefore, we can interpret the coefficient $\tilde{\gamma}$ as the *effective risk aversion* and the coefficient $\tilde{\chi}$ as the *implicit asymmetry aversion* of the investor. The finding that effective risk aversion is endogenous under disappointment-aversion preferences is consistent with the discussions presented by Routledge and Zin (2010) and Bonomo et al. (2011) in an intertemporal consumption-based general equilibrium setting. However, unlike these authors, we explicitly derive the formula of effective risk aversion in our partial equilibrium setting. This provides a novel way to quantify the effect of disappointment aversion on the optimal portfolio choice. The mean-variance-asymmetry problem (19) is similar to that in Mitton and Vorkink (2007) and Harvey et al. (2010), but differs in several ways: the asymmetry measure is not the third central moment of returns; the coefficient governing preference for asymmetry, $\tilde{\chi}$, is not positive a priori; and our solution is analytical. We show in the Online Appendix that the three funds in (17) span the mean-variance-asymmetry efficient frontier, which contains portfolios that minimize portfolio variance, σ_W^2, for a given level of mean, μ_W, and asymmetry, $\sigma_W\delta_W$. Therefore, the optimal portfolios given by (17) are on the efficient frontier.

The lack of asymmetry of asset returns ($\delta = 0$) implies both $w^{\mathbf{AV}} = 0$ and $\tilde{\chi} = 0$. Hence, the optimal portfolio rule simplifies to

$$w = \frac{1}{\tilde{\gamma}} w^{\mathbf{MV}} \ . \tag{20}$$

When returns are symmetric, investors allocate their wealth between the mean-variance fund and the risk-free asset. Consequently, when observing a particular asset allocation, we cannot determine whether it was chosen by a disappointment-averse or a disappointment-neutral investor. In other words, different combinations of the preference parameter values γ, ℓ, and κ lead to the same $\tilde{\gamma}$. Therefore, the concept of effective risk aversion provides a convenient way to compare the effects of different preferences in the presence of return asymmetries. Comparing the optimal choices of different investors (e.g., power-utility versus disappointment-averse investors) who have the same effective risk aversion isolates the effect of return asymmetries, as these investors would choose the same portfolios if returns were symmetric. If the investor has expected utility, the effective risk aversion is simply the curvature parameter of the power utility ($\ell = 0$ implies $\tilde{\gamma} = \gamma$). Disappointment aversion ($\ell > 0$), on the other hand, implies $\tilde{\gamma} > \gamma$; that is, a disappointment-averse investor reduces investment in risky assets, investing a larger fraction of wealth in cash.

2 Empirical Application

2.1 Data and parameter estimation

In this section we investigate how investors who differ in their degree of risk aversion and disappointment aversion allocate their wealth among three assets: cash, bonds, and stocks.[4] We estimate return parameters using monthly U.S. data from July 1952 to December 2014, obtained from the Center for Research in Security Prices (CRSP). We start our sample from 1952 to avoid the period before the 1951 Treasury-Fed Accord similar to Campbell and

[4] We will later comment on other asset classes, including growth and value stocks, international equity, and corporate bonds.

Shiller (1991) and Campbell and Ammer (1993). The risk-free rate is the average of the log return on the 30-day Treasury bill from the CRSP Fama Risk-Free Rates file, referred to simply as "cash". The bond return is the return on the 10-year government bond index from the U.S. Treasury and Inflation Series file in CRSP. The stock return is the value-weighted return on the NYSE, NASDAQ, and AMEX. The excess log bond return is the difference between the log return on bonds and the risk-free rate. Similarly, the excess log stock return is the difference between the log return on stocks and the risk-free rate.

Table 1 presents estimation results for the return distribution of the two risky assets as in (10). Subscript "B" denotes bonds, and subscript "S" denotes stocks. The parameters are estimated by minimizing the distance between model-implied moments and their sample counterparts, using the generalized method of moments (GMM) with an identity-weighting matrix. The GMM estimation is overidentified, fitting the two means, the two volatilities, the correlation, the two skewness values, and the two coskewness values (there are nine moments to fit and seven parameters to estimate). Panel A of Table 1 presents sample and fitted moments together with parameter estimates from the return-generating model. Stock index returns are highly negatively skewed, which is a well-known stylized fact. The proposed model of asset returns captures all key moments, providing a simple characterization of the return distribution. Panel A of Figure 1 shows the sample (kernel-smoothed) density of the stock return distribution, together with the densities implied by the fitted normal and normal-exponential models. The figure confirms that the normal-exponential density is much closer to the empirically observed one.

We use the parameter estimates in Table 1 to compute, via simulations, two additional statistics. Panel B in Figure 1 shows the stocks' expected shortfall at various quantiles. The figure confirms that the normal distribution does not adequately capture the fat left tail of the stock return distribution, though the shortfall values implied by the normal-exponential model are close to the actual sample estimates. Panel C shows the correlation between bonds and stocks conditional on the stock return falling below (for $q \leq 0.5$) or above (for $q > 0.5$)

a given quantile of its distribution. Sample estimates indicate that long-term bonds tend to be negatively correlated with stocks conditional on down markets and positively correlated with stocks conditional on up markets, which is in line with previous literature (see, e.g., Baele, Bekaert, and Inghelbrecht 2010; Campbell, Sunderam, and Viceira 2013; David and Veronesi 2013). The normal-exponential model captures this pattern, while the multivariate normal model fails to do so. The results in Table 1 and Figure 1 illustrate the ability of the model (10) to match key features of asset returns.

In the Online Appendix we demonstrate that the normal-exponential model also captures well the asymmetric correlations between various stock portfolios. In particular, correlations between stocks (e.g., between growth and value stocks or between different international equity portfolios) tend to be greater for downside than for upside moves as documented by Longin and Solnik (2001), Ang and Chen (2002), and Hong, Tu, and Zhou (2007). Ang and Chen (2002) argue that asymmetric conditional correlations are fundamentally different from other measures of asymmetries, such as skewness and coskewness. However, we find that the normal-exponential model, designed to match the third-order moments, also does a good job in matching the asymmetric conditional correlation patterns.

2.2 Optimal portfolios

Given the estimated return distribution, the mean-variance fund, $w^{\mathbf{MV}}$, and the asymmetry-variance fund, $w^{\mathbf{AV}}$, can be calculated according to (18). Each fund can be normalized by the absolute value of the sum of its weights:

$$\bar{w}^{\mathbf{MV}} \equiv \frac{w^{\mathbf{MV}}}{|\iota^\top w^{\mathbf{MV}}|} \qquad \text{and} \qquad \bar{w}^{\mathbf{AV}} \equiv \frac{w^{\mathbf{AV}}}{|\iota^\top w^{\mathbf{AV}}|} \ . \tag{21}$$

Panel B of Table 1 shows the composition of these normalized funds. The mean-variance fund assigns a positive weight to both risky assets as they have positive expected excess returns. As the stocks are negatively skewed, the asymmetry-variance fund assigns a negative weight to them, and the bond weight is positive.[5] Note that the weights in the asymmetry-variance fund are fairly large, though the actual positions investors take in the fund lead to reasonable overall weights (see Table 2). Using the normalized funds from (21), the optimal portfolio rule in (17) can be rewritten as

$$w = \alpha^{\mathbf{MV}} \bar{w}^{\mathbf{MV}} + \alpha^{\mathbf{AV}} \bar{w}^{\mathbf{AV}} \ , \tag{22}$$

where

$$\alpha^{\mathbf{MV}} \equiv \frac{1}{\tilde{\gamma}} \left| \iota^\top w^{\mathbf{MV}} \right| \qquad \text{and} \qquad \alpha^{\mathbf{AV}} \equiv \frac{\tilde{\chi}}{\tilde{\gamma}} \left| \iota^\top w^{\mathbf{AV}} \right| \tag{23}$$

are the weights assigned to the normalized mean-variance and asymmetry-variance funds, respectively. Note that in the current calibration, since $\iota^\top w^{\mathbf{AV}}$ is negative, the optimal investment in cash is $1 - \alpha^{\mathbf{MV}} + \alpha^{\mathbf{AV}}$.

Figure 2 summarizes how investors with different preferences choose their optimal portfolios in our calibrated example. The weight assigned to the mean-variance fund, $\alpha^{\mathbf{MV}}$, is on the horizontal axis and the relative weight of the asymmetry-variance fund, $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, is on the vertical axis. Equations in (23) show that $\alpha^{\mathbf{MV}}$ is a scalar multiple of $1/\tilde{\gamma}$, and $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ is a scalar multiple of $\tilde{\chi}$. Consequently, each optimal portfolio can be equivalently represented in the $(\tilde{\gamma}, \tilde{\chi})$ space. Figure 2 shows the corresponding $\tilde{\gamma}$ and $\tilde{\chi}$ values on its top and right axes, respectively. All curves start at the same point corresponding to the investor with $\gamma = 2$ and $\ell = 0$. The solid line corresponds to the EU investor and shows the effect of increasing γ from 2 to 30. The remaining curves correspond to disappointment-averse investors with different κ values and show the effect of increasing ℓ from 0 to 3, while keeping $\gamma = 2$ fixed. Increasing γ for the EU investor or increasing ℓ for the disappointment-averse investor corresponds to moving left along the horizontal axis.

