

# Consumption Volatility and the Cross-Section of Stock Returns \*

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## Abstract

I derive and test multi-horizon implications of a consumption-based equilibrium model featuring fluctuating expected growth and volatility. My setup allows consumption dynamics to be estimated jointly with covariance risk prices in a single-stage GMM, and then inferences from asset pricing tests reflect uncertainty coming from factor estimation. I show that changes in consumption volatility are the key driver for explaining major asset pricing anomalies across risk horizons, while other factors play no or a secondary role. Value stocks and past long-term losers pay higher average returns mainly because they covary more negatively with these changes than what other stocks do.

**Keywords:** Level Risk, Expected Growth Risk, Consumption Volatility Risk, GARCH, Kalman Filter

**JEL Classification:** G1, G12, G11, C1, C5

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## 1. Introduction

Ongoing developments in consumption-based equilibrium asset pricing provide empirical support for fluctuating macroeconomic uncertainty measured by the volatility of aggregate consumption. For example, empirical evidence in Kandel and Stambaugh (1990) and Bansal et al. (2005) show that the volatility of aggregate consumption is time-varying, predicts and is predictable by the market price-dividend ratio. The price-dividend ratio loads negatively on consumption volatility and vice-versa. Bansal and Yaron (2004) explain several asset market stylized facts by a long-run risk model featuring fluctuating expected consumption growth and consumption volatility, together with Epstein and Zin (1989) recursive preferences. More recently, in a thorough empirical evaluation of long-run risk models, Beeler and Campbell (2012) suggest that persistent consumption volatility shocks have considerable impact on asset prices. Bonomo et al. (2011) show that these persistent shocks to consumption volatility are sufficient when coupled with generalized disappointment aversion preferences of Routledge and Zin (2010) to produce moments of asset prices and predictability patterns that are in line with the data.

Despite these well-established facts linking the aggregate stock market behavior to consumption volatility in the time series dimension, the asset pricing literature has long been silent on the role of consumption volatility in explaining cross-sectional differences in expected stock returns. Consumption volatility measures the imprecision surrounding investors' forecasts of aggregate consumption growth, hence represents a good measure for macroeconomic uncertainty. As well as an investor dislikes a low realization of her current consumption level and a low forecast of her future consumption growth, she also dislikes a high uncertainty about her future consumption growth forecast, as measured by the variance of the forecast error. Therefore, an asset that tends to move downward when macroeconomic uncertainty is high is an unattractive asset because it tends to have low payoffs precisely when investors need resources to save for precaution. Investors who are sensitive to fluctuations in macroeconomic uncertainty require a premium for holding assets that covary negatively with changes in the volatility of aggregate consumption. Hence, in an economy with fluctuating uncertainty about future consumption growth, assets with larger negative sensitivities to changes in consumption volatility will have higher returns on average. Besides, risk-return relations may vary considerably across risk horizons and stock holding periods.

In this article, I examine whether heterogeneity in multi-period expected excess returns across different assets are linked to heterogeneity in their sensitivities to changes in consumption volatility across risk horizons controlling for changes in consumption level and expected consumption growth over the same risk horizon. The empirical investigations are directly motivated by reasonable assumptions and multi-horizon cross-sectional implications of a consumption-based reduced-form general equilibrium model with Epstein and Zin (1989) preferences, incorporating fluctuations in macroeconomic growth forecast and uncertainty as measured respectively by expected growth and volatility of aggregate consumption. The widely studied recursive utility setup allows to disentangle risk aversion from the elasticity of intertemporal substitution. Therefore, changes in consumption growth forecast and volatility endogenously affect the marginal utility of wealth as long as the investor is not neutral on the timing of resolution of uncertainty. The purpose of this paper is to conduct an empirical test of the multi-period implications of the proposed model on the cross-section of stock returns. Such an empirical test is made difficult and may be inconsistent because conditional moments of consumption growth are unobserved and factors must be estimated. Unlike several papers that assume either a pure stochastic volatility (SV) process or a Markov chain to model consumption volatility, the key innovation is that consumption volatility is a generalized autoregressive conditional heteroscedasticity (GARCH) process.<sup>1</sup>

The assumed volatility process achieves two important goals. First, volatility is a well-defined positive process, which is affine so that under the usual log-linear approximation of Campbell and Shiller (1988), log welfare valuation ratios are solved analytically, while the log multi-period pricing kernel of the representative investor depends linearly on three factors: changes in realized consumption, changes in expected consumption growth and changes in consumption volatility over the risk horizon. Second, I can easily implement a joint estimation of factor dynamics and other quantities of interest (cross-sectional factor risk prices on one hand, and factor loadings on the other hand) via an appropriate empirical tool, the generalized method of moments (GMM). So, the reported asset pricing tests account for the estimation error in expected consumption growth and consumption volatility. Such a joint estimation would be otherwise more difficult to deal with empirically, for example if a pure SV or a Markov chain is assumed.

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<sup>1</sup>Pure stochastic volatility of consumption growth includes Bansal and Yaron (2004) among others. Markov chain volatility includes Bhamra et al. (2010), Bonomo et al. (2011) and Boguth and Kuehn (2012) among others.

In my setup, as long as the representative investor has a relative risk aversion parameter larger than one and has preference for early resolution of uncertainty, multi-period investments with undesirable exposures to long-horizon changes in consumption growth forecast and volatility command a positive risk premium. This is because the risk price associated with changes in consumption growth forecast is positive and the risk price associated with changes in consumption volatility is negative. This implies that assets that covary positively with changes in consumption growth forecast earn a higher risk premium in comparison to assets that do not covary or covary negatively with these fluctuations. Similarly, negative covariation with changes in consumption volatility commands a higher risk premium compared to positive or no covariation with volatility fluctuations. Periods of low consumption growth forecast or high consumption volatility usually occur around business cycle contractions, which are usually periods of lower prices, worker layoffs and less consumer spending, therefore less profit for businesses, slow economy and less demand for products. As a consequence, an asset with low returns precisely in such an economic situation, characterized by high marginal utility of consumption, is undesirable for the investors who are willing to require a premium for holding it, thus requiring a higher expected excess return. In this article, I emphasize the long-run consumption volatility risk channel controlling for long-run exposures to changes in consumption level and consumption growth forecast.

I examine the empirical performance of the derived cross-sectional model on standard portfolio sets, the 25 size and book-to-market portfolios (SBM25), and the 25 size and long-term reversal portfolios (SLTR25). In GMM estimations, I add moment conditions related to the pricing of the market (MKT), the size (SMB), the value (HML) and the long-term reversal (LTR) indexes, thus requiring the model not only to fit the cross-section of average excess returns on SBM25 and SLTR25 portfolios, but also to fit common measures of the equity premium, the size effect, the value premium and the long-term reversal premium. The main findings may be summarized as follows. GMM results with identity matrix show that for most risk horizons and stock holding periods, a three-factor model with changes in consumption level and changes in consumption growth forecast and volatility over the risk horizon explains well the cross-sectional dispersion in average multi-period excess returns of SBM25 and SLTR25 portfolios, hence the size effect, the value premium and the long-term reversal premium, in addition to the equity premium. The explanatory power of the model as measured by the cross-sectional adjusted  $R^2$  ranges from 66%

to 86% on size and book-to-market portfolios, and from 43% to 71% on size and long-term reversal portfolios. The model also performs well in efficient GMM estimation, with statistically significant covariance risk price estimates at conventional levels of confidence, and confirming results from estimation with identity weighting matrix that covariance risk prices for changes in consumption level and consumption growth forecast are positive and that consumption volatility covariance risk price is negative, results that are consistent with a preference for early resolution of uncertainty.

I decompose the multi-period value, long-term reversal, size and equity premia into parts attributable to each of the three factors from the model. Explanation of the multi-period value premium is exclusively attributable to changes in consumption volatility over the risk horizon. In other words, the three-factor cross-sectional model explains the returns of the SBM25 portfolios, and hence explains the value premium anomaly, because value stocks have larger consumption volatility risk than growth stocks, making value stocks more riskier. Changes in consumption volatility are also the key factor for explaining the long-term reversal premium anomaly, while changes in consumption growth forecast play a secondary role. To the contrary, channels through which the three-factor cross-sectional model explains the size premium anomaly vary considerably with the risk horizons. While changes in consumption volatility do not contribute in explaining the size effect at shorter risk horizons, the volatility component of the multi-period size premium become substantially important as the risk horizon increases. Estimated loadings of multi-period market returns to changes in consumption volatility do not suggest undesirable exposures, and premium decomposition confirms that changes in consumption volatility do not appear to be the major channel for explaining the multi-period equity premium, although their contribution is increasing with the risk horizon as the market index is held over the full investment period.

Previous research on multi-period consumption risk has mainly focused on payoff exposures to changes in consumption level over the risk horizon, building on implications of the standard consumption-based model with time-separability and constant relative risk aversion (CRRA; see for example, Parker and Julliard, 2005; Bansal et al., 2009), or on the recursive utility model with homoscedastic consumption growth (see for example Hansen et al. 2008). The multi-period implications of the recursive utility setup with both consumption growth predictability and heteroscedasticity has not yet been investigated empirically by previous literature. If the true preferences of the representative investor are recursive and given that fluctuating consumption volatility

is supported in the data, then relying on time-separable CRRA would lead to an omission of macroeconomic uncertainty as a crucial cross-sectional pricing factor. In contrast, if the true preferences are CRRA, then relying on recursive utility, one should precisely find that macroeconomic uncertainty is not priced, controlling for other forms of consumption risks.

Earlier results by Restoy and Weil (2011) show that in an equilibrium model with Epstein and Zin (1989) utility, current returns are only able to predict future conditional means of consumption growth but carry no information about the future conditional variances. This suggests that current asset returns are not exposed to future consumption volatility. However, these results are subject to the strong assumption that log returns and consumption growth have a joint normal conditional distribution whose second-order moments follow GARCH processes of the type of Bollerslev (1986). My empirical tests do not require any distributional assumptions on actual returns.<sup>2</sup> In the online appendix, I rely on an exogenously specified, yet standard dividend process to show in a general equilibrium setting with GARCH consumption volatility that, neither log returns that are endogenously determined within the model and consumption growth do have a joint normal conditional distribution, nor the volatility of returns does follow a GARCH process. Consumption volatility is priced in equilibrium log returns primarily because endogenous asset dividend-payoff ratios load on it, and this is true whether or not volatility is stochastic or GARCH.<sup>3</sup>

The current study of the tradeoff between consumption volatility risk and expected returns differs from earlier work that tests for variance risk factors in cross-sectional asset pricing. Using household data, Jacobs and Wang (2004) find that the cross-sectional dispersion in household consumption growth has some potential to explain asset risk premia. Innovations in market volatility as measured by index option implied volatility is shown to be a significant cross-sectional pricing factor by Ang et al. (2006), while Adrian and Rosenberg (2008) examine the pricing of

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<sup>2</sup>Besides, by imposing the dynamics of the returns (a conditional gaussian dynamics) then solving for the equilibrium expected return through the Euler equation, Restoy and Weil (2011) may not produce truly a general equilibrium price. Eraker and Wang (2011) in a recent paper argue that this method of assuming asset prices dynamics exogenously and solving for the expected rate of return may be inconsistent with present value computations (it produces an asset price that is not equal to the present values of the asset future cash flows).

<sup>3</sup>In the GARCH setting, this is not a surprising result. In a different context, Harvey and Siddique (2000) show that the market return and the squared market return carry two different risks that are all priced, the beta and the coskewness, although innovations in these two factors are otherwise related.

short- and long-run components of market volatility estimated from a two-component GARCH model using daily data. More recently, Boguth and Kuehn (2012) show that the exposure to consumption volatility filtered out from a two-state Markov chain dynamics negatively predicts future returns in the cross-section, and I also largely differ by the empirical methodology by performing a joint factor estimation and asset pricing tests in a single-stage GMM framework while they use two-pass regressions on estimated factors. Finally, while these papers focus on explaining the cross-sectional pattern in one-period returns, I empirically evaluate consumption volatility risk-return relations across various risk horizons and stock holding periods, controlling for consumption level and expected growth risk-return relations.

I also relate to the growing literature that aims at estimating long-run risk models using GMM, but also differ from recent work. Bansal et al. (2012), Constantinides and Ghosh (2011) and Ferson et al. (2013) estimate the original long-run risk model of Bansal and Yaron (2004) by using risk-free rate and market price-dividend ratio as predicted by the model to identify consumption growth forecast and volatility. This approach avoids testing the model assumptions about the dynamics of equilibrium consumption. Instead, I rely directly on the assumed factor dynamics and perform tests from the estimation of these dynamics jointly with equilibrium restrictions imposed by Euler equations, and for risk horizons greater than one period. Besides, the present GMM framework expands the degrees of freedom relative to earlier empirical tests by using a larger asset menu and explicitly disentangles volatility premium from level and expected growth premia through Euler equations in their linear covariance form.

The balance of the paper is organized as follows. Section 2 presents model assumptions and its multi-horizon testable implications. Section 3 presents the data and describes the estimation methodology. Section 4 contains the empirical findings and Section 5 their economic intuition. Section 6 concludes. An internet appendix contains additional materials and proofs.

## **2. Consumption Volatility in Equilibrium**

### 2.1 THEORETICAL BACKGROUND

I derive the stochastic discount factor (SDF) of an economy where investors may demand compensations for undesirable exposures to long-horizon changes in consumption level, consumption

growth forecast and volatility. The proposed model is based on Bansal and Yaron (2004), but differs from previous specifications in that volatility follows a GARCH-type process rather than the gaussian SV used by Bansal and Yaron (2004). Since expected growth and volatility are unobserved, the proposed model aims at easing the estimation of their assumed time series dynamics jointly with cross-sectional model implications via conventional empirical methods such as GMM.