As we argued previously, when returns are jointly lognormal, we cannot differentiate between disappointment-averse and EU investors based on their optimal portfolios. Since

[5] Note that the sum of weights in $\bar{w}^{\mathbf{AV}}$ is –100%. Increasing the weight of $\bar{w}^{\mathbf{AV}}$ in the portfolio corresponds to taking a short position in stocks and a long position in cash and bonds.

the optimal choice is described by the single coefficient $\tilde{\gamma}$, the preference parameters γ, ℓ, and κ cannot be uniquely determined. When returns are asymmetric, the two coefficients $\tilde{\gamma}$ and $\tilde{\chi}$ (or equivalently the weights $\alpha^{\mathbf{MV}}$ and $\alpha^{\mathbf{AV}}$) determine the optimal portfolio. There are two coefficients and three preference parameters, so γ, ℓ, and κ still cannot be uniquely determined. However, a lot more can be learned about the investor's preferences from the observed optimal portfolio than in the lognormal case based on the patterns in Figure 2. First, $\alpha^{\mathbf{MV}}$ reaches zero only in the case of the DA investor ($\kappa = 1$). That is, the DA investor is the only one who, with strong enough disappointment aversion, may choose not to hold risky securities at all. Note that in our calibration any DA investor with $\ell > 0.43$ chooses to invest all her wealth in cash. Second, the EU investor chooses a positive weight in the asymmetry-variance fund for all values of γ, but this weight is very small. Even at its highest value in Figure 2, the relative weight of the asymmetry-variance fund in the EU investor's optimal portfolio is only 0.44%. This emphasizes that EU investors with power utility pay relatively little attention to asymmetries in asset returns. Third, GDA investors' weight in the asymmetry-variance fund (and consequently their implicit asymmetry aversion) can be much larger in magnitude than for any EU or DA investor. Return asymmetries play a much bigger role in the choice of GDA investors. Therefore, an optimal portfolio with a considerable weight in the asymmetry-variance fund belongs to a GDA investor (with $\ell > 0$ and $\kappa \neq 1$). Fourth, a long or a short position in the asymmetry-variance fund can differentiate between GDA investors with $\kappa < 1$ and $\kappa > 1$.

In what follows, we discuss in detail the above results. We start with the observation that DA investors with strong enough disappointment aversion do not hold risky securities. It is important to note that this result has nothing to do with return asymmetries as it also arises when log returns are jointly normal. Ang, Bekaert, and Liu (2005) were the first to demonstrate, in a setting with a single risky asset, that DA preferences lead to nonparticipation in the risky asset market. Farago (2014) demonstrates that $\kappa = 1$ is a knife-edge case in the model, as for GDA investors with $\kappa \neq 1$ investing in the risk-free asset only is never optimal.[6] Figure 2 shows that the results in Farago (2014) extend to the setup with multiple risky assets. Return asymmetries do not have a big impact on a DA investor's optimal choice, since starting from moderate levels of disappointment aversion investors with $\kappa = 1$ do not hold risky securities, regardless of the presence or absence of return asymmetries.

Return asymmetries can have a substantial effect on the portfolios of GDA investors ($\kappa \neq 1$). Panel A of Table 2 presents details of the choice of selected GDA investors. To highlight the effect of GDA preferences, panel B describes the choice of the EU investor who has the same effective risk aversion as does the corresponding GDA investor in the same column. Consequently, their investment in the mean-variance fund is exactly the same and the difference between their optimal portfolios comes from the weights they assign to the asymmetry-variance fund. In other words, the GDA investor and the comparable EU investor would have the same optimal portfolio if return asymmetries were absent, so the difference between their actual portfolios is driven by return asymmetries.

GDA investors pay special attention to disappointing outcomes. When $\kappa < 1$, the disappointment threshold is typically negative and it decreases by roughly one percentage point when κ decreases by 0.01, as shown in Table 2. That is, the GDA investor with $\kappa < 1$ pays special attention to the left tail of the portfolio's return distribution. For a given mean and variance, negative skewness implies a fatter left tail of the return distribution, hence an increased probability of relatively bigger losses. This is illustrated in panel A of Figure 1, where the normal and normal-exponential densities have the same mean and variance, but the latter is negatively skewed and has a fatter left tail. Therefore, the GDA investor with a focus on avoiding left-tail outcomes shifts from negatively skewed assets toward nonskewed

[6] For formal proofs and a detailed discussion of the single-risky-asset case, we refer the reader to Farago (2014), but we provide some intuition here. When the disappointment threshold equals the certainty equivalent ($\kappa = 1$), the cash-only portfolio is the unique portfolio that avoids disappointment in all future states of the world. Therefore, a DA investor who wants to avoid disappointment may prefer this portfolio to any other possible portfolio. When $\kappa < 1$, the disappointment threshold is lower than the certainty equivalent and some other (risky) portfolios also avoid disappointment. The GDA investor with $\kappa < 1$ will prefer some of these portfolios to holding only cash. When $\kappa > 1$, no portfolios avoid disappointment in all future states, so the cash-only portfolio is not a special one.

or positively skewed assets in her risky portfolio. Consequently, she takes a large long position (compared with the EU investor) in the asymmetry-variance fund, indicating a positive implicit asymmetry aversion. This implies that a GDA investor with $\kappa < 1$ shifts from stocks toward bonds in our calibration. As a result, the disappointment probability is much lower than that of the comparable EU investor.[7]

When $\kappa > 1$, the disappointment threshold is positive and it increases by roughly one percentage point when κ increases by 0.01. The GDA investor pays special attention to all outcomes below the threshold; these include not only left-tail outcomes but also central outcomes when the threshold is high. As illustrated in panel A of Figure 1, for a given mean and variance, negative skewness not only leads to a fatter left tail but also implies that the mode of the return distribution shifts to the right. For the GDA investor with $\kappa > 1$, the benefits of having a higher mass on relatively small positive returns outweigh the costs of the fatter left tail. Therefore, she shifts toward negatively skewed assets in her risky portfolio, which is done by taking a short position in the asymmetry-variance fund (and implies a negative implicit asymmetry aversion). A GDA investor with $\kappa > 1$ shifts from bonds toward stocks in our calibration.

In the Online Appendix we provide another portfolio choice example, in which the two risky assets have the same mean and standard deviation and differ only in their skewness. In this case the difference between the choices of different GDA investors can only be driven by the difference in return asymmetries. The same pattern arises as above, namely, that $\kappa < 1$ leads to a positive implicit asymmetry aversion, while $\kappa > 1$ leads to a negative one. These results reassure us that the difference in the choice of GDA investors in our benchmark calibration is indeed driven by return asymmetries.

Figure 2 and Table 2 illustrate how optimal portfolios change with the preference pa-

[7]For example, the disappointment probability of the GDA investor with $\kappa = 0.96$ is 2% in Table 2, while it is 4% for the comparable EU investor. The disappointment probability of the optimal portfolio is $\pi_W \equiv Pr\left[r_{W,t} \leq \ln \kappa + \eta\right]$. Note that the comparable EU investor does not become disappointed, though we can calculate the probability that the portfolio return is below the disappointment threshold $(\ln \kappa + \eta)$ of the corresponding GDA investor.

rameter values. We can also study the sensitivity of the optimal portfolio weights to the asymmetry of returns. In our benchmark calibration, we estimate the skewness of monthly stock returns to be –0.81. However, return asymmetries exhibit significant time variation and there might be periods with much larger skewness. Neuberger (2012), for example, estimates that the expected skewness of the S&P 500 index at the quarterly horizon varies between –1.8 and –1.0 over the 1998-2010 period. In Figure 3 we use the same distributional parameters as in the main calibration, but vary the asymmetry parameter of the stock, δ_S, so that stock skewness varies between –1.8 and –0.3. The vertical line corresponds to our benchmark calibration. The figure presents optimal portfolio weights for an EU investor ($\gamma = 6$) and two GDA investors, one with $\kappa < 1$ ($\gamma = 2$, $\ell = 2$, and $\kappa = 0.96$) and another with $\kappa > 1$ ($\gamma = 2$, $\ell = 2$, and $\kappa = 1.04$). Optimal portfolio weights do not vary much for the EU investor, providing further evidence that return asymmetries have little effect on the portfolio choice of power-utility investors. The GDA investor with a low disappointment threshold further reduces her stock investment and increases her bond investment when the former becomes more negatively skewed. At a stock skewness of –1.8, her stock and bond weights are 28% and 62%, respectively. In contrast, the GDA investor with a high disappointment threshold shifts her risky portfolio from bonds toward stocks even further as the stock skewness becomes more pronounced; at –1.8 skewness, the stock and bond weights are 86% and 1%, respectively.

2.3 Costs of ignoring skewness

Following Das and Uppal (2004), we quantify the certainty-equivalent cost of ignoring return asymmetries. An investor who ignores asymmetry in the distribution of asset returns and chooses her optimal portfolio as if log asset returns were normally distributed with the same mean and variance-covariance matrix as the true distribution, chooses allocation w'. That suboptimal allocation corresponds to a certainty equivalent, $\mathcal{R}'$, under the true return distribution. The cost of ignoring skewness can be measured in absolute terms by $\mathcal{R} - \mathcal{R}'$, or in relative terms by $\frac{\mathcal{R}'-R_f}{\mathcal{R}-R_f}$. The latter is the excess certainty equivalent of the suboptimal allocation relative to the excess certainty equivalent of the optimal allocation.

Table 3 shows the absolute and relative costs of ignoring skewness for different investors using the annualized values of the certainty equivalents $\mathcal{R}$ and $\mathcal{R}'$. Note that the absolute measure is multiplied by 1000, so that it indicates the cost for an investor with an initial wealth of \$1000. For EU investors, the certainty-equivalent cost of ignoring skewness is almost negligible, the annualized cost being less than \$0.02 in all the cases. This is in line with the findings of Das and Uppal (2004).[8] The relative measures indicate that EU investors achieve more than 99.9% of the overall optimal excess certainty equivalent even if they ignore return skewness. For GDA investors, the cost of ignoring skewness is more substantial. When $\kappa = 0.96$, the cost is \$4.20, considerably higher than for the comparable EU investor. In relative terms, this investor achieves only 83.3% of the optimal excess certainty equivalent if she ignores return skewness. As κ increases, the cost of ignoring return asymmetries declines, but it is still much higher for all GDA investors than for the comparable EU investors.

2.4 More than two risky assets

The three-fund separation result lies at the heart of Proposition 1.1. That is, regardless of the number of risky assets, the optimal portfolio is made up of a risk-free asset and two risky funds: the mean-variance and the asymmetry-variance funds. The weights assigned to these funds depend on the investor's effective risk aversion and implicit asymmetry aversion. Therefore, increasing the number of risky assets beyond two or considering different risky assets does not provide further insight into how the model works. We illustrate this in the Online Appendix by considering a portfolio choice problem with three risky assets (treasury bonds, growth stocks, and value stocks) and by considering additional asset classes, including international equities and corporate bonds. Considering additional asset classes illustrates

[8] Das and Uppal (2004) measure the cost of ignoring return asymmetries (caused by jumps that occur simultaneously across assets) for EU investors. They find similar costs for an investor with $\gamma = 5$ and a one-year horizon when the portfolio consists of equity indexes of various developed countries.

the empirical implications of the model regarding the effect of return asymmetries on optimal portfolios. Since various stock portfolios (e.g., portfolios of value stocks, growth stocks, and emerging market equities) and corporate bond portfolios display negative skewness, they enter the asymmetry-variance fund with a short position. Hence, a GDA investor with $\kappa < 1$ ($\kappa > 1$) underweights (overweights) these assets in her optimal portfolio relative to the comparable EU investor.