The underlying environment is a one with complete markets and the representative investor has Epstein and Zin (1989) preferences. These preferences separate risk aversion from the elasticity of intertemporal substitution. Consumption and portfolio choice induces a restriction on the gross return on a typical asset that is given by the Euler equation:

$$E [M_{t,t+1}R_{i,t+1} | \mathcal{J}_t] = 1, \tag{1}$$

where the stochastic discount factor (SDF) derived in Hansen et al. (2007) may be written as:

$$M_{t,t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{C_{t+1} / V_{t+1}}{C_t / \mathcal{R}_t(V_{t+1})} \right)^{\gamma-1/\psi}, \tag{2}$$

and where  $V_t$  is the continuation value of investor's lifetime utility,  $\mathcal{R}_t(V_{t+1})$  is the Kreps and Porteus (1978) certainty equivalent of the next period continuation value conditional on current information,  $\gamma$  is the coefficient of relative risk aversion,  $\psi$  is the elasticity of intertemporal substitution,  $\delta$  is the subjective discount factor and  $\mathcal{J}_t$  is the time  $t$  information set. The preference parameters satisfy  $\gamma \geq 0$ ,  $\psi > 0$  and  $0 < \delta < 1$ .

If  $\gamma = 1/\psi$ , then utility is time-separable with constant relative risk-aversion, and only consumption growth matters in the SDF (2). In this case, the Euler Equation (1) is readily testable by GMM, since both returns and consumption are observed. Earlier results for this test of the standard model are presented in Hansen and Singleton (1982, 1983). Empirical tests of the Euler Equation (1) in the case of recursive utility ( $\gamma \neq 1/\psi$ ) require the observability of the welfare valuation ratios  $C_t/V_t$  and  $C_t/\mathcal{R}_t(V_{t+1})$ .<sup>4</sup> These ratios are not directly observable since actual measures of investor's utility itself are not available from data. To make estimation feasible,

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<sup>4</sup>Epstein and Zin (1991) study Euler equation GMM estimators and tests of the recursive utility model based on an equivalent representation of the SDF that relies on the market return instead of welfare valuation ratios. Small sample properties of these estimators and tests are investigated by Smith (1999).



I exploit the definitions of the recursive utility and of the Kreps and Porteus (1978) certainty equivalent, as well as the dynamics of consumption growth, to solve for welfare valuation ratios.

I postulate that equilibrium consumption growth has the following dynamics:

$$\begin{aligned}
 \Delta c_{t+1} &= \mu_c + x_t + \sigma_t \varepsilon_{c,t+1} \\
 x_{t+1} &= \phi_x x_t + \nu_x \sigma_t \varepsilon_{x,t+1} \\
 \sigma_{t+1}^2 &= (1 - \phi_\sigma) \mu_\sigma + \phi_\sigma \sigma_t^2 + \nu_\sigma \varepsilon_{\sigma,t+1}
 \end{aligned} \tag{3}$$

where  $\varepsilon_{c,t+1}$ ,  $\varepsilon_{x,t+1}$  and  $\varepsilon_{\sigma,t+1}$  are three mutually uncorrelated and serially uncorrelated processes with mean zero and unit variance, and parameters satisfy  $0 < \phi_x < 1$ ,  $\nu_x > 0$ ,  $\mu_\sigma > 0$ ,  $0 < \phi_\sigma < 1$  and  $\nu_\sigma > 0$ . The parameter  $\mu_\sigma$  measures the unconditional mean of volatility, and  $\phi_\sigma$  measures the persistence of volatility while  $\phi_x$  measures the persistence of expected growth. Similar to the original long-run risk model of Bansal and Yaron (2004), and consistent with the empirical results provided by Parker and Julliard (2005), the assumed dynamics allows for serial correlation in consumption growth.<sup>5</sup>

I further assume that volatility shocks satisfy  $\varepsilon_{\sigma,t+1} = (\varepsilon_{c,t+1}^2 - 1) / \sqrt{2}$ , leading to a GARCH process, and that the shocks  $\varepsilon_{c,t+1}$  and  $\varepsilon_{x,t+1}$  are standard normal. The volatility process is thus a well-defined positive process provided  $(1 - \phi_\sigma) \mu_\sigma - \nu_\sigma / \sqrt{2} \geq 0$ , and Equation (3) describes a multivariate affine process. This particular GARCH volatility recursion was first considered for option pricing by Heston and Nandi (2000).<sup>6</sup> So defined, the model belongs to the class of affine general equilibrium models, which under log-linear approximations of the same type as in Campbell and Shiller (1988), can be solved analytically (see Eraker, 2008). In particular log welfare valuation ratios are linear in expected consumption growth and consumption volatility. I explicitly derive the model solution in the internet appendix. Given this solution, the reduced

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<sup>5</sup>To be parsimonious, an earlier version of the paper assumes a GARCH-in-mean dynamics, where consumption growth forecast depends on its volatility through  $x_t = \phi_c (\sigma_t^2 - \mu_\sigma)$ . However, this does not rule out the possibility of interpreting the empirical findings on the pricing of consumption volatility fluctuations as also evidence for priced shocks to expected consumption growth. I thank an anonymous referee for pointing out this potential issue. Following the long-run risk literature, the current model dynamics explicitly disentangle the two factors.

<sup>6</sup>To the contrary of Heston and Nandi (2000), I restrict the leverage effect parameter to zero, explicitly limiting the effect of factor correlation in the empirical results. In model estimation and calibration, without loss of generality, I save one parameter by assuming  $(1 - \phi_\sigma) \mu_\sigma - \nu_\sigma / \sqrt{2} = 0$  so that  $\nu_\sigma = \sqrt{2} (1 - \phi_\sigma) \mu_\sigma$ .

form expression for the log SDF may be written:

$$m_{t,t+1} = p_1 - p_c \Delta c_{t+1} - p_x \Delta_q x_{t+1} - p_\sigma \Delta_q \sigma_{t+1}^2, \quad (4)$$

where  $\Delta_q x_{t+1} = (x_{t+1} - x_t / q_1)$  and  $\Delta_q \sigma_{t+1}^2 = (\sigma_{t+1}^2 - \sigma_t^2 / q_1)$  behaves very much like  $\Delta x_{t+1}$  and  $\Delta \sigma_{t+1}^2$  respectively, since  $q_1$  is close to unity.<sup>7</sup> So the logarithm of the SDF depends on changes in realized consumption level, in expected growth rates, and in consumption volatility.

The coefficients

$$p_c = \gamma \quad \text{and} \quad p_x = - \left( \gamma - \frac{1}{\psi} \right) \beta_{Vx} \quad \text{and} \quad p_\sigma = - \left( \gamma - \frac{1}{\psi} \right) \beta_{V\sigma} \quad (5)$$

are respectively, over one period, the standard price of consumption level risk, measured by the coefficient of risk aversion, the price of expected growth risk, and the price of volatility risk. The coefficients  $\beta_{Vx}$  and  $\beta_{V\sigma}$  are loadings of the log welfare valuation ratio  $\ln(C_t/V_t)$  onto expected consumption growth and consumption volatility. These loadings are negative and positive respectively, if I assume that risk-aversion is greater than unity,  $\gamma > 1$ , as common in the asset pricing literature. When macroeconomic uncertainty is high or expected growth is low, everything else equal, the investor is pessimistic about the future. She then assigns a low valuation to the continuation value and is willing to accept, with certainty, a lower welfare to avoid the risk in future consumption. In consequence, the ratio of current consumption to welfare valuation rises.

Restoy and Weil (2011) assume that equilibrium log returns and consumption growth have a joint normal conditional distribution whose second-order moments follow GARCH processes of the type of Bollerslev (1986), and find that current returns are not exposed to future consumption volatility in an equilibrium model with Epstein and Zin (1989) utility. In a recent article, Eraker and Wang (2011) show that this method of assuming asset prices dynamics exogenously and solving for the expected rate of return is inconsistent with present value computations and produces an asset price that is not equal to the present value of asset future cash flows.

While I model and measure consumption volatility with a GARCH specification, in the internet appendix I discuss channels through which asset returns in the cross-section have a separate

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<sup>7</sup>Expressions of the coefficients  $p_1$  and  $q_1$ , with no interest at this stage, are provided in the internet appendix.

exposure to consumption volatility fluctuations, despite the fact that innovations to consumption volatility may be viewed as nonlinearly related to innovations to consumption growth. Following Bansal and Yaron (2004) and others, I couple the above consumption growth dynamics with a standard asset dividend growth process. I show that the joint conditional distribution of equilibrium log returns and consumption growth that arises endogenously within the model is not gaussian, and the volatility of returns does not follow a GARCH-type process.<sup>8</sup> The exposure of asset returns to changes in consumption volatility can be decomposed into two components: the exposure of asset cash flows to changes in consumption volatility, and the loading of the log payoff-dividend ratio onto consumption volatility. To the contrary of Restoy and Weil (2011), consumption volatility is priced in equilibrium log returns primarily because endogenous asset valuation ratios load on it, and this shall be true whether or not volatility is stochastic or GARCH.

## 2.2 MULTI-PERIOD TESTABLE EQUILIBRIUM ASSET PRICING RESTRICTIONS

Iterating forward and using the law of iterated expectations, the one-period Euler Equation (1) may, in particular and without loss of generality, be generalized to multiple periods as:

$$E [M_{t,t+h} R_{it,k,h} | \mathcal{J}_t] = 1 \quad (6)$$

where

$$M_{t,t+h} = \prod_{j=1}^h M_{t+j-1,t+j} \quad \text{and} \quad R_{it,k,h} = \prod_{j=1}^k R_{i,t+j} \prod_{j=k+1}^h R_{f,t+j} \quad (7)$$

are respectively the  $h$ -period SDF and the multi-period gross return formed by investing from time  $t$  in stock  $i$  for the first  $k$  periods, then reinvesting the payoffs from date  $t+k$  in the safe asset for the remaining  $h-k$  periods. I define the multi-period excess returns

$$R_{it,k,h}^e = R_{it,k,h} - R_{ft,h,h} \quad (8)$$

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<sup>8</sup>A priori non-normality of returns does not matter for asset pricing tests as conducted in the current article. Asset pricing implications of the assumed GARCH are similar to those of the gaussian and square-root stochastic volatility models. All three processes lead to an affine general equilibrium setting as discussed by Eraker (2008). The same factors drive log returns in the cross-section, also implying that multi-period implications are alike. However, the proposed empirical methodology discussed below would be hardly implementable under stochastic volatility.

where  $R_{ft,h,h}$  is the rolling-strategy bond return, and not the return at date  $t$  on a  $h$ -period bond. In fact, the  $h$ -period return  $R_{ft,h,h}$  for  $h > 1$  is risky at date  $t$ , since future one-period risk-free returns are not known at time  $t$  and depend on future consumption growth forecast and volatility.

In the following, I derive a linear version of the above model, as common in the cross-sectional asset pricing literature. Since expected consumption growth is unobserved by the econometrician, I assume that observable economic variables other than consumption growth contain no information about expected growth and volatility beyond that contained in consumption growth. Thus, I can rewrite Equation (6) as a model of expected returns and using the unconditional expectation operator and the definition of covariance to yield:

$$\begin{aligned} E [R_{it,k,h}^e] &= Cov \left( -\frac{M_{t,t+h}}{E[M_{t,t+h}]}, R_{it,k,h}^e \right) \\ &\approx p_{c,h} Cov (\xi_{ct,h}, R_{it,k,h}^e) + p_{x,h} Cov (\xi_{xt,h}, R_{it,k,h}^e) + p_{\sigma,h} Cov (\xi_{\sigma t,h}, R_{it,k,h}^e) \end{aligned} \quad (9)$$

where  $\xi_{ct,h} = \Delta c_{t,h} - E[\Delta c_{t,h}]$ ,  $\xi_{xt,h} = \Delta_q x_{t,h} - E[\Delta_q x_{t,h}]$  and  $\xi_{\sigma t,h} = \Delta_q \sigma_{t,h}^2 - E[\Delta_q \sigma_{t,h}^2]$  are respectively the demeaned  $h$ -horizon changes in consumption level, expected growth and volatility,

$$\Delta c_{t,h} = \sum_{j=1}^h \Delta c_{t+j} = c_{t+h} - c_t \quad (10)$$

$$\Delta_q x_{t,h} = E \left[ \sum_{j=1}^h \Delta_q x_{t+j} \mid \mathcal{G}_{t+h} \right] \approx E [x_{t+h} - x_t \mid \mathcal{G}_{t+h}] = x_{t+h|t+h} - x_{t|t+h} \quad (11)$$

$$\Delta_q \sigma_{t,h}^2 = E \left[ \sum_{j=1}^h \Delta_q \sigma_{t+j}^2 \mid \mathcal{G}_{t+h} \right] \approx E [\sigma_{t+h}^2 - \sigma_t^2 \mid \mathcal{G}_{t+h}] = \sigma_{t+h|t+h}^2 - \sigma_{t|t+h}^2, \quad (12)$$

where  $\mathcal{G}_{t'} = \{\Delta c_{t'}, \Delta c_{t'-1}, \Delta c_{t'-2}, \dots\}$ ,  $x_{t|t'}$  denotes the conditional expectation  $E[x_t \mid \mathcal{G}_{t'}]$  that may be computed through Kalman filtering technique, and  $p_{c,h}$ ,  $p_{x,h}$  and  $p_{\sigma,h}$  are cross-sectional level, expected growth and volatility risk prices defined by

$$p_{c,h} = \gamma \beta_h, \quad p_{x,h} = p_x \beta_h \quad \text{and} \quad p_{\sigma,h} = p_\sigma \beta_h, \quad (13)$$

where  $\beta_h$  is a specific positive constant.<sup>9</sup>

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<sup>9</sup>For the approximation in (9), replace  $M_{t,t+h}$  by  $\widetilde{M}_{t,t+h}$  where  $\frac{\widetilde{M}_{t,t+h}}{E[M_{t,t+h}]} = 1 + \beta_h (m_{t,t+h} - E[m_{t,t+h}])$ . The approximated SDF has the same mean as the true SDF and the coefficient  $\beta_h$  is positive to ensure a positive

Equation (9) postulates that cross-sectional heterogeneity in expected  $k$ -period excess returns depend on three covariances: their covariance with  $h$ -period changes in consumption level; their covariance with  $h$ -period changes in expected consumption growth; and their covariance with  $h$ -period changes in consumption volatility, where  $1 \leq k \leq h$ . I further refer to  $k$  as the stock holding period, and to  $h$  as the risk horizon. As shown by Equations (5) and (13), the prices of these covariance risks depend on investor preference parameters. The consumption risk price is always positive. The separation between risk aversion and elasticity of intertemporal substitution is necessary for priced expected growth and volatility fluctuations, since  $p_x = 0$  and  $p_\sigma = 0$  if  $\gamma = 1/\psi$  (CRRA). By assuming the representative investor prefers early resolution of uncertainty, that is  $\gamma > 1/\psi$ , we have  $p_x > 0$  and  $p_\sigma < 0$ , and the risk price for changes in expected growth is positive while the risk price for changes in consumption volatility is negative.

In Equation (9), an asset with negative covariation with changes in consumption volatility pays a higher risk premium than an asset with lower negative covariation or with positive covariance with volatility changes. Similarly, an asset that covaries positively with changes in expected consumption growth pays a higher risk premium than an asset that covaries less positively or covaries negatively with expected growth fluctuations. Such assets pay less in bad states of the economy characterized by sharp increases in macroeconomic uncertainty or decreases in growth forecasts, when investors fear the repercussion on their future wealth and would like to increase their precautionary savings or smooth their consumption. Investors will then require a relatively high premium for holding those assets.

The covariances  $Cov(\xi_{ct,h}, R_{it,k,h}^e)$ ,  $Cov(\xi_{xt,h}, R_{it,k,h}^e)$  and  $Cov(\xi_{\sigma t,h}, R_{it,k,h}^e)$  are also crucial from an empirical standpoint. Rather than measure risk by the contemporaneous (one-period) covariance of an asset's return to consumption growth ( $k = 1 = h$ ), Parker and Julliard (2005) measure risk by the covariance of an asset's return to future consumption growth. Specifically, 

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 correlation between the SDF and its approximation. In particular, the coefficient  $\beta_h$  can be chosen so that the SDF and its approximation have the same variance,  $Var[\widetilde{M}_{t,t+h}] = Var[M_{t,t+h}]$ , or so as to minimize the mean squared error  $E\left[\left(M_{t,t+h} - \widetilde{M}_{t,t+h}\right)^2\right]$ . They can be thought of as second-order approximations of the true SDF. Theoretically, I show for these two special cases that  $\beta_h > 1$  and increases with  $h$ . In consequence, an estimate of  $p_{c,h}$  would be higher than the risk aversion parameter  $\gamma$ . Alternatively, in a single-period setting ( $h = 1$ ), although in a different context, Yogo (2006) considers a similar SDF approximation with  $\beta_1 = 1$ .

they examine whether a  $h$ -period return formed by investing in stocks for one period and then transforming to bonds for  $h - 1$  periods is priced by  $h$ -period consumption growth ( $k = 1 < h$ ). They find that, while the contemporaneous covariance explains little of the variation in average returns across size and book-to-market portfolios, the multi-period covariance at the risk horizon of twelve quarters explains a large fraction of this variation. In fact, they study the multi-period moment condition (6) for  $k = 1$ , in the case of CRRA utility ( $\gamma = 1/\psi$ ). They argue that this moment condition is robust to measurement errors in consumption of the type highlighted in Wilcox (1992), and simple errors by consumers. For example, if consumers adjust consumption slowly to news in returns, the moment condition (6) will work better than (1). Then, multi-period covariances provide a better measure of asset risk as the horizon  $h$  increases.

Bansal et al. (2009) also emphasize that the risk-return relationship varies considerably across investment horizons. To the contrary of Parker and Julliard (2005), they focus on the multi-period moment condition (6) for  $k = h$ , and similar to Parker and Julliard (2005) in the case of CRRA utility. As their main finding, they show that as the investment horizon increases, the consumption beta of an asset almost converges to the cointegration parameter between dividends and consumption, which in turn explains a large part of cross-sectional variation in average returns on size and book-to-market portfolios. These two papers typically examine the pricing of long-run consumption level risk in the cross-section of asset returns. I differ from these studies by examining the Euler equation (6) for  $1 \leq k \leq h$ , and I focus on the pricing of long-run changes in consumption volatility, controlling for long-run changes in level and expected growth.

Constantinides and Ghosh (2011) examine empirically the joint pricing and time series implications of the long-run risk model. However, they use equity dividend growth dynamics and exploit the fact that equity price-dividend ratio and interest rate are affine functions of the unobservable state variables, to express the pricing kernel as an affine function of equity price-dividend ratio, interest rate and their lags. Bansal et al. (2012) also exploit the same argument to extract the time series of unobservable expected growth and conditional volatility from linear projections on actual equity price-dividend ratio and interest rate as predicted by the original long-run risk model. In what follows, I rely on an equivalent cross-sectional representation of the Euler Equation (1), that explicitly relates expected returns to covariances between asset payoffs and risk factors. I do not exploit model implications for asset prices to solve for expected growth and volatility processes,

but instead I rely directly on model assumptions about the dynamics of these state variables. I finally differ from these studies by testing multi-period implications of the model. As in Constantinides and Ghosh (2011), and Ferson et al. (2013), I do not directly estimate the preference parameters  $\gamma$ ,  $\psi$  and  $\delta$ , but instead I estimate the reduced-form cross-sectional risk prices  $p_{c,h}$ ,  $p_{x,h}$  and  $p_{\sigma,h}$  which are functions of these preference parameters. Details on the data and the estimation methodology are provided in the next section.

### 3. Data and Estimation Methodology

#### 3.1 DATA

I jointly test model assumptions (3) and cross-sectional implications (9) using quarterly data spanning the period from 1963:I to 2008:IV. I use two alternative asset menus, with 28 components each. The first asset menu consists of the market (MKT), the Fama-French size (SMB) and value (HML) factors, and the 25 Fama and French (1992, 1993) portfolios sorted on size and book-to-market (SBM25). The second asset menu consists of the market, the size factor, the long-term reversal (LTR) factor, and the 25 portfolios sorted on size and long-term reversal (SLTR25). More specifically, the proxy for market return is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The proxy for the risk free rate used to compute excess returns is the one-month Treasury Bill rate from Ibbotson Associates. Return data are obtained from the webpage of Kenneth French.

The attractiveness of SBM25 and SLTR25 portfolios in empirical studies is due to the fact that stocks show significant differences in their average excess returns. In particular, long-term reversal portfolios are used by De Bondt and Thaler (1985), Fama and French (1996), Da (2009), Da and Warachka (2009) and Lioui and Maio (2012), amongst others, to advocate that past long-term losers (portfolios with lower returns in the long-term past) tend to have subsequent higher returns, while past long-term winners tend to have lower future returns. This pattern corresponds to a long-term mean reversion in stock returns, which is not explained by standard cross-sectional asset pricing models. The use of the SLTR25 portfolios represents an important empirical check on the model, since there is some recent criticism on the validity of cross-sectional asset pricing tests that rely exclusively on the SBM25 portfolios (e.g., Lewellen et al., 2010). Data on returns

are available monthly. They are aggregated to obtain quarterly returns.

Following earlier work (e.g., Hansen and Singleton, 1983), aggregate consumption is measured as the seasonally adjusted real per capita consumption of nondurables plus services. The quarterly real per capita consumption data are taken from the NIPA tables available from the Bureau of Economic Analysis. To convert returns and other nominal quantities, I also obtain the associated personal consumption expenditures (PCE) deflator from the NIPA tables.

### 3.2 ESTIMATION METHODOLOGY

Following recent empirical studies of cross-sectional asset pricing (e.g., Jagannathan and Wang, 1996; Cochrane, 1996; Jacobs and Wang, 2004; Parker and Julliard, 2005; Yogo, 2006), I use the generalized method of moments (GMM, Hansen 1982) to evaluate the statistical significance and the economic importance of consumption volatility in the cross-section of stock returns. Cochrane (2001, Chapter 15) demonstrates that the GMM approach works well for linear asset pricing models. The cross-sectional model (9) satisfies a moment condition of the form:

$$E[e_{t,k,h}(\theta, b, p)] = 0 \quad \text{where} \quad e_{t,k,h}(\theta, b, p) = -\iota b + \left(1 - \xi_{t,h}^\top(\theta)p\right) R_{t,k,h}^e \quad (14)$$

where  $\xi_{t,h}(\theta)$  is the  $3 \times 1$  vector of demeaned factors,  $R_{t,k,h}^e$  is the  $N \times 1$  vector of portfolio excess returns,  $p$  is the  $3 \times 1$  vector of covariance risk prices and the intercept  $b$  is introduced to measure by how much the cross-sectional model fails to predict returns by the same amount. Demeaned factors depend on  $\theta = (\mu_c, \phi_x, \nu_x, \mu_\sigma, \phi_\sigma, \nu_\sigma)^\top$ , the parameter vector that governs the full dynamics (3) of consumption growth. The vector  $\iota$  is of same size as  $R_{t,k,h}^e$  and has all its components equal to one.

The maximization of the log likelihood  $L(\theta; \mathcal{G}_T) = \sum_{t=1}^T \ln f(\Delta c_t | \mathcal{G}_{t-1}; \theta)$  of dynamics (3) with respect to  $\theta$ , using a sample of size  $T$ , has a first-order condition equivalent to the sample counterpart of a population moment condition

$$E[\ell_t(\theta)] = 0 \quad \text{where} \quad \ell_t(\theta) = \frac{\partial \ln f(\Delta c_t | \mathcal{G}_{t-1}; \theta)}{\partial \theta}. \quad (15)$$

I estimate the parameters  $\theta$ ,  $b$  and  $p$  in a full single-stage GMM system that uses the moment



conditions (14) and (15). I perform the GMM estimation by placing the weighting matrices  $W$  and  $\lambda \hat{S}_{\ell\ell}^{-1}(\theta)$  respectively on the moments (14) and (15), and a null matrix on any product of these moments, where  $\hat{S}_{\ell\ell}^{-1}(\theta)$  is the long-run variance-covariance matrix of  $\ell_t(\theta)$  and where the number  $\lambda$  is large enough to ensure that estimates fit well consumption growth dynamics (3), and minimize the gap between actual and fitted returns (e.g., Parker and Julliard, 2005; Yogo, 2006). Explicit closed-form formulas for  $\ell_t(\theta)$  are derived from the Kalman Filter log likelihood, and full details about the estimation procedure can be found in the internet appendix.

The main results are derived from the identity matrix  $W = I$  that puts equal weight on initial portfolios. I also report results of an efficient GMM estimation where  $W = [S(b, p)]^{-1}$ , and where I estimate the parameters and the spectral density matrix simultaneously, following Hansen et al. (1996). They conduct Monte Carlo experiments to show that this estimator, which is a member of a class of generalized empirical likelihood (GEL) estimators as demonstrated by Newey and Smith (2004), may have small-sample advantages. For each estimation, I compute two alternative and robust goodness-of-fit measures to evaluate the overall pricing ability of the model. The first measure is the distance  $\hat{d}$  between average actual returns and average predicted returns with respect to the positive definite matrix  $W$ , and the second measure is the cross-sectional coefficient of determination  $R^2$ , given by

$$\hat{d} = \sqrt{\hat{e}^\top \hat{W} \hat{e}} \quad \text{and} \quad R^2 = 1 - \frac{N-1}{N-K-1} \frac{\hat{e}^\top \hat{A} \hat{e}}{\hat{\mu}_R^\top \hat{A} \hat{\mu}_R}, \quad \text{with} \quad \hat{A} = \hat{W} - \hat{W} \iota \left( \iota^\top \hat{W} \iota \right)^{-1} \iota^\top \hat{W},$$

where  $\hat{e}$  is the vector of pricing errors, and  $\hat{\mu}_R$  is the vector of actual average excess returns. If  $W = I$ , the identity matrix, then the formula  $R^2$  gives the adjusted central R-squared calculated as if I were doing a linear regression of the average returns on risks measured by covariances between returns and factors. In this case,  $\hat{d}/\sqrt{N}$  is the root-mean-square of pricing errors and measures how much the expected return based on the fitted model is off for a typical portfolio.

As an additional check on the results, I also provide Monte Carlo evidence on the finite sample properties of the  $t$ -statistics of estimated covariance risk prices ( $\hat{p}$ ), of the distance  $\hat{d}$  and of the cross-sectional  $R^2$ , from which I report the probability of obtaining a statistic at least as extreme as the actual one. This experiment attempts to show that the empirical results reflect economic content rather than some random chance. To do this, I simulate 1,000 samples of quarterly

time series observations of aggregate consumption growth, expected consumption growth and consumption volatility.<sup>10</sup> Changes in these simulated consumption level, expected growth and volatility are independent from the observed returns. Note that by definition, this means that the values of the  $t$ -statistics of covariance risk prices, and of the cross-sectional  $R^2$  from the regression (9) under this hypothesis are then zero. For each draw, which is of the same length as the data, I estimate the cross-sectional regression (9) via GMM as described above and I compute the  $t$ -statistics of covariance risk prices and the cross-sectional  $R^2$ . To interpret this Monte-Carlo, for example, a  $p$ -value of say 0.10 means that 10% of the  $t$ -stats of the Monte-Carlo based empirical distribution are larger (resp. smaller) than the  $t$ -stat of the estimate of consumption level or expected growth (resp. volatility) risk price in the data.

## 4. Empirical Evidence

### 4.1 EMPIRICAL TESTS: GMM WITH IDENTITY WEIGHTING MATRIX

The results for estimating Equation (9) by GMM based on the identity matrix, and using the first asset menu consisting of MKT, SMB, HML and SBM25 portfolios are displayed in Table 1. Similar results that use the second asset menu consisting of MKT, SMB, LTR and SLTR25 portfolios are displayed in Table 2. Adding MKT, SMB, HML and LTR factors requires the model not only to fit the cross-section of average returns on SBM25 and SLTR25 portfolios, but also the equity premium, the size effect, the value premium and the long-term reversal premium. For a maximum horizon  $h_{\max}$ , I estimate in total  $h_{\max}(h_{\max} + 1)/2$  cross-sectional models, corresponding to all couples  $(h, k)$  with  $1 \leq h \leq h_{\max}$  and  $1 \leq k \leq h$ . For example, this means 78 different specifications for  $h_{\max} = 12$  and 210 for  $h_{\max} = 20$ . Due to space limitations, I choose  $h_{\max} = 12$ , and only report results when  $h$  is a multiple of 4, so that the risk horizon may be interpretable in annual terms as I use quarterly data, and for even values of  $k$ . This set of  $h$  and  $k$  values is sufficient to draw conclusions about risk-return patterns as both risk horizon and stock holding period vary.