3 Common Portfolio Recommendations

3.1 Asset-allocation puzzle of Canner, Mankiw, and Weil (1997)

The two-fund separation strategy arising from standard models implies that all investors should hold risky assets in the same proportion, and should change only their relative weights in the risky portfolio and in cash according to their risk appetite. Consequently, all investors should have the same bond/stock ratio in their portfolios. The asset-allocation puzzle of Canner, Mankiw, and Weil (1997) is that, in contrast with the above predictions, financial advisors recommend different ratios for different investors: a high bond/stock ratio for "conservative" investors and a low ratio for "aggressive" investors.[9] Table 4 is adapted from Canner, Mankiw, and Weil (1997) and presents the recommendations of four financial advisors, together with the assumed asset returns from the original paper. Note that Canner, Mankiw, and Weil (1997) do not report asset skewness, so we use values from our calibration. The results are not sensitive to moderate changes in these skewness values. For each advisor in Table 4, the bond/stock ratio (w_B/w_S) increases as we move from aggressive toward conservative portfolios.

[9] Evidence suggests that individual investors consider the recommendations of a financial advisor when making portfolio decisions. Bluethgen et al. (2008) report that 66% of the individual investors consult their advisors for investment advice from a sample provided by a large German retail bank. Guiso and Sodini (2013) refer to a survey of Italian individual investors with checking accounts at a large European banking group, where 60% of the investors report relying on the help of an advisor or intermediary when making financial decisions.

Given the distributional assumptions in panel A of Table 4, we can determine the composition of the mean-variance and asymmetry-variance funds using (18). The resulting normalized funds, $\bar{w}^{\mathbf{MV}}$ and $\bar{w}^{\mathbf{AV}}$, are given in the last two columns of panel A. Equation (22) shows that each recommended portfolio can be constructed using these normalized funds. Since there are two equations (one for the stock weight, w_S, and one for the bond weight, w_B), there is a unique pair of fund weights, $\alpha^{\mathbf{MV}}$ and $\alpha^{\mathbf{AV}}$, that yields a given recommended portfolio. Equation (23) further shows that a given pair of $\alpha^{\mathbf{MV}}$ and $\alpha^{\mathbf{AV}}$ corresponds to a unique pair of effective risk aversion ($\tilde{\gamma}$) and implicit asymmetry aversion ($\tilde{\chi}$). The last four columns of panel B present these quantities ($\alpha^{\mathbf{MV}}$, $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, $\tilde{\gamma}$, and $\tilde{\chi}$) for each recommended portfolio. As we move from aggressive toward conservative portfolios, the weight in the mean-variance fund decreases, which is consistent with increasing effective risk aversion. At the same time, the relative weight in the asymmetry-variance fund increases, leading to an increase in the bond/stock allocation ratio and consistent with increasing implicit asymmetry aversion. Comparing the $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ and $\tilde{\chi}$ values from Table 4 with those in Figure 2, we can see that the values corresponding to moderate and conservative portfolios are much higher than those implied by EU preferences, though they are in line with the choice of GDA investors for whom $\kappa < 1$. That is, return asymmetries together with increasing disappointment aversion from aggressive to conservative investors offers an explanation for the asset allocation puzzle of Canner, Mankiw, and Weil (1997).[10]

3.2 Short- and long-term portfolios

Next, we examine the effect of the investment horizon on optimal portfolios assuming that the one-period returns are independent and identically distributed (IID). Consider the H-period

[10]Other potential explanations have been proposed. Bajeux-Besnainou, Jordan, and Portait (2001) explain the puzzle by assuming that the investor's horizon may exceed the maturity of the cash asset. Shalit and Yitzhaki (2003) use conditional stochastic dominance arguments to demonstrate that advisors, acting as agents for numerous clients, recommend portfolios that are efficient for some risk-averse investors. Campbell and Viceira (2001) rationalize the popular advice in the context of intertemporal asset allocation models with time-varying expected returns.

log return between dates t and $t+H$,

$$r_{t,t+H} \equiv \sum_{h=1}^{H} r_{t+h} \ . \tag{24}$$

The H-period returns follow

$$r_{t,t+H} = \mu_H - \sqrt{H}\left(\sigma_H \circ \delta\right) + \left(\sigma_H \circ \delta\right)\varepsilon_{0,t,t+H} + \left(\sigma_H \circ \sqrt{\iota - \delta \circ \delta}\right) \circ \varepsilon_{t,t+H} \ , \tag{25}$$

where,

$$\varepsilon_{0,t,t+H} \sim \Gamma\left(H, 1/\sqrt{H}\right) \ , \qquad \varepsilon_{t,t+H} \sim N\left(0, \Psi\right) \ , \qquad \mu_H = H\mu \ , \qquad \sigma_H = \sqrt{H}\sigma \ . \tag{26}$$

Note that the parameters δ and Ψ do not have an H subscript, since their values are independent of the horizon.[11] The moments of the H-period return of asset i are given by

$$\begin{aligned} E\left(r_{i,t,t+H}\right) &= H\mu_i \ , & Var\left(r_{i,t,t+H}\right) &= H\sigma_i^2 \ , \\ Skew\left(r_{i,t,t+H}\right) &= \frac{2\delta_i^3}{\sqrt{H}} \ , & Xkurt\left(r_{i,t,t+H}\right) &= \frac{6\delta_i^4}{H} \ . \end{aligned} \tag{27}$$

Both the mean and variance of the assets grow by H as the horizon increases. Asset skewness, on the other hand, is scaled by $1/\sqrt{H}$. That is, skewness diminishes as H increases and the distribution of long-horizon returns is closer to normal than is the distribution of short-horizon returns. In fact, for large values of H the distribution of the common shock $\varepsilon_{0,t,t+H}$ converges to a normal distribution, and the asset log returns become jointly normal.

The optimal asset allocation for an investor with horizon H may be written as

$$w_H = \frac{1}{\tilde{\gamma}_H}\left(w^{\mathbf{MV}} + \tilde{\chi}_H w_H^{\mathbf{AV}}\right) \ . \tag{28}$$

[11] The key result for deriving the return-generating model of the H-period returns is that the sum of H IID exponential variables is a random variable that follows a gamma distribution with a shape parameter H. Note also that for $H=1$, the model in (25) is equivalent to the model in (10), since $\Gamma\left(1,1\right) \equiv \exp\left(1\right)$.

Note that $w^{\mathbf{MV}}$ does not have an H subscript. The mean-variance fund has the same composition, regardless of the investor's horizon. Although the second risky fund, $w_H^{\mathbf{AV}}$, does have a horizon subscript, note that $w_H^{\mathbf{AV}} = w^{\mathbf{AV}}/\sqrt{H}$. That is, investors with different horizons will use the same asymmetry-variance fund in their portfolios, but the size of their investment will be different. The parameters describing the risk attitude of the investor, $\tilde{\gamma}_H$ and $\tilde{\chi}_H$, depend on the horizon.[12]

Consider again our example with a stock, a bond, and a risk-free asset. We take the parameters μ, σ, δ, and ψ from the estimation using monthly data in Table 1, and assume that the H-month returns are generated by the model in (25). The "stock" is the value-weighted portfolio of the CRSP stocks held for H months. The "bond" is a bond portfolio that is rolled over at the end of each month during the H-month holding period to keep the maturity at the rollover date fixed at 10 years. Finally, we simply assume that the H-period risk-free return is Hr_f.

For a given level of effective risk aversion, $\tilde{\gamma}_H$, the only part of the optimal portfolio rule (22) that changes with the investment horizon is $\alpha^{\mathbf{AV}}$, i.e., the weight assigned to $\bar{w}^{\mathbf{AV}}$. To illustrate the effect of horizon in our calibration, for each H we fix $\tilde{\gamma}_H = 5$ (consequently, fix $\alpha^{\mathbf{MV}}$) and calculate the corresponding weight in the asymmetry-variance fund for different investors. For an EU investor, $\tilde{\gamma}_H = 5$ implies $\gamma = 5$. For a GDA investor, a given value of $\tilde{\gamma}_H$ can correspond to different sets of parameter values. We fix $\gamma = 2$ and $\ell = 2$, and choose the value of κ that leads to $\tilde{\gamma}_H = 5$. Note that two different κ values lead to $\tilde{\gamma}_H = 5$, one such that $\kappa < 1$ and the other such that $\kappa > 1$. We report results for both cases.

Figure 4A shows how $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, the relative weight in the normalized asymmetry-variance fund, changes with the horizon. Return asymmetries do not have a large effect on the EU investor's portfolio as her relative weight barely changes with H. For a GDA investor, however, the investment horizon is an important factor determining the optimal portfolio. Over short horizons, GDA investors hold different $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ ratios than do EU

[12] Analytical formulas for the horizon-dependent effective risk aversion $\tilde{\gamma}_H$ and implicit asymmetry aversion $\tilde{\chi}_H$ are given in the Online Appendix.

investors due to asymmetries in returns. However, these return asymmetries become less pronounced as the horizon increases. Hence, the $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ of the GDA investors approaches that of the EU investors. Disappointment aversion (with $\kappa < 1$) together with skewness prompts a shift from bonds to stocks as the investment horizon increases. This is a different mechanism from the often-emphasized effect of mean reversion in prices (see, e.g., Campbell and Viceira 2002, 2005) or nontradable human capital (see, e.g., Jagannathan and Kocherlakota 1996; Cocco, Gomes, and Maenhout 2005).