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<sup>10</sup>Note that GARCH volatility modeling ensures positivity of the conditional variance process so that in model simulations (which I perform for example to compute empirical  $p$ -values of asset pricing tests), to the contrary of the gaussian and square-root stochastic volatility models used for example by Bansal and Yaron (2004) and Tauchen (2011), it avoids the drawback of replacing negative realizations with an arbitrary small positive number to ensure positivity of simulated variance series.

I first examine parameter estimates of the consumption dynamics. In both Table 1 and Table 2, the magnitudes of these parameters are stable across horizons. This is not surprising given the large weight ( $\lambda$ ) assigned to the likelihood scores of the consumption dynamics in the joint asset pricing test and factor estimation, as discussed in Section 3.2. These parameters are all statistically significant at conventional confidence levels, based on their asymptotic  $t$ -statistics. In particular, the estimated persistence of volatility is about 0.85, corresponding to a monthly value of about 0.95, and the estimated persistence of expected growth is about 0.70, corresponding to a monthly value of about 0.89. These values appear to be low compared to the value of about 0.98 or larger, used in standard calibration of long-run risk models. Higher persistence are crucial for the ability of these models to match actual asset price moments and predictability regressions, as illustrated for example in Beeler and Campbell (2012) and Bonomo et al. (2011).

Given above parameter estimates of the consumption dynamics, I analyse the time series patterns of changes in consumption level, expected growth and volatility. This preliminary analysis is necessary as business cycle indicators have the potential of explaining the cross-section of stock returns, as payoff heterogeneity across assets, and especially across value and growth stocks on one hand, or past long-term losers and winners on the other hand, may potentially exacerbate during recessions. Focusing on a risk horizon of twelve quarters, the top-left panel of Figure 1 shows the time series plot of changes in the level of aggregate consumption. The top-right panel shows a similar plot for changes in expected consumption growth, and the bottom-left panel for changes in consumption volatility. These graphs evidence the procyclical nature of both realized consumption level and expected growth, as well as the countercyclical behavior of consumption volatility. Specifically, changes in consumption volatility peak up during business cycle recessions, and a notable case is the 2007-2008 economic downturn where changes in volatility start to increase a couple of quarters before the recession until they reach their peak during the recession. This peak also represents the second largest over the sample period, the largest occurring during the 1990-1991 business cycle contraction.

I also examine factor correlations, given the multi-factor nature of the cross-sectional asset pricing model under study. Notice that the specification (3) assumes a zero correlation of shocks to consumption level and expected consumption growth with volatility innovations, implying in population that changes in consumption level and in expected growth are uncorrelated to changes

in consumption volatility. However, factors that are estimated from the data are not necessarily uncorrelated. The bottom-right panel of Figure 1 shows small sample correlations between these changes as the risk horizon increases. These correlation coefficients are moderate across risk horizons from four to twenty quarters, with negative values lying between -35% and 0%.

I now examine estimates of factor risk prices. In tests with SBM25 portfolios, the covariance risk price for changes in consumption volatility is negative as predicted by theory, with values that range between  $-1.70 \times 10^5$  ( $h = 12, k = 4$ ) and  $-6.55 \times 10^5$  ( $h = 4, k = 4$ ). These volatility risk price estimates from the first-step GMM with identity weighting matrix are not statistically significant at conventional levels of confidence, based on their asymptotic  $t$ -statistics. The empirical  $p$ -values of these estimates are however close to a 10% significance level. The magnitude of the estimated covariance risk price for changes in consumption volatility tends to decrease with the risk horizon  $h$ , and increase when estimation is performed with longer stock holding period  $k$ . The covariance risk price associated with changes in expected growth is estimated to be positive as predicted, as the stock holding period increases. This is almost the case for the covariance risk price associated with changes in consumption level. As the risk horizon increases, differences in factor risk price estimates become minor as the stock holding period approaches the risk horizon. These observations are consistent with consumption-based asset pricing models performing well over long horizons. Also note that the reported estimates of the covariance risk price associated with changes in consumption level for longer risk horizons and stock holding periods suggest values of the risk aversion parameter  $\gamma$  that are consistent with those used in existing and standard calibrations of consumption-based models. This parameter is set at 10 in Bansal and Yaron (2004) and at a very high value of 30 in Lettau et al. (2008). To rationalize the current empirical evidence, I use the value  $\gamma = 25$  in the calibration assessment of the model as reported in the internet appendix.

Large asymptotic standard errors of covariance risk price estimates are typical to consumption-based asset pricing tests via GMM with pre-specified weighting matrix (see for example reported standard errors in panels B and C of Table 5 in Parker and Julliard, 2005), and does not alter the economic significance of the results as I further discuss. The identity weighting matrix forces the model to try to explain the size and the value premia by minimizing the root-mean-square of pricing errors. Standard errors on factor risk prices are large because the GMM objective function is quite flat in some components of the vector  $p$  near the estimates, due to low magnitude and variance of

the corresponding factors versus returns, and to the fact that the weighting scheme, by giving each portfolio the same weight, does not attempt to balance the relative importance of all moments. Subsequent tests via efficient GMM (weighting moments by the inverse of their spectral covariance matrix) mitigate these results and show a major improvement on the statistical significance of covariance risk price estimates. Efficient GMM minimizes the root-mean-square pricing errors on weighted combinations of the portfolios, giving more weight to linear combinations of returns that have low variance, and often ignoring value premium or size effect. To the contrary of efficient GMM, GMM with identity matrix is suitable for model comparison across investment horizons as the weighting matrix is invariant across tested models.

Table 1 shows that long-horizon consumption level, expected growth and volatility risks fit expected returns on SBM25 portfolios very well. Their joint explanatory power ranges from an adjusted  $R^2$  of 66% ( $h = 4, k = 2$ ) to 86% ( $h = 12, k = 12$ ). This economic significance tends to increase both with the risk horizon  $h$  and the stock holding period  $k$ . The empirical  $p$ -values of the cross-sectional  $R^2$  in the data are all significant at conventional levels of confidence, suggesting that long-horizon changes in consumption level, expected growth and volatility are economically significant cross-sectional pricing factors. The intercept estimates have relatively small magnitudes. Sorting on the  $R^2$  from Table 1, these estimates are 1.19% ( $h = 4, k = 2$ ) and 5.67% ( $h = 12, k = 12$ ). To get a sense of the economic significance of these values, the cross-sectional means of the 28 average excess returns corresponding to the same risk horizons and stock holding periods are 4.49% ( $h = 4, k = 2$ ) and 28.39% ( $h = 12, k = 12$ ). Based on the reported distances, 0.91% ( $h = 4, k = 2$ ) and 4.24% ( $h = 12, k = 12$ ), the corresponding root-mean-squares of pricing errors are 0.17% ( $h = 4, k = 2$ ) and 0.80% ( $h = 12, k = 12$ ).

The results for the test with SLTR25 portfolios show that the global fit of the model is still high, with cross-sectional  $R^2$  ranging from 43% ( $h = 4, k = 2$ ) to 71% ( $h = 12, k = 12$ ), values that are somewhat lower than those obtained with SBM25 portfolios. Based on the reported distances, 1.11% ( $h = 4, k = 2$ ) and 5.26% ( $h = 12, k = 12$ ), the corresponding root-mean-squares of pricing errors are 0.21% ( $h = 4, k = 2$ ) and 0.99% ( $h = 12, k = 12$ ) and are closed to similar values in the test with SBM25 portfolios. The risk price estimates of changes in consumption volatility are smaller in magnitude compared to the test with SBM25 portfolios, ranging between  $-1.05 \times 10^5$  ( $h = 12, k = 6$ ) and  $-3.46 \times 10^5$  ( $h = 4, k = 2$ ). All other findings regarding the sign and

the magnitude of factor risk premia, the magnitude of intercept estimates and the cross-sectional  $R^2$ , as well their patterns across risk horizons and stock holding periods remain valid with this alternative asset menu. For example, sorting on the  $R^2$  from Table 2, the intercept estimates are 1.84% ( $h = 4, k = 2$ ) and 8.88% ( $h = 12, k = 12$ ), values that are relatively small compared to the cross-sectional means of the 28 average excess returns corresponding to the same risk horizons and stock holding periods, 4.70% ( $h = 4, k = 2$ ) and 29.89% ( $h = 12, k = 12$ ).

#### 4.2 EMPIRICAL TESTS: EFFICIENT GMM

I now turn to the efficient GMM estimation of the cross-sectional asset pricing model (9). This alternative estimation method is necessary for at least two reasons. First, to compare with the GMM estimation based on identity weighting matrix in order to provide a more accurate analysis of the statistical significance of the cross-sectional pricing factors, and second, as a robustness check on the presented empirical evidence. Efficient GMM results using the first asset menu consisting of MKT, SMB, HML and SBM25 portfolios are displayed in Table 3. Similar results that use the second asset menu consisting of MKT, SMB, LTR and SLTR25 portfolios are displayed in Table 4. In both tables, estimates of parameters governing the consumption growth dynamics are significant at conventional confidence levels, and their magnitudes are stable across horizons. Results from both tables also show that intercept estimates are closed to zero across risk horizons and stock holding periods, whether or not they are significant at conventional confidence levels.

The results with the SBM25 portfolios in Table 3 show that for longer risk horizons and stock holding periods, point estimates of the consumption volatility risk price are negative and statistically significant at conventional confidence levels based on both the asymptotic  $t$ -statistics and the empirical  $p$ -values. Focusing on the largest risk horizon  $h = 12$  as reported in the table, the point estimate of consumption volatility risk price varies from  $-1.80 \times 10^5$  ( $t$ -stat of -2.81 and  $p$ -value of 0.03) at the 6-quarter stock holding period to  $-3.76 \times 10^5$  ( $t$ -stat of -2.41 and  $p$ -value of 0.02) at the 12-quarter stock holding period. These magnitudes are quite closed to those of the corresponding estimates with the identity weighting matrix. Efficient GMM thus clearly improves the statistical significance of the results without affecting much the magnitudes of the estimates. Consumption volatility is both an economically and statistically significant cross-sectional pricing factor based on the two alternative GMM estimations. Point estimates for the covariance risk

prices of changes in consumption level and expected growth are also positive and statistically significant at conventional levels, for longer risk horizons and stock holding periods, with values that are also quite comparable to the estimates based on the identity weighting matrix. Focusing again on  $h = 12$ , the cross-sectional  $R^2$  varies from 32% ( $k = 6$ ) to 66% ( $k = 12$ ), with empirical  $p$ -values suggesting a 10% significance level.

Efficient GMM tests with SLTR25 portfolios as shown in Table 4 globally leads to similar conclusions. Theoretically consistent signs of factor risk price estimates and their statistical significance at conventional confidence levels, as well as high cross-sectional  $R^2$  arise at even shorter risk horizons and stock holding periods compared to the efficient GMM test with SBM25 portfolios. Overall, the efficient GMM results evidence that the three-factor model with  $h$ -horizon changes in consumption level, expected growth and volatility is able to price mean-variance efficient combinations of  $h$ -period returns formed by investing in SBM25, SLTR25, MKT, SMB, HML and LTR portfolios for the first  $k$  periods, then reinvesting the payoffs in the safe asset for the remaining  $h - k$  periods. The model is thus able to explain Sharpe ratios of such multi-period returns.

## 5. Economic Intuition

A fundamental tenet in finance is that there is a tradeoff between risk and return, so that higher expected returns are justified by higher undesirable exposures to factors that are priced in the securities market. The test assets used in the previous empirical investigation present four well-documented asset price anomalies: the value premium, the long-term reversal premium, the size effect and the equity premium. The value, the long-term reversal, the size and the equity indexes refer to portfolios which one-period returns in excess of the risk-free rate are equal to the HML, the LTR, the SMB and the MKT factors respectively. Specifically,  $HML_{t+1} + R_{f,t+1}$  is the gross value index return,  $LTR_{t+1} + R_{f,t+1}$  is the gross long-term reversal index return,  $SMB_{t+1} + R_{f,t+1}$  is the gross size index return and  $MKT_{t+1} + R_{f,t+1}$  is the gross market index return. The exposure of the value index can typically be interpreted as the difference in exposures between value and growth firms. Similarly, the exposure of the long-term reversal index can be interpreted as the difference in exposures between past long-term loser and winner stocks, and the exposure of the size index can be interpreted as the difference in exposures between small and big firms. To provide

an intuitive account of the previous empirical findings, I use risk price estimates based on GMM with identity weighting matrix to decompose the value, the long-term reversal, the size and the market premia into different contributions based on the cross-sectional model.

Table 5 displays portfolio average multi-period excess returns and factor loadings. The annualized time series average of multi-period value index excess returns in the sample is about 5.70%, the amount by which value stocks outperformed growth stocks on average on a yearly basis.<sup>11</sup> The question whether value stocks have higher undesirable exposures than growth stocks on changes in consumption level, expected growth and volatility may be answered by looking at loadings on multi-period value index returns on these factors. Multiperiod value index returns load negatively on changes in consumption volatility and these loadings are highly statistically significant as the risk horizon increases. For example, for a 4-quarter stock holding period, exposures of multi-period value index returns to changes in consumption volatility are  $-6.04 \times 10^3$  ( $t$ -stat of -1.42),  $-1.22 \times 10^4$  ( $t$ -stat of -3.60) and  $-1.03 \times 10^4$  ( $t$ -stat of -2.69) for risk horizons of 4, 8 and 12 quarters respectively. These negative numbers represent undesirable exposures of the value index to changes in volatility, and suggest that value stocks enjoy higher average returns than growth stocks because they load more negatively or less positively on changes in consumption volatility than growth stocks.

Similar to changes in consumption volatility, exposures of the value index to changes in consumption level and expected consumption growth are negative, and to the contrary, they do not represent undesirable exposures to these factors, as the marginal investor would instead prefer assets with negative covariance with changes in consumption level and expected consumption growth. This shows that the current three-factor cross-sectional model is away from explaining the value premium through these two channels. This also contrasts with the findings from the one-factor model of Parker and Julliard (2005) that long-horizon consumption growth explains the value premium. My results suggest that this may not be the case anymore in a multi-factor setting that controls for long-horizon changes in consumption volatility as a cross-sectional pricing factor.

Regarding long-term reversal index excess returns, their annualized time series average in the

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<sup>11</sup>Notice that for a given stock holding period  $k$  the average multi-period excess returns increases slightly as the risk horizon  $h$  increases. Roughly, an annualized value of the multi-period average excess return can then be obtained by simply dividing it by  $k/4$ .



sample is about 5.00%, showing that on average past long-term losers outperformed past long-term winners by this same amount on a yearly basis. Factor loadings reported in Table 5 show that multi-period long-term reversal index returns load negatively on changes in consumption volatility and these loadings are highly statistically significant as the risk horizon increases. They also load positively on changes in expected consumption growth, but these loadings are not statistically significant at conventional levels of confidence. For example, for a 4-quarter stock holding period, exposures of multi-period long-term reversal index returns to changes in consumption volatility are  $-5.01 \times 10^3$  ( $t$ -stat of -1.54),  $-8.99 \times 10^3$  ( $t$ -stat of -3.17) and  $-6.19 \times 10^3$  ( $t$ -stat of -2.16) for risk horizons of 4, 8 and 12 quarters respectively. As for the value index, these negative numbers represent undesirable exposures of the long-term reversal index to changes in volatility. The positive loadings of the long-term reversal index on changes in expected growth represent undesirable exposures as well. These findings suggest that long-term losers enjoy higher average returns than long-term winners because they load more negatively or less positively on changes in volatility, and more positively or less negatively on changes in expected growth than long-term winners.

I now turn to the size and the market index excess returns, which annualized time series averages in the sample are about 3.70% and 5.70% respectively. This shows that on average small stocks outperformed large stocks by 3.70% and that the aggregate stock market earns 5.70% over the risk-free rate on a yearly basis. Factor loadings reported in Table 5 show that multi-period size index returns load negatively on changes in consumption volatility for relatively longer risk horizons. To the contrary, loadings on changes in expected consumption growth are positive for relatively shorter risk horizons and seem to be negative for longer risk horizons. Size index returns load positively on changes in consumption level. This suggests that channels through which the current three-factor cross-sectional model explains the size premium vary considerably with the risk horizons. At very short horizons, changes in expected consumption growth matter more, while at very long horizons, changes in consumption volatility are the key channel. Loadings of multi-period market returns on all three factors are positive, suggesting undesirable exposures of the aggregate stock market to changes in consumption level and expected growth, but not to changes in consumption volatility. The current three-factor cross-sectional model thus cannot explain the equity premium primarily through the consumption volatility channel.

Portfolio premium decomposition further enhances our understanding of the major sources of

the premium required to invest in stocks across risk horizons and stock holding periods. The first three panels of Figure 2 and Figure 3 provide this decomposition for market, size, value and long-term reversal indexes, using covariance risk price estimates of Section 4.1, when portfolios are held over the full investment period, that is when  $k = h$ . Table 6 further provides specific numbers corresponding to stock holding periods for which portfolio average multi-period excess returns and factor loadings are displayed in Table 5.

The negative exposure of the value index to changes in consumption volatility together with a negative covariance risk price for this factor leads to a positive volatility component of the value premium. For a 4-quarter stock holding period, this component accounts for 90% and 67% of the multi-period value premium at risk horizons of 8 and 12 quarters respectively. When the value index is held over the full investment period, these contributions are about 80% and larger for risk-horizons of three to twenty quarters as shown in the third panel of Figure 2. Numbers in Table 6 show that they amount to 102%, 86% and 78% at risk horizons of 4, 8 and 12 quarters respectively. Corresponding contributions for expected growth are -41%, -22% and 6% respectively, while those for consumption level are -13%, 1% and -1%. Notice that the sum of contributions from different sources is less than 100%, the remaining accounting for the part of the premium that remains unexplained by the cross-sectional model. Summing up the three contributions over the full risk horizon, the cross-sectional model explains 48%, 65% and 83% of the multi-period value premium at risk horizons of 4, 8 and 12 quarters, which are exclusively attributable to changes in volatility. In other words, the current three-factor cross-sectional model explains the returns of the SBM25 portfolios, and hence explains the value premium anomaly, because value stocks have larger consumption volatility risk than growth stocks, making value stocks more riskier.

Transposing the above decomposition to the long-term reversal premium, the third panel of Figure 3 justifies a volatility component of about 50% and larger for risk-horizons of four to twenty quarters, and Table 6 shows that it accounts for 49%, 53% and 57% of the multi-period long-term reversal premium at risk horizons of 4, 8 and 12 quarters respectively, when the long-term reversal index is held over the full investment period. These contributions amount to 20%, 20% and 24% respectively for expected growth, and to -10%, -11% and -4% respectively for consumption level. These numbers confirm my previous discussion that changes in consumption volatility and expected consumption growth are the two major channels for explaining the long-term reversal

premium, while changes in consumption level do not. The cross-sectional model explains 59%, 62% and 77% of the multi-period long-term reversal premium at risk horizons of 4, 8 and 12 quarters, and changes in consumption volatility are the key factor. Hence, the current three-factor cross-sectional model explains the returns of the SLTR25 portfolios, and the long-term reversal premium anomaly, because long-term losers enjoy higher average returns than long-term winners, specifically due to the larger undesirable exposure of the former to changes in consumption volatility.

While changes in consumption volatility do not contribute in explaining the size effect at shorter risk horizons, the volatility component of the multi-period size premium become substantially important as the risk horizon increases. For example, based on SBM25 estimates, Table 6 shows that the volatility component of the multi-period size premium for a 4-quarter stock holding period accounts for -20%, 48% and 35% at risk horizons of 4, 8 and 12 quarters respectively. These contributions amount to 75% and 73% at risk horizons of 8 and 12 quarters respectively, when the size index is held over the full investment period. To the contrary, consumption level and expected growth components are larger at shorter risk horizons and become considerably lower at longer risk horizons. The size premium decomposition patterns in the second panels of Figure 2 and Figure 3 also confirm these observations. For example, Table 6 shows that when the size index is held over the full investment period, contributions for expected growth are 54%, 31% and -3% for risk horizons of 4, 8 and 12 quarter respectively, while those for consumption level are 35%, -2% and 7% respectively, based on based on SBM25 estimates. A similar pattern is observed based on SLTR25 estimates, although contributions of the volatility component are relatively smaller, compared to those based on SBM25 estimates. Table 6 and the first panels of Figure 2 and Figure 3 finally show that changes in consumption volatility do not appear to be the major channel for explaining the multi-period equity premium at short risk horizons, but their contribution is increasing steadily with the risk horizon as the market index is held over the full investment period, and will exceeds 50% at very long horizons.

The last five panels of Figure 2 plot for each book-to-market quintile, the average compensations across size portfolios, for level, expected growth and volatility risks as percentages of total asset risk premium. The last five panels of Figure 3 show similar plots for each long-term reversal quintile. Overall, percentages of the premium attributable to level and expected growth risks exhibits a rapid or steady decline as the risk horizon grows, while the part that compensates for

volatility fluctuations increases steadily. Changes in consumption volatility increasingly becomes the major source of the premium required to invest in stocks as the risk horizon gets longer. In summary, these results provide empirical evidence that changes in consumption volatility are the key factor for explaining major cross-sectional asset pricing anomalies across risk horizons and stock holding periods, controlling for changes in consumption level and expected growth. This also suggests that future research on consumption-based asset pricing models with time-varying macroeconomic uncertainty, as measured by the volatility of aggregate consumption, should emphasize a relatively more important volatility channel and examine its long-horizon implications.<sup>12</sup>

While previous empirical results give a clear understanding of the fact that consumption volatility risk is priced and is relevant for understanding differences in risk compensations across investment horizons and across assets, another comprehensive evidence of asset exposures to consumption volatility may be built on fundamentals, where we may also see heterogeneity of asset dividends with respect to volatility risk. Measuring consumption volatility risk embodied in asset dividends is a very challenging issue that I do not address in this article. In the internet appendix, I show how heterogeneity in exposures of asset dividend growths to consumption growth forecast and volatility translates into risks in returns and, into cross-sectional differences in risk premia. I then use equilibrium arguments to prove that the presented empirical evidence, although based on exposure of returns to consumption volatility, is mainly due to endogenous asset dividend-payoff ratios loading positively on volatility and more so for value stocks compared to growth stocks. I also show that this is not always true if value and growth stocks have different cash flow exposures to volatility fluctuations, and in this case, simply measuring consumption volatility risk in returns through sensitivity of the asset valuation ratio may lead to a wrong cross-sectional consumption volatility risk-return pattern. Modeling innovations in dividend growth as correlated with innovations in consumption volatility is a property entirely missed out in previous long-run risk models and uncovered in the present article.

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<sup>12</sup>In their thorough empirical assessment of long-run risk models, Beeler and Campbell (2012) show that, allowing for a small stochastic and predictable component in consumption growth exacerbates its predictability by the dividend-price ratio inside the model, and deteriorates the predictability of excess returns. They also realize that the limited effect of volatility in these long-run risk models explains the model's wrong predictability pattern. They finally advise that time-varying volatility rather than predictable variation in expected consumption growth should be the focus of attention in future research.

## 6. Conclusion

Investors have serious concerns about consumption volatility because they fear repercussions of macroeconomic uncertainty on their future wealth. They forecast future consumption growth and want to do so with as low imprecision as possible. Hence, in equilibrium, agents who are averse to large fluctuations in uncertainty about future consumption growth forecast demand a greater compensation for these volatility fluctuations, in the form of higher expected returns for holding stocks with large negative covariation with changes in consumption volatility.

I derive the multi-period cross-sectional implications of an affine consumption-based general equilibrium model with fluctuating consumption growth forecast and volatility, featuring generalized autoregressive conditional heteroscedasticity and recursive preferences of Epstein and Zin (1989). In this model, there are three factors that exposures to help explain the heterogeneity in expected excess returns across assets: changes in consumption level and changes in consumption growth forecast and volatility. I estimate the model in its linear covariance form via GMM with identity matrix and efficient GMM, and the model performs well in both estimation strategies. My empirical results support an economically and statistically significant cross-sectional relation between stock returns and changes in consumption volatility. The data evidence that, in a three-factor model with changes in consumption level, consumption growth forecast and volatility, changes in consumption volatility a the key factor for explaining well-documented cross-sectional asset pricing anomalies such as value, long-term reversal and size premia, and to a lesser extent the equity premium, across various risk horizons and stock holding periods. Growth stocks and past long-term winners have a low volatility risk compared to value stocks and past long-term losers, for most risk horizons and stock holding periods.

Another comprehensive evidence of asset exposures to consumption volatility may be built on fundamentals, where we may also see heterogeneity of asset dividends with respect to volatility risk. I use model-based arguments to prove that the volatility risk-return relation as observed empirically may depend critically on the fact that asset dividends are exposed to consumption volatility and that volatility risk in dividends must be more larger for value stocks than for growth stocks. Measuring volatility risk in dividends is a very challenging issue in the data, and exploring further empirical evidence in that direction would be a valuable contribution to the literature.

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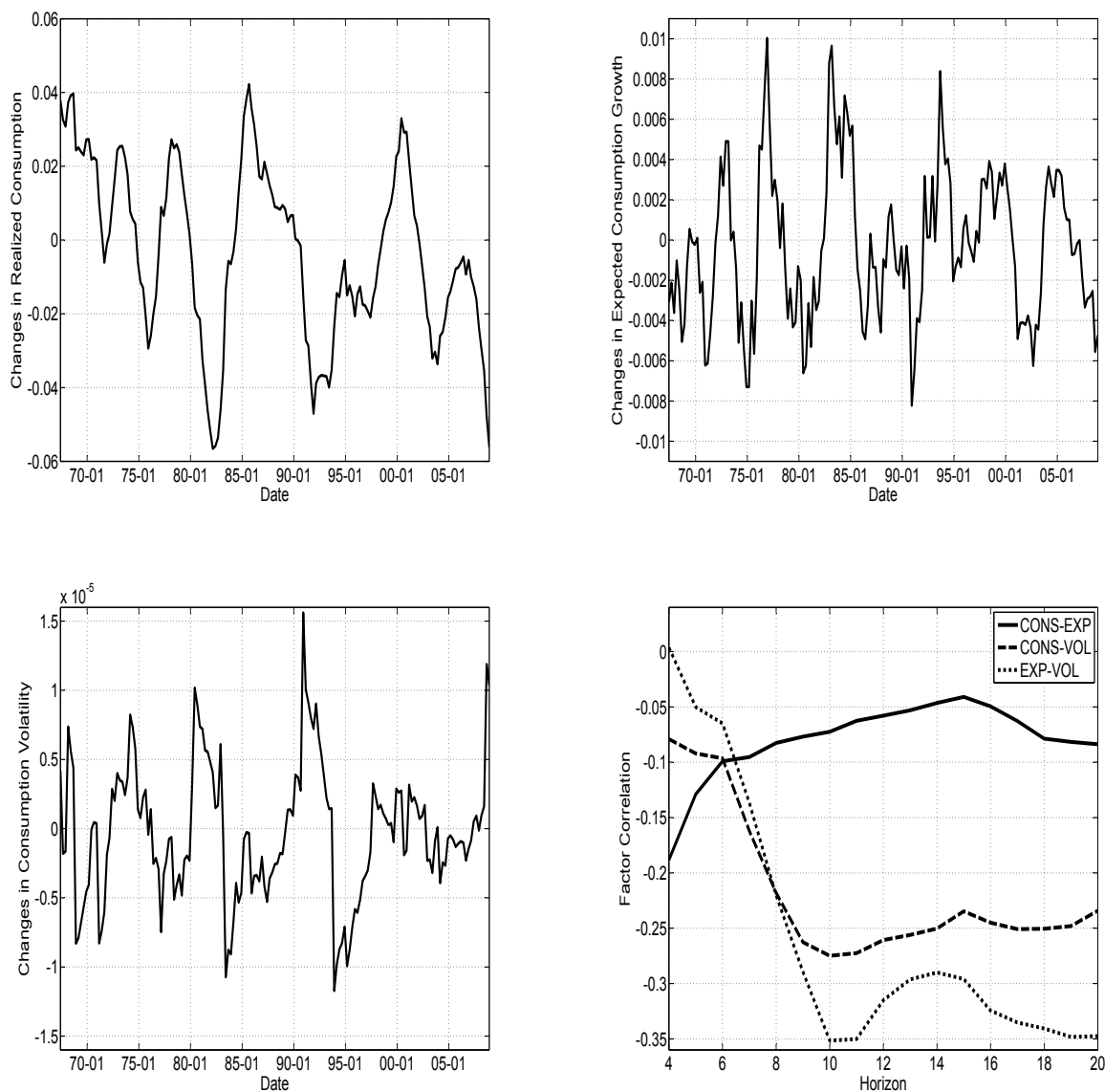


Figure 1: **Factor Time Series and Correlations**

The top-left panel shows the time series plot of twelve-quarter changes in the level of aggregate consumption,  $\Delta c_{t,12}$ . The top-right panel shows a similar plot for changes in expected consumption growth,  $\Delta \hat{x}_{t,12}$ , and the bottom-left panel for changes in consumption volatility,  $\Delta \hat{\sigma}_{t,12}^2$ . The bottom-right panel plots the small sample correlation  $\widehat{Corr}(\Delta c_{t,h}, \Delta \hat{x}_{t,h})$  between  $h$ -horizon changes in consumption level and expected growth estimates, the small sample correlation  $\widehat{Corr}(\Delta \hat{x}_{t,h}, \Delta \hat{\sigma}_{t,h}^2)$  between  $h$ -horizon changes in expected growth and consumption volatility estimates, and the small sample correlation  $\widehat{Corr}(\Delta c_{t,h}, \Delta \hat{\sigma}_{t,h}^2)$  between  $h$ -horizon changes in consumption level and consumption volatility estimates.