When returns are IID, the effect of skewness quickly disappears as the horizon increases. However, there is evidence that return skewness does not diminish with the investment horizon as quickly as implied by the IID assumption.[13] To illustrate the effect of persistence in skewness, instead of relying on the IID assumption to calculate H-period returns, we fit our return-generating model (10) to returns aggregated over $H = 1, ..., 12$ months. Figure 4B shows the stock's skewness over different horizons. The sample estimates are further from zero than the values implied by the IID assumption. In fact, the skewness of the H-month return is greater in magnitude than that of the one-month return for all $H > 1$. Figures 4C and 4D show the optimal stock weight and bond/stock allocation ratio, respectively, for the EU investor and two GDA investors (one with $\kappa < 1$ and the other with $\kappa > 1$) in the IID case (labeled "IID") and for the same GDA investors using the estimated H-period returns (labeled "Sample"). The optimal choice of the GDA investor with $\kappa < 1$ does not converge to that of the EU investor for holding periods up to one year when the estimated H-period returns are used. Since the $\kappa < 1$ case is arguably more relevant empirically, this evidence suggests that return asymmetry may have a larger effect on optimal portfolios for longer investment horizons than in an IID calibration.

[13]Neuberger (2012) develops an unbiased estimate of the third moment of long-horizon returns from high-frequency returns. He finds that the skewness of U.S. equity index returns does not diminish with the horizon; it actually increases with horizons up to a year, and its magnitude is economically important. Ghysels, Plazzi, and Valkanov (2014) introduce an asymmetry measure based on conditional quantiles and find that the return asymmetry is more pronounced at the quarterly frequency than at the monthly frequency for the United States and many other countries in their sample.

4 Other Reference-Dependent Preference Frameworks

Disappointment aversion is not the only preference framework building on the idea that investors are more sensitive to outcomes below a certain reference point. Other notable examples are the reference-dependent preferences of Kőszegi and Rabin (2006, 2007) and the (cumulative) prospect theory of Kahneman and Tversky (1979, 1992). In this section we compare the portfolio choice implications of GDA and these alternative preferences.

4.1 Reference-dependent preferences

Kőszegi and Rabin (2006, 2007) introduce a model of reference-dependent preferences in which the agent is loss averse around a reference point that is determined endogenously by the economic environment. Similar to Kőszegi and Rabin (2007), we let utility be defined over the terminal wealth level, or, without loss of generality, over the portfolio return:

$$\tilde{U}(F \mid G) = \int \int \tilde{u}(R_W \mid R_z)\, dG(R_z)\, dF(R_W) = E^F\left[E^G\left[\tilde{u}(R_W \mid R_z)\right]\right], \qquad (29)$$

where R_W is the return on the portfolio with probability distribution F and R_z is a stochastic reference point with probability distribution G. Let $\tilde{u}(R_W \mid R_z)$ be defined as

$$\tilde{u}(R_W \mid R_z) = (1-\vartheta)\, U(R_W) + \vartheta L\left(U(R_W) - U(R_z)\right) \qquad (30)$$

with

$$L(x) = x + (\lambda - 1)\, x I(x \leq 0) = \begin{cases} x & x > 0 \\ \lambda x & x \leq 0 \end{cases}, \qquad (31)$$

where $U(\cdot)$ is a standard utility function and $L(\cdot)$ is the "gain-loss utility". The functional form of $L(\cdot)$ captures Kahneman and Tversky's (1979) loss aversion with λ as the loss-aversion parameter. The parameter $\vartheta \in [0,1]$ can be interpreted as the relative weight attached to the gain-loss utility. Using the first expression for the gain-loss utility in (31),

$$\tilde{U}(F \mid G) = E^F[U(R_W)] - \vartheta E^G[U(R_z)] - \vartheta(\lambda - 1) E^F\left[E^G[(U(R_z) - U(R_W)) I(R_W \leq R_z)]\right] \ . \tag{32}$$

The reference point, R_z, can either be stochastic or be constant (taking a fixed value with probability one). The reference point also can be either exogenous or endogenous (depending on the portfolio weights). Considering a constant reference point is highly relevant to the current paper.

When the reference point is constant and exogenous, the portfolio choice problem, i.e., choosing the portfolio weights to maximize $\tilde{U}(F \mid G)$, simplifies to

$$\max_w \ E^F[U(R_W)] - \underbrace{\vartheta(\lambda - 1)}_{\equiv \ell^{KR}} E^F[(U(R_z) - U(R_W)) I(R_W \leq R_z)] \ . \tag{33}$$

Berkelaar, Kouwenberg, and Post (2004) refer to these preferences as the "kinked power utility" when $U(\cdot)$ is the power utility, and use them to isolate the effect of loss aversion in prospect theory. Comparing the above formula with (7), we can see that (33) is equivalent to the GDA portfolio problem with disappointment-aversion parameter ℓ^{KR}, except that the reference point is exogenously given in (33), while it is endogenously determined in the GDA problem. Since the two problems are closely related, our results for the GDA problem also provide an analytical solution to (33). The result in Proposition 1.1 holds if we replace the endogenous (log) reference point $\ln \kappa + \eta$ with the exogenously given reference point $\ln R_z$. We demonstrate in the Online Appendix that if the reference point is defined as $R_z \equiv \kappa R_f$ and we set $\ell = \ell^{KR}$, the two problems lead to almost identical optimal portfolios in our benchmark calibration.

When the reference point is constant and endogenous, the portfolio choice problem becomes

$$\max_w \ E^F[U(R_W)] - \vartheta U(R_z) - \ell^{KR} E^F[(U(R_z) - U(R_W)) I(R_W \leq R_z)] \ . \tag{34}$$

We show in Appendix C that if $U(\cdot)$ is the power utility and the reference point is defined as $R_z \equiv \kappa \mathcal{R}$, where $\mathcal{R}$ is the GDA certainty equivalent from (7), every GDA optimal portfolio problem can be formulated as the problem in (34). To the best of our knowledge, this connection between the two preference frameworks has not been pointed out previously.

The central idea of Kőszegi and Rabin (2006, 2007) is that the reference point is stochastic and it is described by the probability distribution G. They assume that this distribution represents the investor's fully probabilistic rational expectations about the portfolio return, i.e., $G = F$. In this case, the portfolio choice can be rewritten as (if $\vartheta \neq 1$)

$$\max_{w} \; E^F\left[U\left(R_W\right)\right] - \frac{\vartheta\left(\lambda-1\right)}{1-\vartheta} E^F\left[E^F\left[\left(U\left(R_z\right) - U\left(R_W\right)\right) I\left(R_W \leq R_z\right)\right]\right] \; . \tag{35}$$

In the Online Appendix we solve this problem numerically and demonstrate that it leads to nonparticipation in risky asset markets similar to the GDA preferences with $\kappa = 1$ and to the kinked power-utility preferences in (33) with $R_z = R_f$. That is, return asymmetries in these cases do not have a big impact on the optimal portfolio.

4.2 Cumulative prospect theory

Kahneman and Tversky (1979) introduced prospect theory (PT) and Tversky and Kahneman (1992) consider the modified (cumulative) version (CPT). Four building blocks of CPT distinguish it from expected utility theory. These can be summarized as follows:

1. the CPT investor is more sensitive to losses than to gains of the same magnitude; this is also known as loss aversion.

2. the CPT investor's reference point for distinguishing gains from losses is exogenously given.

3. the CPT investor's utility function is concave over gains and convex over losses.

4. the CPT investor uses transformed (or subjective) probabilities instead of objective probabilities.

To isolate the effect of the first two items in the above list, Berkelaar, Kouwenberg, and Post (2004) suggest the kinked power utility described in (33), which deviates from EU by assuming that the investor is more sensitive to outcomes below an exogenously given reference point. As we discussed in the previous section, this setup is very similar to GDA. The only difference is that the reference point is exogenously given for the kinked power utility, while it is endogenously determined for the GDA. We also demonstrated that they lead to similar optimal portfolios, at least in our static portfolio choice problem. That is, the effect of loss aversion only is similar to that of disappointment aversion.

However, CPT is a collection of deviations from EU theory that contains not only loss aversion but also the convexity of the utility function over losses and the use of transformed probabilities. With all these deviations, CPT does not nest EU as a special case, and it is difficult to provide a direct analytical comparison between CPT and GDA. To compare their implications, we numerically solve the portfolio choice problem of various CPT investors when returns are generated from the normal-exponential model using our benchmark calibration. The Online Appendix contains the detailed assessment of the results, and we summarize the key lessons here.

First, for certain parameterizations, when the degree of loss aversion is low enough, the CPT portfolio problem does not have a finite optimal solution. He and Zhou (2011) refer to this as the problem being "illposed" and provide a detailed analysis for the case with a single risky asset in the portfolio. Our results indicate that the CPT portfolio choice problem also may be illposed if there are multiple risky assets in the portfolio. Note that this issue does not arise in the case of EU and GDA preferences.

Second, when the reference point is the initial wealth grown at the risk-free rate, which is an important benchmark in the literature, the CPT portfolio problem has an inconvenient implication: whenever there is a unique finite solution, the CPT investor's optimal strategy is to invest all her wealth in the risk-free asset.[14] This is discussed in detail by Ang, Bekaert, and Liu (2005) and He and Zhou (2011) in the case with one risk-free and one risky asset. Our numerical analysis in the Online Appendix suggests that this result extends to the case of multiple risky assets.

Third, when the reference point is lower than the initial wealth grown at the risk-free rate, CPT has similar portfolio choice implications as do GDA preferences with $\kappa < 1$ (which also corresponds to a low reference point). A CPT investor increases the weight of bonds relative to stocks in the optimal portfolio as her (1) degree of loss aversion increases and (2) subjective probability distortion becomes more pronounced. This is similar to the effect of increasing disappointment aversion on the GDA investor's portfolio. It is not surprising that loss aversion and disappointment aversion have the same effect based on the similarity of kinked-power-utility and GDA preferences discussed previously. However, it is not a priori evident that probability distortion also should have a similar effect. We demonstrate in the Online Appendix that the mechanism through which the probability distortion component of CPT works is different. Probability distortion works through the subjective mean and variance of the return distributions, while loss aversion and disappointment aversion work through return asymmetries.

Fourth, when the reference point is higher than the initial wealth grown at the risk-free rate, CPT and GDA (with $\kappa > 1$, which also corresponds to a high reference point) have different portfolio choice implications. This is not surprising given that when the reference point is high, the CPT utility function is convex on most of the outcomes (as they are considered losses), while the GDA utility is concave everywhere.