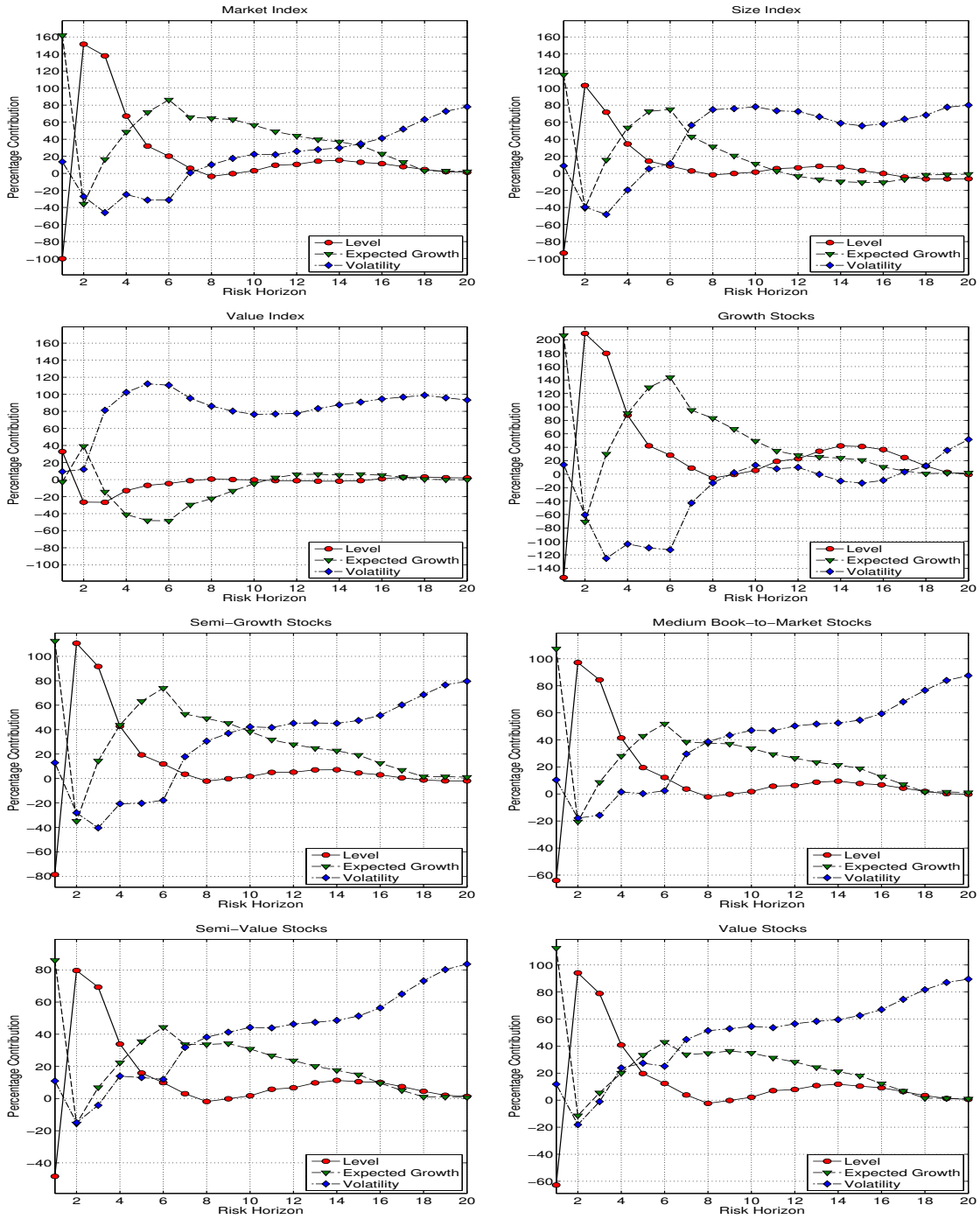


Figure 2: Risk Premium Decomposition of Book-to-Market Portfolios

The figure plots contributions (in percentage) of consumption level, expected growth and volatility risk premia to the market (MKT), the size (SMB) and the value (HML) premia, as well as their average contributions across size portfolios in each book-to-market quintile. Portfolios are held over the full investment period. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Data are converted to real using the PCE deflator. Contributions are computed from GMM estimates with identity weighting matrix and using the SBM25 portfolios.

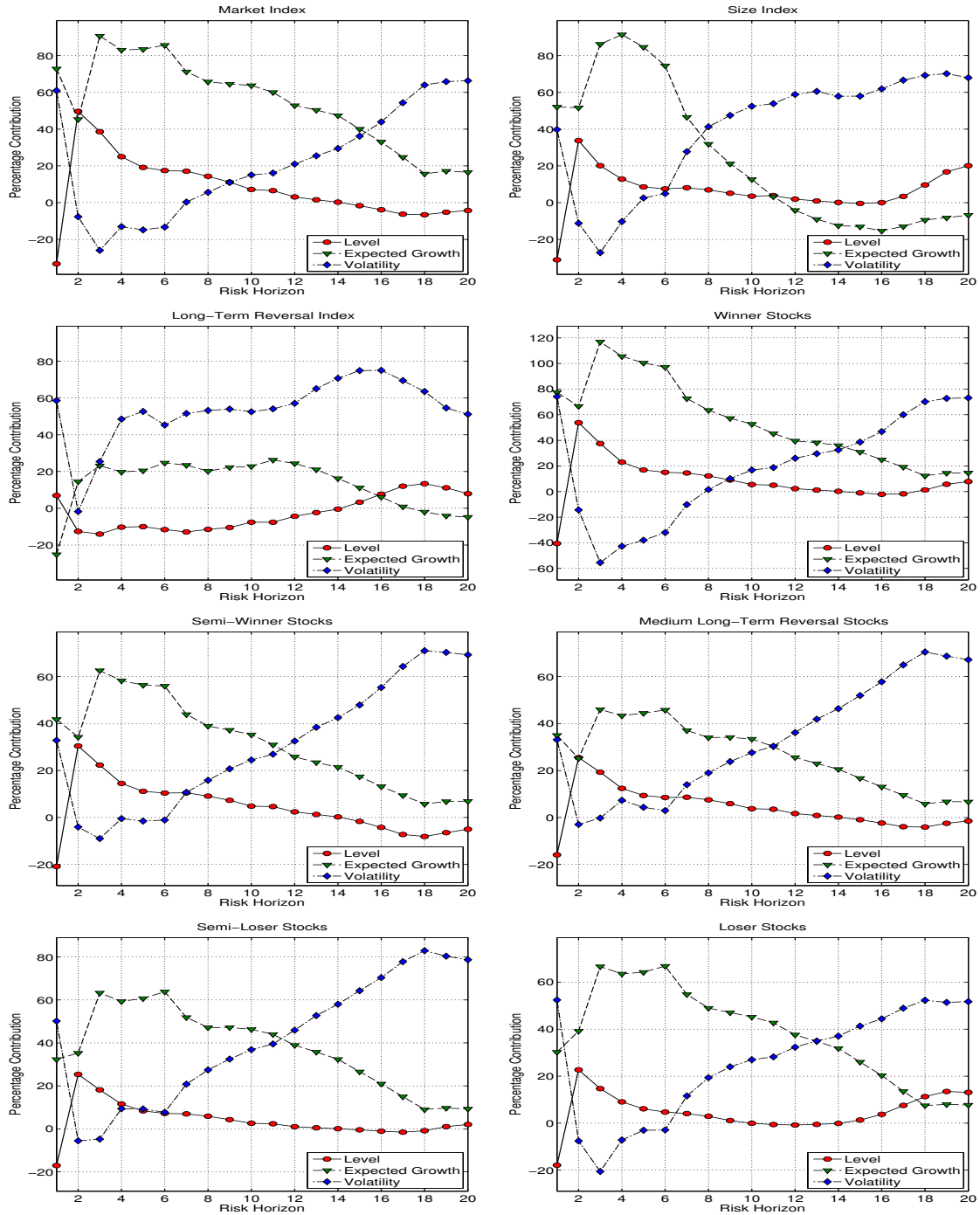


Figure 3: Risk Premium Decomposition of Long-Term Reversal Portfolios

The figure plots contributions (in percentage) of consumption level, expected growth and volatility risk premia to the market (MKT), the size (SMB) and the long-term reversal (LTR) premia, as well as their average contributions across size portfolios in each long-term reversal quintile. Portfolios are held over the full investment period. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Data are converted to real using the PCE deflator. Contributions are computed from GMM estimates with identity weighting matrix and using the SLTR25 portfolios.

Table 1: **GMM with Identity Matrix (SBM25).**

This table reports the estimation and evaluation results for the following model:

$$E \left[ R_{it,k,h}^e \right] = b + p_{c,h} Cov \left( \xi_{ct,h}, R_{it,k,h}^e \right) + p_{x,h} Cov \left( \xi_{xt,h}, R_{it,k,h}^e \right) + p_{\sigma,h} Cov \left( \xi_{\sigma t,h}, R_{it,k,h}^e \right),$$

jointly with the consumption growth dynamics (3) specified in Section 2.1. The test assets consist of the value-weighted excess market return (MKT), the size factor (SMB), the value factor (HML) and the 25 size/book-to-market portfolios (SBM25). Consumption is measured as the seasonally adjusted real per capita consumption of nondurables plus services. Data are converted to real using the associated PCE deflator. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Parameter estimates and robust standard errors are obtained in a single step via GMM as described in Section 3.2. The weighting matrix is the identity matrix. The  $R^2$  is adjusted for degrees of freedom. For the  $t$ -statistic of estimates (in parentheses), the distance between actual and fitted expected returns in the column labeled “*dist*”, and the cross-sectional  $R^2$  in the column labeled “ $R^2$ ”, the respective  $p$ -values (in square brackets) correspond to the empirical probability distribution of obtaining a quantity at least as extreme as the one that was actually observed (left-tailed one-sided test for volatility risk price and right-tailed one-sided tests for level and expected growth risk prices). This  $p$ -value is based on a Monte-Carlo with 1,000 replications. The Monte-Carlo is designed so that level, expected growth and volatility risk prices are zero. Further details regarding the Monte-Carlo are given in the text in Section 3.2.

$h$	$k$	$b$	$p_{c,h}$	$p_{x,h}$	$p_{\sigma,h}$	$\mu_c$	$\phi_x$	$\nu_x$	$\mu_\sigma$	$\phi_\sigma$	<i>dist</i>	$R^2$
4	2	0.01 (1.05) [0.15]	78.41 (1.21) [0.09]	-152.71 (-0.44) [0.55]	$-4.82 \times 10^5$ (-1.02) [0.14]	$5.59 \times 10^{-3}$ (10.65)	0.70 (7.84)	0.92 (7.10)	$3.64 \times 10^{-6}$ (2.16)	0.85 (20.71)	0.009 [0.09]	0.66 [0.09]
4	4	0.02 (0.97) [0.14]	78.65 (1.01) [0.10]	261.70 (0.77) [0.19]	$-6.55 \times 10^5$ (-1.07) [0.10]	$5.59 \times 10^{-3}$ (10.54)	0.70 (6.98)	0.92 (7.03)	$3.64 \times 10^{-6}$ (2.16)	0.85 (20.32)	0.017 [0.06]	0.72 [0.06]
8	2	0.02 (1.59) [0.06]	19.52 (0.71) [0.19]	-136.78 (-1.06) [0.67]	$-2.65 \times 10^5$ (-1.30) [0.02]	$5.49 \times 10^{-3}$ (11.82)	0.70 (12.28)	0.92 (7.24)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.62)	0.009 [0.10]	0.66 [0.10]
8	4	0.03 (1.70) [0.04]	42.29 (1.37) [0.06]	12.37 (0.05) [0.41]	$-2.48 \times 10^5$ (-1.15) [0.09]	$5.49 \times 10^{-3}$ (11.84)	0.70 (12.70)	0.92 (7.26)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.71)	0.017 [0.06]	0.73 [0.06]
8	6	0.05 (1.65) [0.04]	23.24 (0.90) [0.13]	200.41 (0.60) [0.22]	$-3.18 \times 10^5$ (-1.17) [0.06]	$5.49 \times 10^{-3}$ (11.72)	0.70 (11.17)	0.92 (7.18)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.41)	0.025 [0.04]	0.76 [0.04]
8	8	0.04 (1.55) [0.06]	-4.32 (-0.07) [0.55]	266.71 (0.99) [0.11]	$-3.54 \times 10^5$ (-1.19) [0.06]	$5.49 \times 10^{-3}$ (11.64)	0.70 (10.55)	0.92 (7.15)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.32)	0.029 [0.02]	0.83 [0.02]
12	4	0.03 (1.65) [0.03]	51.20 (1.27) [0.07]	4.74 (0.04) [0.48]	$-1.70 \times 10^5$ (-0.84) [0.15]	$5.39 \times 10^{-3}$ (13.33)	0.70 (16.39)	0.93 (7.23)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.76)	0.016 [0.04]	0.77 [0.04]
12	6	0.05 (1.90) [0.01]	40.99 (1.20) [0.07]	235.90 (1.57) [0.07]	$-1.82 \times 10^5$ (-0.87) [0.15]	$5.39 \times 10^{-3}$ (13.36)	0.70 (16.04)	0.93 (7.22)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.70)	0.024 [0.05]	0.79 [0.05]
12	8	0.05 (1.13) [0.09]	27.93 (0.63) [0.22]	320.54 (0.99) [0.10]	$-1.97 \times 10^5$ (-0.92) [0.12]	$5.39 \times 10^{-3}$ (13.39)	0.70 (15.59)	0.93 (7.21)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.62)	0.029 [0.02]	0.83 [0.02]
12	10	0.05 (0.68) [0.19]	17.09 (0.29) [0.41]	298.83 (0.60) [0.18]	$-2.27 \times 10^5$ (-0.93) [0.12]	$5.39 \times 10^{-3}$ (13.44)	0.70 (13.92)	0.93 (7.17)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.34)	0.035 [0.01]	0.86 [0.01]
12	12	0.06 (0.65) [0.17]	20.85 (0.28) [0.37]	255.34 (0.56) [0.20]	$-2.81 \times 10^5$ (-0.87) [0.10]	$5.39 \times 10^{-3}$ (13.43)	0.70 (13.32)	0.93 (7.15)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.25)	0.042 [0.01]	0.86 [0.01]

Table 2: **GMM with Identity Matrix (SLTR25).**

This table reports the estimation and evaluation results for the following model:

$$E \left[ R_{it,k,h}^e \right] = b + p_{c,h} Cov \left( \xi_{ct,h}, R_{it,k,h}^e \right) + p_{x,h} Cov \left( \xi_{xt,h}, R_{it,k,h}^e \right) + p_{\sigma,h} Cov \left( \xi_{\sigma t,h}, R_{it,k,h}^e \right),$$

jointly with the consumption growth dynamics (3) specified in Section 2.1. The test assets consist of the value-weighted excess market return (MKT), the size factor (SMB), the long-term reversal factor (LTR) and the 25 size/long-term reversal portfolios (SLTR25). Consumption is measured as the seasonally adjusted real per capita consumption of nondurables plus services. Data are converted to real using the associated PCE deflator. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Parameter estimates and robust standard errors are obtained in a single step via GMM as described in Section 3.2. The weighting matrix is the identity matrix. The  $R^2$  is adjusted for degrees of freedom. For the  $t$ -statistic of estimates (in parentheses), the distance between actual and fitted expected returns in the column labeled “*dist*”, and the cross-sectional  $R^2$  in the column labeled “ $R^2$ ”, the respective  $p$ -values (in square brackets) correspond to the empirical probability distribution of obtaining a quantity at least as extreme as the one that was actually observed (left-tailed one-sided test for volatility risk price and right-tailed one-sided tests for level and expected growth risk prices). This  $p$ -value is based on a Monte-Carlo with 1,000 replications. The Monte-Carlo is designed so that level, expected growth and volatility risk prices are zero. Further details regarding the Monte-Carlo are given in the text in Section 3.2.

$h$	$k$	$b$	$p_{c,h}$	$p_{x,h}$	$p_{\sigma,h}$	$\mu_c$	$\phi_x$	$\nu_x$	$\mu_\sigma$	$\phi_\sigma$	<i>dist</i>	$R^2$
4	2	0.02 (2.36) [0.01]	67.14 (0.94) [0.11]	387.71 (1.06) [0.12]	$-3.46 \times 10^5$ (-1.07) [0.10]	$5.59 \times 10^{-3}$ (10.90)	0.70 (9.02)	0.92 (7.20)	$3.64 \times 10^{-6}$ (2.16)	0.85 (20.88)	0.011 [0.11]	0.43 [0.11]
4	4	0.03 (1.42) [0.04]	29.26 (0.58) [0.22]	447.20 (1.76) [0.08]	$-3.45 \times 10^5$ (-1.07) [0.09]	$5.59 \times 10^{-3}$ (10.91)	0.70 (9.04)	0.92 (7.20)	$3.64 \times 10^{-6}$ (2.16)	0.85 (20.89)	0.020 [0.11]	0.54 [0.11]
8	2	0.02 (3.41) [0.00]	34.11 (1.17) [0.05]	78.97 (1.15) [0.07]	$-1.52 \times 10^5$ (-0.84) [0.14]	$5.49 \times 10^{-3}$ (11.85)	0.70 (15.32)	0.92 (7.39)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.32)	0.010 [0.09]	0.53 [0.09]
8	4	0.04 (3.11) [0.00]	39.48 (1.15) [0.07]	124.41 (0.98) [0.10]	$-1.43 \times 10^5$ (-0.81) [0.13]	$5.49 \times 10^{-3}$ (11.84)	0.70 (15.55)	0.92 (7.41)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.39)	0.020 [0.10]	0.53 [0.10]
8	6	0.05 (2.95) [0.00]	30.83 (0.88) [0.12]	239.53 (1.63) [0.05]	$-1.57 \times 10^5$ (-0.77) [0.19]	$5.49 \times 10^{-3}$ (11.86)	0.70 (15.18)	0.92 (7.39)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.29)	0.029 [0.10]	0.59 [0.10]
8	8	0.06 (2.78) [0.00]	17.79 (0.45) [0.33]	272.04 (1.60) [0.06]	$-1.95 \times 10^5$ (-0.76) [0.17]	$5.49 \times 10^{-3}$ (11.87)	0.70 (14.09)	0.92 (7.33)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.02)	0.035 [0.07]	0.65 [0.07]
12	4	0.05 (3.02) [0.00]	33.50 (0.94) [0.14]	-116.52 (-1.23) [0.88]	$-1.44 \times 10^5$ (-0.82) [0.13]	$5.39 \times 10^{-3}$ (13.26)	0.70 (17.18)	0.93 (7.24)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.90)	0.022 [0.12]	0.48 [0.12]
12	6	0.07 (2.94) [0.00]	37.93 (1.20) [0.06]	154.27 (1.52) [0.06]	$-1.05 \times 10^5$ (-0.55) [0.23]	$5.39 \times 10^{-3}$ (13.14)	0.70 (18.25)	0.93 (7.24)	$3.53 \times 10^{-6}$ (2.18)	0.85 (20.13)	0.032 [0.09]	0.52 [0.09]
12	8	0.09 (2.26) [0.01]	29.97 (0.98) [0.09]	231.33 (1.59) [0.05]	$-1.25 \times 10^5$ (-0.61) [0.19]	$5.39 \times 10^{-3}$ (13.21)	0.70 (17.72)	0.93 (7.25)	$3.53 \times 10^{-6}$ (2.17)	0.85 (20.01)	0.040 [0.07]	0.56 [0.07]
12	10	0.09 (1.81) [0.02]	16.38 (0.54) [0.37]	293.17 (1.37) [0.07]	$-1.63 \times 10^5$ (-0.64) [0.18]	$5.39 \times 10^{-3}$ (13.32)	0.70 (16.60)	0.93 (7.24)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.80)	0.046 [0.04]	0.65 [0.04]
12	12	0.09 (1.64) [0.04]	6.18 (0.17) [0.77]	307.03 (1.13) [0.07]	$-2.28 \times 10^5$ (-0.61) [0.15]	$5.39 \times 10^{-3}$ (13.42)	0.70 (14.69)	0.93 (7.19)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.46)	0.053 [0.02]	0.71 [0.02]

Table 3: **Efficient GMM (SBM25).**

This table reports the estimation and evaluation results for the following model:

$$E \left[ R_{it,k,h}^e \right] = b + p_{c,h} Cov \left( \xi_{ct,h}, R_{it,k,h}^e \right) + p_{x,h} Cov \left( \xi_{xt,h}, R_{it,k,h}^e \right) + p_{\sigma,h} Cov \left( \xi_{\sigma t,h}, R_{it,k,h}^e \right),$$

jointly with the consumption growth dynamics (3) specified in Section 2.1. The test assets consist of the value-weighted excess market return (MKT), the size factor (SMB), the value factor (HML) and the 25 size/book-to-market portfolios (SBM25). Consumption is measured as the seasonally adjusted real per capita consumption of nondurables plus services. Data are converted to real using the associated PCE deflator. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Parameter estimates and robust standard errors are obtained in a single step via GMM as described in Section 3.2. The weighting matrix is the spectral density matrix. The  $R^2$  is adjusted for degrees of freedom. For the  $t$ -statistic of estimates (in parentheses), the distance between actual and fitted expected returns in the column labeled “ $dist$ ”, and the cross-sectional  $R^2$  in the column labeled “ $R^2$ ”, the respective  $p$ -values (in square brackets) correspond to the empirical probability distribution of obtaining a quantity at least as extreme as the one that was actually observed (left-tailed one-sided test for volatility risk price and right-tailed one-sided tests for level and expected growth risk prices). This  $p$ -value is based on a Monte-Carlo with 1,000 replications. The Monte-Carlo is designed so that level, expected growth and volatility risk prices are zero. Further details regarding the Monte-Carlo are given in the text in Section 3.2.

$h$	$k$	$b$	$p_{c,h}$	$p_{x,h}$	$p_{\sigma,h}$	$\mu_c$	$\phi_x$	$\nu_x$	$\mu_\sigma$	$\phi_\sigma$	$dist$	$R^2$
4	2	0.00 (0.23) [0.69]	-31.87 (-1.71) [0.48]	-282.53 (-2.08) [0.41]	$-1.68 \times 10^5$ (-1.31) [0.05]	$5.59 \times 10^{-3}$ (11.27)	0.70 (11.91)	0.92 (7.47)	$3.64 \times 10^{-6}$ (2.17)	0.85 (21.79)	0.247 [0.39]	0.19 [0.42]
4	4	0.00 (0.00) [0.87]	-24.94 (0.00) [0.97]	-143.67 (0.00) [0.96]	$1.31 \times 10^5$ (0.00) [0.96]	$5.59 \times 10^{-3}$ (11.28)	0.70 (12.57)	0.92 (7.55)	$3.64 \times 10^{-6}$ (2.17)	0.85 (22.08)	0.000 [0.93]	0.02 [0.97]
8	2	0.00 (1.64) [0.18]	25.59 (1.60) [0.08]	-147.10 (-1.78) [0.44]	$-1.69 \times 10^5$ (-2.60) [0.02]	$5.49 \times 10^{-3}$ (11.87)	0.70 (14.82)	0.92 (7.37)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.20)	0.566 [0.09]	0.55 [0.11]
8	4	-0.00 (-0.52) [0.82]	21.27 (1.21) [0.08]	-18.13 (-0.19) [0.92]	$-0.10 \times 10^5$ (-0.11) [0.70]	$5.49 \times 10^{-3}$ (11.77)	0.70 (17.15)	0.92 (6.83)	$3.60 \times 10^{-6}$ (2.67)	0.85 (20.81)	0.101 [0.95]	-0.01 [1.00]
8	6	-0.00 (-0.76) [0.98]	23.66 (1.38) [0.11]	235.25 (2.88) [0.04]	$-1.77 \times 10^5$ (-1.30) [0.05]	$5.49 \times 10^{-3}$ (11.87)	0.70 (14.61)	0.92 (7.36)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.15)	0.235 [0.17]	0.12 [0.23]
8	8	0.00 (0.43) [0.47]	7.98 (0.37) [0.61]	27.14 (0.31) [0.37]	$-2.35 \times 10^5$ (-1.73) [0.03]	$5.49 \times 10^{-3}$ (11.86)	0.70 (13.02)	0.92 (7.28)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.78)	0.287 [0.13]	0.19 [0.17]
12	4	0.00 (0.94) [0.16]	26.38 (1.13) [0.11]	510.57 (3.12) [0.02]	$1.61 \times 10^5$ (1.01) [0.88]	$5.39 \times 10^{-3}$ (13.31)	0.70 (16.68)	0.93 (7.24)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.82)	0.219 [0.07]	0.74 [0.07]
12	6	0.00 (0.52) [0.39]	12.19 (0.58) [0.25]	79.24 (0.86) [0.11]	$-1.80 \times 10^5$ (-2.81) [0.03]	$5.39 \times 10^{-3}$ (13.36)	0.70 (16.08)	0.93 (7.22)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.70)	0.463 [0.10]	0.32 [0.10]
12	8	0.00 (0.67) [0.24]	17.91 (2.44) [0.05]	272.50 (3.91) [0.02]	$-1.82 \times 10^5$ (-1.67) [0.04]	$5.39 \times 10^{-3}$ (13.36)	0.70 (16.02)	0.93 (7.22)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.70)	0.833 [0.08]	0.37 [0.11]
12	10	0.00 (0.96) [0.18]	20.15 (2.73) [0.03]	317.81 (4.53) [0.02]	$-2.27 \times 10^5$ (-1.65) [0.04]	$5.39 \times 10^{-3}$ (13.42)	0.70 (14.72)	0.93 (7.19)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.48)	0.796 [0.08]	0.46 [0.09]
12	12	0.00 (1.04) [0.16]	33.19 (1.58) [0.05]	434.21 (1.91) [0.04]	$-3.76 \times 10^5$ (-2.41) [0.02]	$5.39 \times 10^{-3}$ (13.30)	0.70 (11.46)	0.93 (7.09)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.02)	0.454 [0.06]	0.66 [0.06]

Table 4: **Efficient GMM (SLTR25).**

This table reports the estimation and evaluation results for the following model:

$$E \left[ R_{it,k,h}^e \right] = b + p_{c,h} Cov \left( \xi_{ct,h}, R_{it,k,h}^e \right) + p_{x,h} Cov \left( \xi_{xt,h}, R_{it,k,h}^e \right) + p_{\sigma,h} Cov \left( \xi_{\sigma t,h}, R_{it,k,h}^e \right),$$

jointly with the consumption growth dynamics (3) specified in Section 2.1. The test assets consist of the excess market return (MKT), the size factor (SMB), the long-term reversal factor (LTR) and the 25 size/long-term reversal portfolios (SLTR25). Consumption is measured as the seasonally adjusted real per capita consumption of nondurables plus services. Data are converted to real using the associated PCE deflator. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Parameter estimates and robust standard errors are obtained in a single step via GMM as described in Section 3.2. The weighting matrix is the spectral density matrix. The  $R^2$  is adjusted for degrees of freedom. For the  $t$ -statistic of estimates (in parentheses), the distance between actual and fitted expected returns in the column labeled “*dist*”, and the cross-sectional  $R^2$  in the column labeled “ $R^2$ ”, the respective  $p$ -values (in square brackets) correspond to the empirical probability distribution of obtaining a quantity at least as extreme as the one that was actually observed (left-tailed one-sided test for volatility risk price and right-tailed one-sided tests for level and expected growth risk prices). This  $p$ -value is based on a Monte-Carlo with 1,000 replications. The Monte-Carlo is designed so that level, expected growth and volatility risk prices are zero. Further details regarding the Monte-Carlo are given in the text in Section 3.2.