[14]For example, Ang, Bekaert, and Liu (2005) and Barberis and Huang (2008) consider a setup in which the reference point is the initial wealth grown at the risk-free rate. Note that with the same reference point, the kinked power utility from (33) also may lead to nonparticipation in risky asset markets, as well as GDA preferences with $\kappa = 1$. However, the solution is either nonparticipation or no unique solution in case of CPT, and for the other two preferences there are parameterizations with a unique solution and risky investment.

5 Conclusion

We studied the joint impact of two types of asymmetries on portfolio choice: in asset returns and in investor attitudes toward risk. We modeled asymmetric preferences using generalized disappointment aversion, and we modeled asymmetric return distributions using a normal-exponential model. We have found that the two types of asymmetries jointly yield qualitatively different optimal portfolios from those of the standard model, in which these asymmetries are ignored. On the one hand, when asset returns are symmetric, all investors hold the same risky portfolio. Consequently, when observing a particular asset allocation, it is impossible to determine whether it was chosen by a disappointment-averse or a disappointment-neutral investor. On the other hand, standard preferences imply that return asymmetry only marginally affects the composition of optimal portfolios. However, when both asymmetries are taken into account, the composition of the optimal portfolio changes.

In our calibrated main example involving three assets (cash, bonds, and stocks), a disappointment-averse investor with a reference point lower than the certainty equivalent of the investment shifts from negatively skewed stocks toward bonds to avoid the occasional large losses that negatively skew the stock returns. On the other hand, a disappointment-averse investor with a reference point higher than the certainty equivalent prefers to hold stocks in her optimal portfolio. We also have demonstrated that the portfolio choice of an investor with a longer investment horizon is less affected by return asymmetries.

Return asymmetries only have a marginal effect on the portfolio choice of investors with standard preferences. Expected utility investors achieve more than 99.9% of their overall optimal excess certainty equivalent even if they ignore return asymmetries. However, cross-sectional asset pricing studies suggest that systematic and idiosyncratic return asymmetries are priced (e.g., Harvey and Siddique 2000; Dittmar 2002; Boyer, Mitton, and Vorkink 2010). Our results suggest that preference asymmetries, and disappointment aversion in particular, may help to reconcile the portfolio choice implications with the asset pricing evidence.

Appendix

A Log Certainty Equivalent

Recall that $r_W = \ln R_W$, $\eta = \ln(\mathcal{R})$, and $U(X) = \frac{X^{1-\gamma}}{1-\gamma}$. We can accordingly rewrite

$$\begin{aligned} U(\kappa\mathcal{R}) - U(R_W) &= U(\kappa\mathcal{R})\left(1 - \frac{U(R_W)}{U(\kappa\mathcal{R})}\right) = U(\kappa\mathcal{R})\left(1 - \left(\frac{R_W}{\kappa\mathcal{R}}\right)^{1-\gamma}\right) \\ &= \kappa^{1-\gamma}U(\mathcal{R})\left(1 - \exp\left((\gamma-1)\left(\ln\kappa + \eta - r_W\right)\right)\right). \end{aligned} \tag{A1}$$

Noting that $\forall a, X \in \mathbb{R}$

$$(1 - \exp(aX))\, I(X > 0) = 1 - \exp(aXI(X > 0)) = 1 - \exp(a \max(X, 0)) \ , \tag{A2}$$

Equation (A1) implies

$$E\left[(U(\kappa\mathcal{R}) - U(R_W))\, I(R_W < \kappa\mathcal{R})\right] = \kappa^{1-\gamma}U(\mathcal{R})\left(1 - E\left[\exp\left((\gamma-1)\, p_W\right)\right]\right) \ , \tag{A3}$$

where $p_W \equiv \max(\ln\kappa + \eta - r_W, 0)$.

Substituting (A3) into (7) and solving for $U(\mathcal{R})$, we arrive at

$$\begin{aligned} U(\mathcal{R}) &= \frac{E\left[U(R_W)\right]}{\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_W\right)\right]\right)} \ , \\ \ln\mathcal{R}^{1-\gamma} &= \ln E\left[R_W^{1-\gamma}\right] - \ln\left(\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_W\right)\right]\right)\right) \ . \end{aligned} \tag{A4}$$

This finally leads to the first case in Equation (8). The second case in (8) directly derives from the first case by taking the limit and applying l'Hôpital's rule.

B Proof of Proposition 1.1

To simplify notation, we drop the t subscript and use r instead of r_t, and r_W instead of $r_{W,t}$. Equation (8) defines an implicit function:

$$G(w,\eta) \equiv -\eta + \frac{1}{1-\gamma}\ln E\left[\exp\left((1-\gamma)r_W\right)\right] - \frac{1}{1-\gamma}\ln\left(\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)p_W\right)\right]\right)\right) = 0\ . \quad \text{(A5)}$$

Implicit differentiation of (A5) implies that

$$\frac{\partial\eta}{\partial w} = -\frac{G_1'(w,\eta)}{G_2'(w,\eta)}\ , \quad \text{(A6)}$$

where G_1' (G_2') is the partial derivative of G with respect to its first (second) argument. If an optimal allocation policy exists, it satisfies the necessary condition $\frac{\partial\eta}{\partial w} = 0$, implying that

$$G_1'(w,\eta) = 0\ . \quad \text{(A7)}$$

From (A5),

$$G_1'(w,\eta) = \frac{E\left[\exp\left((1-\gamma)r_W\right)\left(\partial r_W/\partial w\right)\right]}{E\left[\exp\left((1-\gamma)r_W\right)\right]} - \frac{\ell\kappa^{1-\gamma}E\left[\exp\left((\gamma-1)p_W\right)\left(\partial p_W/\partial w\right)\right]}{\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)p_W\right)\right]\right)}\ . \quad \text{(A8)}$$

Equation (14) implies

$$\frac{\partial r_W}{\partial w} = \left(r - r_f\iota + \frac{1}{2}\sigma^2\right) - \Sigma w \ \text{ and } \ \frac{\partial p_W}{\partial w} = -\frac{\partial r_W}{\partial w}I\left(r_W < \ln\kappa + \eta\right), \quad \text{(A9)}$$

which is substituted into (A8) to yield

$$\begin{aligned} G_1'(w,\eta) = {} & \frac{E\left[\exp\left((1-\gamma)r_W\right)r\right]}{E\left[\exp\left((1-\gamma)r_W\right)\right]} + \frac{\nu}{1-\nu}\frac{E\left[\exp\left((1-\gamma)r_W\right)rI\left(r_W < \ln\kappa + \eta\right)\right]}{E\left[\exp\left((1-\gamma)r_W\right)I\left(r_W < \ln\kappa + \eta\right)\right]} \\ & + \frac{1}{1-\nu}\left(-r_f\iota + \frac{1}{2}\sigma^2 - \Sigma w\right)\ , \end{aligned} \quad \text{(A10)}$$

where

$$\nu \equiv \frac{\ell\kappa^{1-\gamma}\exp\left((\gamma-1)\left(\ln\kappa + \eta\right)\right)E\left[\exp\left((1-\gamma)r_W\right)I\left(r_W < \ln\kappa + \eta\right)\right]}{\theta + \ell\kappa^{1-\gamma}E\left[I\left(r_W < \ln\kappa + \eta\right)\right]}\ . \quad \text{(A11)}$$

Define

$$M(u, v; x) \equiv E\left[\exp\left(ur_W + v^\top r\right) I(r_W < x)\right] \ . \tag{A12}$$

Then (A10) can be rewritten as

$$G_1'(w, \eta) = \frac{M_2'(1-\gamma, 0; \infty)}{M(1-\gamma, 0; \infty)} + \frac{\nu}{1-\nu} \frac{M_2'(1-\gamma, 0; \ln\kappa + \eta)}{M(1-\gamma, 0; \ln\kappa + \eta)} + \frac{1}{1-\nu}\left(-r_f\iota + \frac{1}{2}\sigma^2 - \Sigma w\right) \ , \tag{A13}$$

while the log certainty equivalent and ν can be rewritten as

$$\begin{aligned} \eta =& \frac{1}{1-\gamma} \ln M((1-\gamma), 0; \infty) \\ &- \frac{1}{1-\gamma} \ln\left(\theta + \ell\kappa^{1-\gamma} M(0, 0; \ln\kappa + \eta) - \ell\kappa^{1-\gamma} e^{-(1-\gamma)(\ln\kappa+\eta)} M(1-\gamma, 0; \ln\kappa + \eta)\right) \ , \end{aligned} \tag{A14}$$

and

$$\nu = \frac{\ell\kappa^{1-\gamma} e^{-(1-\gamma)(\ln\kappa+\eta)} M(1-\gamma, 0; \ln\kappa + \eta)}{\theta + \ell\kappa^{1-\gamma} M(0, 0; \ln\kappa + \eta)} \ . \tag{A15}$$

Finding an analytical formula for $M(u, v; x)$ and $M_2'(u, v; x)$ allows us to calculate all the quantities of interest from (A13), (A14), and (A15). The Online Appendix contains the derivation of the formulas for these quantities in a more general case, in which the investment horizon is H periods and the H-period log return on risky assets between dates t and $t+H$ is defined as

$$r_{t,t+H} \equiv \sum_{h=1}^{H} r_{t+h} \ . \tag{A16}$$

In Proposition 1.1, we consider the case in which $H = 1$ (and $r_{t,t+1}$ is simply referred to as r). Using the results in the Online Appendix for $H = 1$,

$$M(1-\gamma, 0; \ln\kappa + \eta) = \exp\left((1-\gamma)(\mu_W - \sigma_W \delta_W) + \frac{(1-\gamma)^2 \sigma_W^2 \left(1 - \delta_W^2\right)}{2}\right) \frac{\Phi(c_0) + C}{c_2} \ , \tag{A17}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and

$$C \equiv \begin{cases} \exp\left(\frac{c_2^2 + 2c_0c_1c_2}{2c_1^2}\right) \Phi\left(-\frac{c_2 + c_0c_1}{c_1}\right) & \text{if } c_1 > 0 \\ -\exp\left(\frac{c_2^2 + 2c_0c_1c_2}{2c_1^2}\right) \Phi\left(\frac{c_2 + c_0c_1}{c_1}\right) & \text{if } c_1 < 0 \end{cases} \ , \tag{A18}$$

and

$$c_0 \equiv \frac{\ln \kappa + \eta - \mu_W + \sigma_W \delta_W - (1-\gamma)\,\sigma_W^2\left(1-\delta_W^2\right)}{\sigma_W \sqrt{1-\delta_W^2}}$$
$$c_1 \equiv \frac{-\delta_W}{\sqrt{1-\delta_W^2}} \tag{A19}$$
$$c_2 \equiv 1-(1-\gamma)\,\sigma_W \delta_W \ .$$

Note also that

$$M(1-\gamma, 0; \infty) = \exp\left((1-\gamma)\left(\mu_W - \sigma_W \delta_W\right) + \frac{(1-\gamma)^2 \sigma_W^2\left(1-\delta_W^2\right)}{2}\right)\frac{1}{c_2} \ . \tag{A20}$$

It also is shown in the Online Appendix that

$$\frac{M_2'(1-\gamma, 0; \ln \kappa + \eta)}{M(1-\gamma, 0; \ln \kappa + \eta)} = \mu + \left(1-\gamma + \frac{\xi_{\Sigma,0}^B}{\Phi(c_0) + C}\right)\Sigma w + \left(\frac{(1-\gamma)^2 \sigma_W^2 \delta_W^2}{c_2} + \frac{\xi_{a,0}^B}{\Phi(c_0)+C}\right)(\sigma \circ \delta) \ , \tag{A21}$$

with

$$\begin{aligned} \xi_{a,0}^B &= \exp\left(\frac{c_2^2 + 2c_0 c_1 c_2}{2c_1^2}\right)\Phi\left(-\frac{c_2 + c_0 c_1}{c_1}\right)\left(-c_2 - \frac{c_2 + c_0 c_1}{c_1^2}\right) \\ &\quad + \exp\left(\frac{c_2^2 + 2c_0 c_1 c_2}{2c_1^2}\right)\phi\left(-\frac{c_2 + c_0 c_1}{c_1}\right)\left(\frac{1}{c_1} + c_1\right) - c_1 \phi(c_0) \\ \xi_{\Sigma,0}^B &= \exp\left(\frac{c_2^2 + 2c_0 c_1 c_2}{2c_1^2}\right)\Phi\left(-\frac{c_2 + c_0 c_1}{c_1}\right)\frac{c_2}{\sigma_W \delta_W} \\ &\quad + \exp\left(\frac{c_2^2 + 2c_0 c_1 c_2}{2c_1^2}\right)\phi\left(-\frac{c_2 + c_0 c_1}{c_1}\right)\frac{1}{\sigma_W\sqrt{1-\delta_W^2}} - \frac{\phi(c_0)}{\sigma_W\sqrt{1-\delta_W^2}} \ , \end{aligned} \tag{A22}$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution. Also,

$$\frac{M_2'(1-\gamma, 0; \infty)}{M(1-\gamma, 0; \infty)} = \mu + (1-\gamma)\,\Sigma w + \frac{(1-\gamma)^2 \sigma_W^2 \delta_W^2}{c_2}(\sigma \circ \delta) \ . \tag{A23}$$

Substituting (A21) and (A23) into (A13), setting it to zero, and solving for w, we arrive at the optimal portfolio rule:

$$w = \frac{1}{\tilde{\gamma}}\left(\Sigma^{-1}\left(\mu - r_f \iota + \frac{1}{2}\sigma^2\right) + \tilde{\chi}\Sigma^{-1}(\sigma \circ \delta)\right) \ , \tag{A24}$$

with

$$\tilde{\gamma} = \gamma - \nu \frac{\xi^B_{\Sigma,0}}{\Phi(c_0) + C}$$
$$\tilde{\chi} = \frac{(1-\gamma)^2 \sigma_W^2 \delta_W^2}{1 - (1-\gamma)\sigma_W \delta_W} + \nu \frac{\xi^B_{a,0}}{\Phi(c_0) + C} \ , \tag{A25}$$

which corresponds to (17) in the paper. Note that $\ell = 0$ implies $\nu = 0$, which easily can be seen from (A15).

C Equivalence of the GDA and the KR Problem

When the reference point is constant and endogenously determined, $\tilde{U}(F \mid G)$ from (32) can be written as

$$\tilde{U}(F \mid G) = E^F[U(R_W)] - \vartheta U(R_z) - \underbrace{\vartheta(\lambda - 1)}_{\equiv \ell^{KR}} E^F[(U(R_z) - U(R_W)) I(R_W \leq R_z)] \ . \tag{A26}$$

Assume that the reference point is $R_z = \kappa\mathcal{R}$, where $\mathcal{R}$ is the GDA certainty equivalent from (7). Then using the definition in (7)

$$\begin{aligned}\tilde{U}(F \mid G) &= E^F[U(R_W)] - \vartheta U(\kappa\mathcal{R}) - \ell^{KR} E^F[(U(\kappa\mathcal{R}) - U(R_W)) I(R_W \leq \kappa\mathcal{R})] \\ &= \theta U(\mathcal{R}) - \vartheta U(\kappa\mathcal{R}) \\ &= \left(\theta - \vartheta\kappa^{1-\gamma}\right) U(\mathcal{R}) \ ,\end{aligned} \tag{A27}$$

where the last equality uses the assumption that $U(\cdot)$ has the power-utility form. Maximizing $\tilde{U}(F \mid G)$ is the same as maximizing the GDA certainty equivalent, $\mathcal{R}$, if $\theta - \vartheta\kappa^{1-\gamma} > 0$. That is, for every GDA problem described by the set of preference parameters $\{\gamma, \ell, \kappa\}$, we can find a corresponding Kőszegi and Rabin (2006, 2007) problem with parameters $\{\gamma, \vartheta, \lambda, R_z\}$ such that they lead to the same optimization problem. We need to pick the parameters so that $R_z = \kappa\mathcal{R}$, $\vartheta(\lambda - 1) = \ell$ and the inequality $\theta - \vartheta\kappa^{1-\gamma} > 0$ is satisfied.

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Table 1. Parameter estimates

Panel A. Moment and parameter estimates

	Sample		GMM	
	Est.	95% CI	Est.	SE
r_f (%)	0.37			
$\mu_B - r_f$ (%)	0.13	{0.00, 0.28}	0.13	(0.08)
$\mu_S - r_f$ (%)	0.48	{0.17, 0.79}	0.48	(0.17)
σ_B (%)	2.11	{1.97, 2.25}	2.11	(0.12)
σ_S (%)	4.34	{4.01, 4.70}	4.34	(0.24)
$corr_{BS}$	0.10	{0.00, 0.19}	0.10	(0.06)
$skew_B$	0.20	{–0.11, 0.50}	0.02	(0.06)
$skew_S$	–0.82	{–1.32, –0.33}	–0.81	(0.38)
$coskew_{BS}$	–0.07	{–0.27, 0.13}	–0.07	(0.02)
$coskew_{SB}$	0.18	{–0.07, 0.45}	0.23	(0.13)
$xkurt_B$	1.48	{0.82, 2.15}	0.01^i	
$xkurt_S$	2.92	{0.60, 5.49}	1.80^i	
ψ			0.39	(0.16)
δ_B			0.21	(0.08)
δ_S			–0.74	(0.12)

Panel B. Composition of the normalized funds

	Bond	Stock
$\bar{w}^{\mathbf{MV}}$ (%)	48.4	51.6
$\bar{w}^{\mathbf{AV}}$ (%)	332.5	–432.5

Panel A presents parameter and moment estimates for the calibration of the normal-exponential model described in (10). The data used for the calibration are monthly log returns on three assets: 30-day Treasury bills (f), the 10-year government bond index (B), and the value-weighted index of the CRSP stocks (S). The period used is from July 1952 to December 2014. The first two columns present sample moment estimates together with their bootstrapped 95% confidence intervals. The last two columns show the results of the GMM estimation. The GMM estimation is overidentified and fits the two means (μ), two volatilities (σ), correlation ($corr$), two skewnesses ($skew$), and two coskewnesses ($coskew$). Values with superscript i are not estimated but are implied by the fitted distribution. Panel B shows the composition of the mean-variance fund ($\bar{w}^{\mathbf{MV}}$) and asymmetry-variance fund ($\bar{w}^{\mathbf{AV}}$) calculated using (18) and then normalized using (21).

Table 2. Risk attitudes and optimal portfolios

κ	0.96	0.97	0.98	0.99	1.01	1.02	1.03	1.04
Panel A. GDA investor ($\gamma = 2$ and $\ell = 2$)								
Effective risk aversion, $\tilde{\gamma}$	6.2	8.1	11.8	23.0	21.1	11.0	7.6	5.9
Implicit asymmetry aversion, $\tilde{\chi}$ ($\times 100$)	4.88	4.98	5.07	5.15	–5.73	–5.03	–4.41	–3.85
Disappointment threshold, $\ln \kappa + \eta$ (%)	–3.52	–2.52	–1.55	–0.58	1.45	2.51	3.55	4.57
Disappointment probability, π_W (%)	2.0	2.2	2.5	2.8	88.6	90.1	91.4	92.6
Cash weight, w_f (%)	12.3	32.4	53.7	76.2	72.0	46.4	22.9	1.1
Bond weight, w_B (%)	54.6	42.3	29.1	15.0	9.3	18.8	28.3	37.7
Stock weight, w_S (%)	33.1	25.3	17.2	8.8	18.7	34.8	48.8	61.2
MV fund weight, $\alpha^{\mathbf{MV}}$ (%)	90.9	70.1	48.0	24.7	26.9	51.7	74.7	96.2
AV fund weight, $\alpha^{\mathbf{AV}}$ (%)	3.19	2.51	1.75	0.92	–1.11	–1.87	–2.37	–2.67
Panel B. Comparable EU investor								
Effective risk aversion, $\tilde{\gamma}$	6.2	8.1	11.8	23.0	21.1	11.0	7.6	5.9
Implicit asymmetry aversion, $\tilde{\chi}$ ($\times 10^3$)	0.47	0.51	0.56	0.60	0.60	0.55	0.50	0.46
Disappointment probability, π_W (%)	4.0	4.4	4.8	5.2	94.0	94.6	95.1	95.6
Cash weight, w_f (%)	9.4	30.2	52.2	75.4	73.2	48.5	25.5	4.1
Bond weight, w_B (%)	45.0	34.8	23.9	12.3	13.4	25.7	37.0	47.6
Stock weight, w_S (%)	45.6	35.1	24.0	12.3	13.4	25.8	37.4	48.3
MV fund weight, $\alpha^{\mathbf{MV}}$ (%)	90.9	70.1	48.0	24.7	26.9	51.7	74.7	96.2
AV fund weight, $\alpha^{\mathbf{AV}}$ (%)	0.31	0.26	0.19	0.11	0.12	0.20	0.27	0.32

The table presents detailed information about the optimal portfolio choice of specific investors. For the GDA investors in panel A, $\gamma = 2$ and $\ell = 2$ are used, and κ varies across columns. The choice of the DA investor ($\kappa = 1$) is not presented as the investor chooses not to participate in risky asset markets when $\ell = 2$. The investment horizon is one month. The distribution of asset returns is calibrated using the values reported in Table 1. The log certainty equivalent, expected shortfall, and upside potential are presented in monthly percentage values. Panel B presents values for a comparable EU investor. The effective risk aversion, $\tilde{\gamma}$, of the investor is exactly the same as that of the GDA investor in the same column, but the implicit asymmetry aversion is the one implied by EU preferences. The disappointment probabilities (π_W) in panel B are calculated using the corresponding threshold ($\ln \kappa + \eta$) reported in panel A.