$h$	$k$	$b$	$p_{c,h}$	$p_{x,h}$	$p_{\sigma,h}$	$\mu_c$	$\phi_x$	$\nu_x$	$\mu_\sigma$	$\phi_\sigma$	<i>dist</i>	$R^2$
4	2	0.00 (1.07) [0.21]	79.92 (2.19) [0.01]	83.25 (0.73) [0.17]	$-1.78 \times 10^5$ (-1.28) [0.06]	$5.59 \times 10^{-3}$ (11.26)	0.70 (11.74)	0.92 (7.45)	$3.64 \times 10^{-6}$ (2.17)	0.85 (21.72)	0.354 [0.10]	0.36 [0.13]
4	4	0.01 (3.43) [0.00]	67.46 (2.68) [0.02]	159.90 (1.93) [0.04]	$-3.22 \times 10^5$ (-1.75) [0.05]	$5.59 \times 10^{-3}$ (10.97)	0.70 (9.33)	0.92 (7.23)	$3.64 \times 10^{-6}$ (2.16)	0.85 (20.95)	0.427 [0.09]	0.44 [0.09]
8	2	0.00 (0.96) [0.14]	43.46 (3.70) [0.00]	-37.43 (-0.77) [0.81]	$-1.01 \times 10^5$ (-1.91) [0.06]	$5.49 \times 10^{-3}$ (11.79)	0.70 (16.57)	0.92 (7.44)	$3.60 \times 10^{-6}$ (2.18)	0.85 (20.69)	0.437 [0.05]	0.67 [0.06]
8	4	0.00 (1.89) [0.08]	45.84 (3.39) [0.00]	16.21 (0.15) [0.44]	$-1.57 \times 10^5$ (-1.64) [0.04]	$5.49 \times 10^{-3}$ (11.86)	0.70 (15.15)	0.92 (7.39)	$3.60 \times 10^{-6}$ (2.17)	0.85 (20.28)	0.510 [0.02]	0.80 [0.02]
8	6	0.01 (1.78) [0.08]	30.79 (3.97) [0.00]	121.99 (1.28) [0.10]	$-3.38 \times 10^5$ (-2.31) [0.01]	$5.49 \times 10^{-3}$ (11.68)	0.70 (10.81)	0.92 (7.16)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.36)	0.565 [0.04]	0.71 [0.07]
8	8	0.00 (0.37) [0.45]	32.54 (2.33) [0.01]	6.84 (0.18) [0.34]	$-2.10 \times 10^5$ (-2.22) [0.02]	$5.49 \times 10^{-3}$ (11.87)	0.70 (13.69)	0.92 (7.31)	$3.60 \times 10^{-6}$ (2.17)	0.85 (19.93)	0.570 [0.00]	0.87 [0.00]
12	4	0.00 (1.12) [0.17]	39.40 (2.56) [0.02]	-30.60 (-0.60) [0.90]	$0.47 \times 10^5$ (0.64) [0.78]	$5.39 \times 10^{-3}$ (13.03)	0.70 (19.17)	0.93 (7.07)	$3.53 \times 10^{-6}$ (2.23)	0.85 (20.35)	0.484 [0.09]	0.38 [0.13]
12	6	0.00 (2.55) [0.02]	39.51 (4.42) [0.00]	83.20 (2.08) [0.02]	$-0.37 \times 10^5$ (-0.74) [0.12]	$5.39 \times 10^{-3}$ (13.04)	0.70 (19.21)	0.93 (6.99)	$3.53 \times 10^{-6}$ (2.26)	0.85 (20.34)	0.912 [0.03]	0.71 [0.03]
12	8	0.01 (4.54) [0.00]	29.45 (5.93) [0.00]	339.47 (3.90) [0.02]	$-3.05 \times 10^5$ (-4.83) [0.00]	$5.39 \times 10^{-3}$ (13.41)	0.70 (12.77)	0.93 (7.13)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.18)	0.673 [0.00]	0.82 [0.00]
12	10	0.01 (3.62) [0.01]	17.03 (1.67) [0.03]	79.24 (0.66) [0.22]	$-1.53 \times 10^5$ (-1.33) [0.08]	$5.39 \times 10^{-3}$ (13.29)	0.70 (16.91)	0.93 (7.24)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.86)	0.843 [0.09]	0.34 [0.10]
12	12	-0.00 (-0.33) [0.86]	28.48 (3.19) [0.00]	235.35 (2.17) [0.03]	$-2.00 \times 10^5$ (-1.53) [0.04]	$5.39 \times 10^{-3}$ (13.39)	0.70 (15.49)	0.93 (7.21)	$3.53 \times 10^{-6}$ (2.17)	0.85 (19.60)	0.723 [0.08]	0.52 [0.08]



Table 5: **Indexes: Average Excess Returns and Factor Loadings**

The table presents measures of consumption level, expected growth and volatility risks for the market (MKT), the size (SMB), the value (HML) and the long-term reversal (LTR) factors. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Data are converted to real using the PCE deflator. For given stock holding period  $k$  and risk horizon  $h$ , the rows labeled “ $\mu_R$ ”, “ $\beta_{Rc}$ ”, “ $\beta_{Rx}$ ” and “ $\beta_{R\sigma}$ ” present the average excess returns and factor loading estimated from the following multivariate time series regression model:

$$R_{it,k,h}^e = \alpha_i(k, h) + \beta_{i,Rc}(k, h) \times \Delta c_{t,h} + \beta_{i,Rx}(k, h) \times \Delta \hat{x}_{t,h} + \beta_{i,R\sigma}(k, h) \times \Delta \hat{\sigma}_{t,h}^2 + u_{R,t+h}(k, h),$$

jointly with the consumption growth dynamics (3) specified in Section 2.1. The reported  $t$ -statistics (in parentheses below the estimates) are corrected for heteroskedasticity and autocorrelation using the procedure in Newey and West (1987). Adjusted R-squared are reported in square brackets.

	$k = 4$								$k = 8$			
	$h = 8$				$h = 12$				$h = 12$			
	MKT	SMB	HML	LTR	MKT	SMB	HML	LTR	MKT	SMB	HML	LTR
$\mu_R$	5.87	3.67	5.72	5.03	6.06	3.74	5.77	5.07	11.94	7.49	11.91	10.26
$\beta_{Rc}$ (t-stat)	3.28 (2.91)	1.38 (1.25)	-1.01 (-0.84)	-0.50 (-0.75)	2.05 (1.97)	0.52 (0.51)	-0.21 (-0.23)	0.01 (0.02)	3.40 (2.16)	0.52 (0.27)	-0.53 (-0.32)	-1.12 (-1.03)
$\beta_{Rx}$ (t-stat)	8.17 (1.54)	4.87 (1.05)	-6.50 (-1.59)	2.10 (0.65)	1.95 (0.35)	-1.39 (-0.31)	-4.39 (-1.05)	2.53 (0.73)	17.76 (2.37)	-1.20 (-0.13)	-6.94 (-1.08)	3.99 (0.60)
$\beta_{R\sigma}$ (t-stat)	$7.31 \times 10^3$ (2.00)	$-1.63 \times 10^3$ (-0.65)	$-1.22 \times 10^4$ (-3.60)	$-8.99 \times 10^3$ (-3.17)	$6.27 \times 10^3$ (1.33)	$-2.53 \times 10^3$ (-0.88)	$-1.03 \times 10^4$ (-2.69)	$-6.19 \times 10^3$ (-2.16)	$9.88 \times 10^3$ (1.58)	$-8.86 \times 10^3$ (-1.65)	$-1.86 \times 10^4$ (-3.54)	$-1.52 \times 10^4$ (-3.20)
$[R^2]$	[12.92]	[4.76]	[12.19]	[13.52]	[6.21]	[0.78]	[8.58]	[8.35]	[12.96]	[3.31]	[15.16]	[17.48]
	$k = h$											
	$h = 4$				$h = 8$				$h = 12$			
	MKT	SMB	HML	LTR	MKT	SMB	HML	LTR	MKT	SMB	HML	LTR
$\mu_R$	5.69	3.61	5.67	5.00	11.57	7.37	11.81	10.19	18.31	11.42	18.04	15.49
$\beta_{Rc}$ (t-stat)	4.28 (2.39)	1.74 (1.30)	-1.02 (-0.68)	-0.83 (-0.76)	3.72 (2.27)	0.95 (0.51)	-0.94 (-0.59)	-1.93 (-1.63)	2.74 (1.57)	0.26 (0.09)	-0.85 (-0.49)	-2.41 (-1.73)
$\beta_{Rx}$ (t-stat)	20.96 (3.59)	12.98 (2.75)	-14.16 (-2.04)	2.28 (0.60)	27.75 (3.86)	7.12 (0.93)	-10.64 (-1.24)	2.15 (0.33)	30.08 (2.30)	-3.54 (-0.31)	-2.16 (-0.22)	3.50 (0.37)
$\beta_{R\sigma}$ (t-stat)	$1.98 \times 10^3$ (0.45)	$8.68 \times 10^2$ (0.27)	$-6.04 \times 10^3$ (-1.42)	$-5.01 \times 10^3$ (-1.54)	$6.47 \times 10^3$ (0.98)	$-5.70 \times 10^3$ (-0.96)	$-1.60 \times 10^4$ (-3.37)	$-1.47 \times 10^4$ (-2.95)	$5.33 \times 10^3$ (0.65)	$-1.14 \times 10^4$ (-1.40)	$-2.00 \times 10^4$ (-2.91)	$-1.59 \times 10^4$ (-2.87)
$[R^2]$	[14.21]	[6.59]	[7.95]	[3.42]	[20.47]	[2.87]	[11.13]	[14.70]	[13.49]	[2.14]	[14.88]	[13.21]

Table 6: **Indexes: Risk Premium Decomposition**

The table shows contributions (in percentage) of consumption level, expected growth and volatility risk premia to the market (MKT), the size (SMB), the value (HML) and the long-term reversal (LTR) premia. The data are sampled at the quarterly frequency, and cover the second quarter of 1963 through fourth quarter of 2008. Data are converted to real using the PCE deflator. For a given stock holding period  $k$  and risk horizon  $h$ , the rows labeled “ $\mu_{Rc}$ ”, “ $\mu_{Rx}$ ” and “ $\mu_{R\sigma}$ ” represent the consumption level, expected growth and volatility premia, respectively. In panels label “SBM25”, contributions are computed from GMM estimates with identity weighting matrix and using the SBM25 portfolios. In panels label “SLTR25”, contributions are computed from GMM estimates with identity weighting matrix and using the SLTR25 portfolios.

	$k = 4$						$k = 8$		
	$h = 8$			$h = 12$			$h = 12$		
	MKT	SMB	HML	MKT	SMB	HML	MKT	SMB	HML
SBM25									
$\mu_{Rc}$	0.72	0.52	-0.16	0.76	0.45	0.11	0.32	0.17	0.03
$\mu_{Rx}$	0.01	0.02	-0.01	0.00	0.00	0.00	0.36	0.04	-0.07
$\mu_{R\sigma}$	-0.24	0.48	0.90	-0.16	0.35	0.67	-0.01	0.61	0.70
SLTR25									
	MKT	SMB	LTR	MKT	SMB	LTR	MKT	SMB	LTR
$\mu_{Rc}$	0.67	0.49	-0.11	0.50	0.30	0.06	0.34	0.18	-0.11
$\mu_{Rx}$	0.09	0.16	0.12	0.07	0.05	-0.12	0.26	0.03	0.27
$\mu_{R\sigma}$	-0.14	0.28	0.50	-0.13	0.29	0.49	-0.01	0.39	0.46
	$k = h$								
	$h = 4$			$h = 8$			$h = 12$		
	MKT	SMB	HML	MKT	SMB	HML	MKT	SMB	HML
SBM25									
$\mu_{Rc}$	0.67	0.35	-0.13	-0.03	-0.02	0.01	0.11	0.07	-0.01
$\mu_{Rx}$	0.49	0.54	-0.41	0.64	0.31	-0.22	0.44	-0.03	0.06
$\mu_{R\sigma}$	-0.25	-0.20	1.02	0.10	0.75	0.86	0.26	0.73	0.78
SLTR25									
	MKT	SMB	LTR	MKT	SMB	LTR	MKT	SMB	LTR
$\mu_{Rc}$	0.25	0.13	-0.10	0.14	0.07	-0.11	0.03	0.02	-0.04
$\mu_{Rx}$	0.83	0.91	0.20	0.66	0.32	0.20	0.53	-0.04	0.24
$\mu_{R\sigma}$	-0.13	-0.10	0.49	0.06	0.41	0.53	0.21	0.59	0.57