Table 3. Cost of ignoring skewness

κ	0.96	0.97	0.98	0.99	1.01	1.02	1.03	1.04
Panel A. GDA investor ($\gamma = 2$ and $\ell = 2$)								
Relative cost, $\frac{\mathcal{R}'-R_f}{\mathcal{R}-R_f}$ (%)	83.27	83.17	83.24	83.47	94.15	95.12	95.98	96.73
Absolute cost, $\mathcal{R} - \mathcal{R}'$ ($\times 10^3$)	4.201	3.303	2.285	1.175	0.649	1.000	1.144	1.150
Panel B. Comparable EU investor								
Relative cost, $\frac{\mathcal{R}'-R_f}{\mathcal{R}-R_f}$ (%)	99.93	99.91	99.90	99.88	99.88	99.90	99.92	99.93
Absolute cost, $\mathcal{R} - \mathcal{R}'$ ($\times 10^3$)	0.015	0.014	0.011	0.007	0.007	0.012	0.014	0.016

The table presents measures for the cost of ignoring return asymmetries. The preference parameters used are the same as in the corresponding columns of Table 2. The investment horizon is one month for all investors and the distribution of asset returns is calibrated using the parameters from Table 1. The cost in absolute terms is measured as $(\mathcal{R} - \mathcal{R}')$, and in relative terms it is measured as $(\mathcal{R}' - R_f) / (\mathcal{R} - R_f)$, $\mathcal{R}$ being the annualized certainty equivalent of the optimal portfolio. An investor who ignores return asymmetry and chooses the optimal portfolio as if log asset returns were normally distributed with the same mean and variance-covariance matrix as the true distribution, chooses the suboptimal allocation w'. $\mathcal{R}'$ is the annualized certainty equivalent of the suboptimal allocation under the true distribution.

Table 4. Asset allocations recommended by financial advisors

Panel A. Assumed asset returns

	Mean	SD	Skew	Correlation with bonds	Correlation with stocks	$\bar{w}^{\mathbf{MV}}$	$\bar{w}^{\mathbf{AV}}$
Cash	0.05%						
Bonds	0.18%	2.9%	0.01	1.00	0.23	0.27	9.2
Stocks	0.75%	6.0%	–0.81	0.23	1.00	0.73	–10.2

Panel B. Portfolio recommendations

	Percent of portfolio (%)							
	Cash	Bonds	Stocks					
	w_C	w_B	w_S	$\frac{w_B}{w_S}(\times 100)$	$\alpha^{\mathbf{MV}}(\%)$	$\frac{\alpha^{\mathbf{AV}}}{\alpha^{\mathbf{MV}}}(\times 100)$	$\tilde{\gamma}$	$\tilde{\chi}(\times 100)$
Advisor A								
Conservative	50	30	20	150	52	3.4	6.2	8.1
Moderate	20	40	40	100	82	2.4	3.9	5.7
Aggressive	5	30	65	46	95	0.5	3.4	1.2
Advisor B								
Conservative	20	35	45	78	81	1.8	3.9	4.2
Moderate	5	40	55	73	97	1.6	3.3	3.8
Aggressive	5	20	75	27	94	–0.6	3.4	–1.5
Advisor C								
Conservative	50	30	20	150	52	3.4	6.2	8.1
Moderate	10	40	50	80	92	1.8	3.5	4.4
Aggressive	0	0	100	0	97	–2.9	3.3	–7.0
Advisor D								
Conservative	20	40	40	100	82	2.4	3.9	5.7
Moderate	10	30	60	50	91	0.7	3.5	1.6
Aggressive	0	20	80	25	99	–0.7	3.2	–1.7

Panel A presents assumptions regarding the distribution of monthly asset returns based on Table 2 (p. 185) of Canner, Mankiw, and Weil (1997). Note that Canner, Mankiw, and Weil (1997) do not report asset skewness, so we use values from our calibration. Panel B presents the recommendations of four financial advisors. The first four columns are taken from Table 1 (p. 183) of Canner, Mankiw, and Weil (1997). The last four columns present the weight in the mean-variance fund $\bar{w}^{\mathbf{MV}}$ ($\alpha^{\mathbf{MV}}$), the relative weight in the asymmetry-variance fund $\bar{w}^{\mathbf{AV}}$ ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$), and the corresponding effective risk aversion ($\tilde{\gamma}$) and implicit asymmetry aversion ($\tilde{\chi}$) for each portfolio.

Figure 1. Conditional bond–stock correlations and expected stock shortfalls

Figure A plots the probability density function (pdf) of the log stock return distribution. Figure B plots the expected stock return shortfall defined as $E\left[r_S \mid r_S < Q_S\left(q\right)\right]$ and expressed in monthly percentages (%), where $Q_S\left(q\right)$ denotes the qth quantile of the stock return distribution. Figure C plots conditional bond–stock correlations defined as $Corr\left(r_S, r_B | r_S < Q_S\left(q\right)\right)$ if $q \leq 0.5$ and $Corr\left(r_S, r_B | r_S > Q_S\left(q\right)\right)$ if $q > 0.5$. All figures display values estimated from the sample (Sample), values simulated from a normal distribution fitted to the data (Normal), and values simulated using model (10) fitted to the data by GMM (Normal-Exponential).

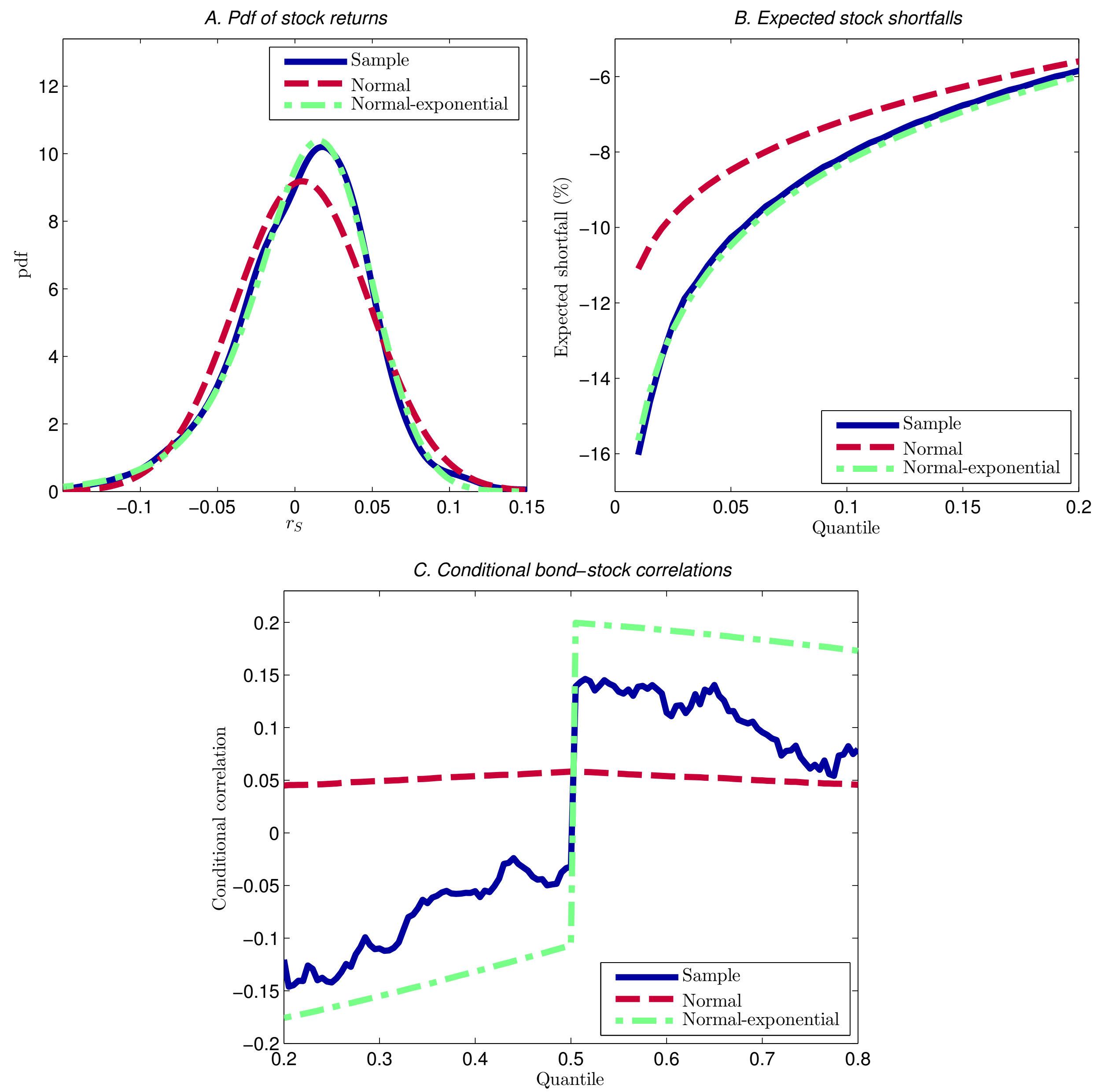

Figure 2. Optimal portfolios for different investors

The figure presents the relative weights in the mean-variance fund ($\alpha^{\mathbf{MV}}$, on the bottom axis) and the asymmetry-variance fund ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, on the left axis) for optimal portfolios of investors with different preferences. The corresponding effective risk aversion ($\tilde{\gamma}$, on the top axis) and implicit asymmetry aversion ($\tilde{\chi}$, on the right axis) are also presented. All curves start at the same point corresponding to the investor for whom $\gamma = 2$ and $\ell = 0$. The line corresponding to the EU investor shows the effect of increasing γ from 2 to 30 (moving left along the horizontal axis). The rest of the curves correspond to disappointment-averse investors with different κ values (see the legend) and show the effect of increasing ℓ from 0 to 3 (moving left along the horizontal axis), while keeping γ fixed at 2. The investment horizon is one month, and the distribution parameters for the asset returns are given in Table 1.

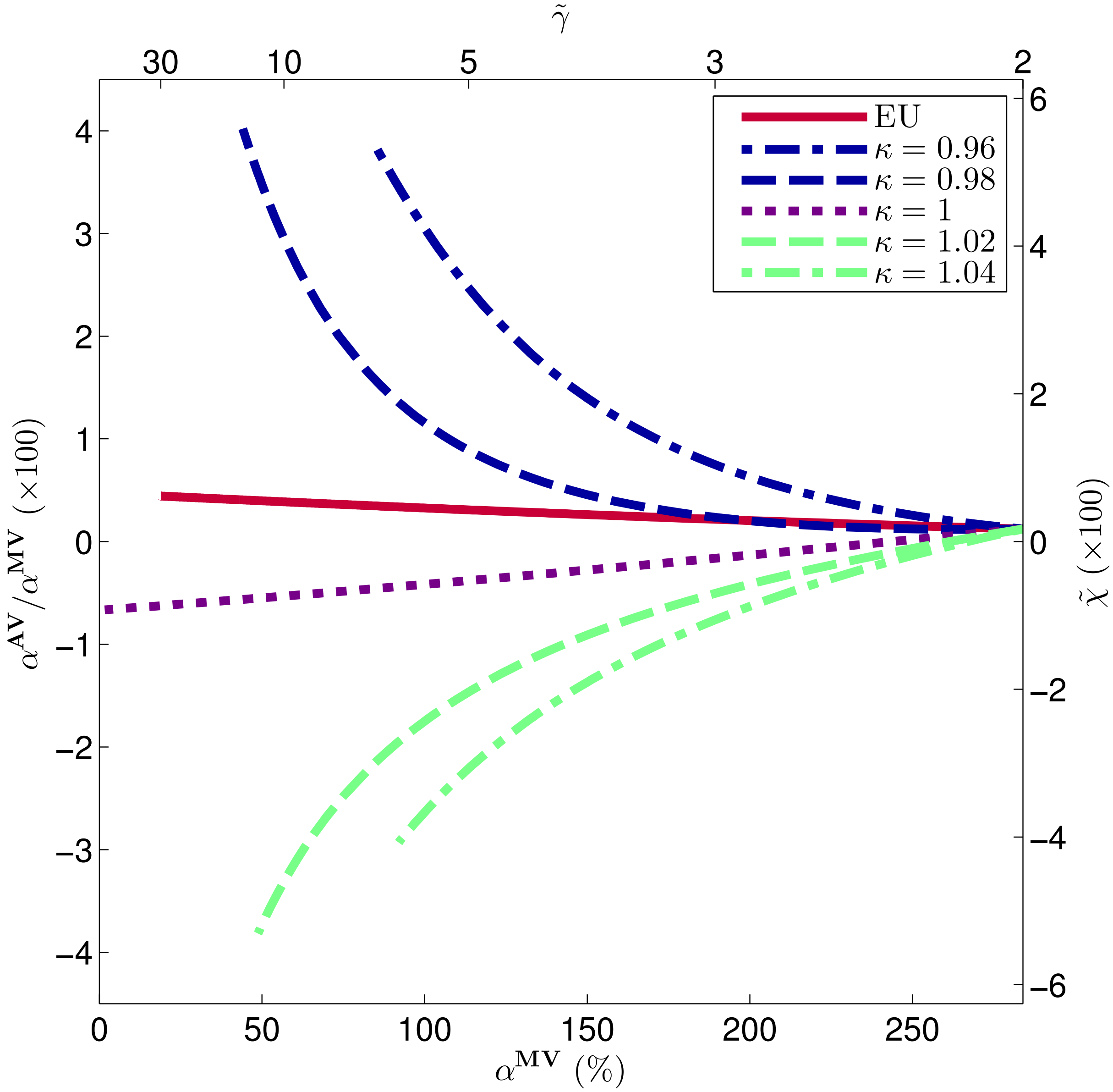

Figure 3. Optimal portfolios for different levels of stock skewness

The figure presents optimal portfolio weights in stocks and bonds for an EU investor ($\gamma = 6$) and two GDA investors (one with $\gamma = 2$, $\ell = 2$, and $\kappa = 0.96$, and the other with $\gamma = 2$, $\ell = 2$, and $\kappa = 1.04$). The investment horizon is one month and the distribution parameters, except for δ_S, are given in Table 1. The parameter δ_S is varied along the horizontal axis leading to different stock skewness values. The vertical line corresponds to our benchmark calibration with a stock skewness of –0.81.

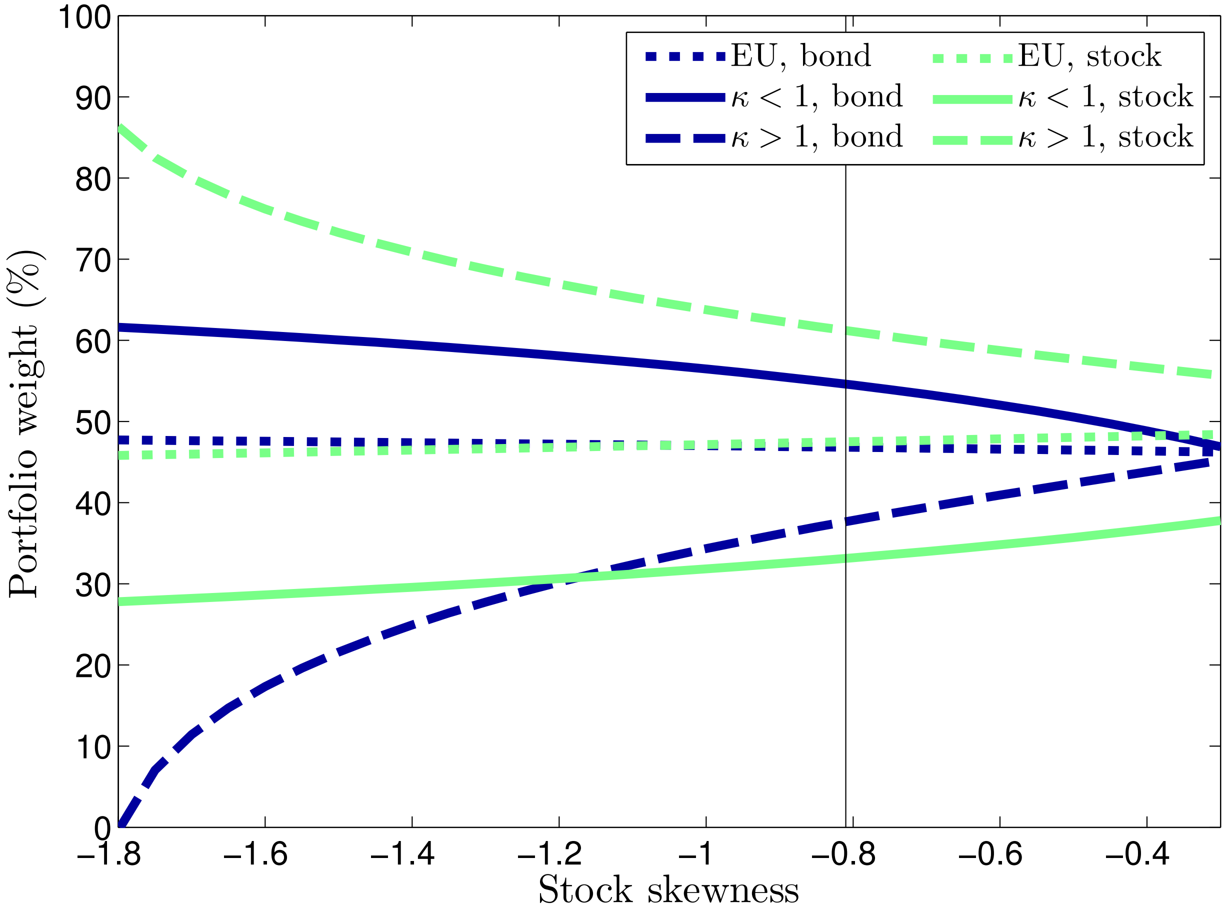

Figure 4. Effect of increasing the investment horizon

Figure A illustrates how the optimal portfolio changes with the investment horizon if returns are IID. The distribution parameters for the one-period returns are given in Table 1, while the H-period parameters are calculated according to (25). The figure shows the relative weight in the asymmetry-variance fund ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$) for the EU ($\ell = 0$) investor and two GDA investors ($\ell = 2$ for both and $\kappa < 1$ for one and $\kappa > 1$ for the other). The preference parameters are chosen so that the effective risk aversion is $\tilde{\gamma} = 5$ for all investors and horizons. Figures B to D compare the IID assumption with the case in which the return-generating model is fit to returns aggregated over $H = 1, ..., 12$ months. Figure B shows the stock's skewness when returns are aggregated over H months (round markers) and in the IID case (solid line). Figures C and D show the optimal stock weight and bond/stock allocation ratio, respectively, for the EU investor and two GDA investors (one with $\kappa < 1$ and the other with $\kappa > 1$) in the IID case, and for the same GDA investors with the estimated H-period returns.