Disappointment Aversion, Term Structure, and Predictability Puzzles in Bond Markets*

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Abstract

We solve a dynamic general equilibrium model with generalized disappointment aversion preferences and continuous state endowment dynamics. We apply the framework to the term structure of interest rates and show that the model generates an upward sloping term structure of nominal interest rates, a downward sloping term structure of real interest rates, and that it accounts for the failure of the expectations hypothesis. The key ingredients are preferences with disappointment aversion, preference for early resolution of uncertainty, and an endowment economy with three state variables: time-varying macroeconomic uncertainty, time-varying expected inflation and inflation uncertainty.

Keywords: Asset Pricing, Macrofinance, Numerical solution methods

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1 Introduction

Asymmetric preferences over losses and gains influence investors to make choices that are inconsistent with the predictions of expected utility theory. Such a behavior is rationally explained through a relatively greater risk aversion to disappointing events.[1] Models that incorporate such asymmetries into preferences have proved successful in rationalizing several asset pricing anomalies. Routledge and Zin (2010) generalize the disappointment aversion preference framework and show how the endogenous variation in disappointment probability produces countercyclical risk aversion, a necessary ingredient for resolving asset pricing puzzles. Bonomo et al. (2011) show that persistent fluctuations in macroeconomic uncertainty and asymmetric preferences are sufficient for generating realistic stock return predictability patterns, and for explaining first and second moments of asset returns and price-dividend ratios. Augustin and Tédongap (2016) illustrate how asymmetric preferences help improve the quantitative implications of the conditional moments of credit default swap spreads, while Campanale et al. (2010) study the implications of disappointment aversion preferences for asset prices in a production economy. Other successful applications relate to cross-sectional pricing anomalies (Delikouras; 2017) and portfolio choice problems (Dahlquist et al.; 2017).[2]

The discontinuity in the preferences, characterized through a kink in the indifference curves, complicates the solutions of analytical asset pricing formulas. As a consequence, a common denominator to prior references is a discrete state space approximation of the economy.[3] While discrete regime-switching models approximate continuous processes well in population (Timmermann; 2000), they are not useful for studying the properties of highly

[1] Gul (1991) introduces an axiomatic model of decision making under uncertainty with disappointment aversion that can rationalize the Allais paradox, i.e. realized choices that are inconsistent with expected utility theory.

[2] Further applications include Dolmas (2013), who combines disappointment aversion with rare disasters, Farago and Tédongap (2018), who use the framework to decompose asset risk premia into regular and downside risk premia, Schreindorfer (2016) with an application to option prices, and Delikouras (2014), who applies preferences with disappointment to explain the credit spread puzzle.

[3] In a special case with central disappointment aversion and homoscedastic consumption growth, Delikouras (2017) provides a closed-form solution to the welfare valuation ratios using the Campbell and Shiller (1988b) log-linear approximation of the return to the claim on aggregate consumption.

persistent processes in small samples. However, asset-pricing frameworks with recursive utility do often rely on highly persistent processes, in particular for the dynamics of consumption growth volatility. The incorporation of asymmetric preferences into asset pricing models through the mechanism of generalized disappointment aversion is growing because of its success in resolving multiple asset pricing puzzles jointly. We thus find it necessary to suggest a method that allows for solving such models when the state of the economy is continuous, consistent with empirically observed dynamics. It is not obvious that some standard approximation methods such as projection onto Chebyshev polynomials or perturbation methods handle the discontinuity in the preferences well.

We solve a general framework that allows for a solution to asset prices when preferences feature non-linearities and the endowment dynamics are continuous in state-dependent outcomes. We apply the framework to the term structure of real and nominal interest rates and specify explicit dynamics for real growth and inflation. We propose a parsimonious model of real aggregate consumption growth with only one single state variable, the volatility of aggregate consumption growth. Consumption growth is non-predictable and features an affine GARCH model for macroeconomic uncertainty. Thus, we assume that realized consumption growth and economic uncertainty are impacted by the same shock. This allows us to limit the resolution of asset prices to a one-dimensional numerical integration. We also specify an exogenous process for inflation, which is necessary to price nominal assets. Realized inflation has a time-varying mean, and the innovations in both realized and expected inflation are perfectly positively correlated, making realized inflation an ARMA(1,1) process. Like macroeconomic uncertainty, inflation uncertainty follows affine GARCH dynamics.[4] While inflation innovations are not allowed to affect future consumption growth, innovations in consumption growth can affect realized and expected inflation. Nominal prices thus rely on only three state variables: time-varying macroeconomic uncertainty, time-varying expected inflation and inflation uncertainty.

[4] Realized inflation thus follows ARMA(1,1)-GARCH(1,1) dynamics.

The model matches an upward (downward) sloping term structure of nominal interest rates (volatilities) estimated in the data. An upward sloping nominal yield curve is obtained if inflation is negatively correlated with innovations in aggregate consumption growth and the agent prefers early resolution of uncertainty. If consumption is negatively correlated with expected inflation, agents will borrow from future consumption by issuing bonds. This drives down nominal bond prices and increases nominal yields. Long-term bonds are, however, less sensitive to expected inflation shocks than short-term bonds. On the other hand, bond yields respond negatively to a rise in inflation uncertainty, and more so for longer-maturity bonds. This suggests a flight-to-quality effect in response to nominal uncertainty. Similarly, an increase in real uncertainty increases nominal bond prices and lowers nominal yields. The magnitude of the impact depends on the asset horizon and is greater for shorter maturities. Thus, the flight-to quality effect dominates the intertemporal substitution effect whereby higher consumption volatility lowers nominal yields. The slope of the term structure of real interest rates is negative, consistent with the intuition that inflation-indexed bonds provide a hedge against future consumption. Thus, agents are willing to pay a premium to hold such assets, which implies a negative risk premium.

Our model also accounts for the failure of the expectations hypothesis. We replicate different versions of the regressions that have confirmed the existence of predictability in bond returns using simulated data with 300,000 monthly observations. We quantitatively match the regression coefficients and explanatory power implied by the projection of holding period returns on the single Cochrane and Piazzesi (2005) factor, the Fama and Bliss (1987) regressions of holding period returns on forward-spot spreads, the Campbell and Shiller (1991) regressions of changes in long rate spreads on yield-spot spreads, and by the Dai and Singleton (2002) regressions of adjusted changes in long rate spreads on yield-spot spreads.

The success of the model relies partly on the model's ability to generate both time-varying prices and quantities of risk. This is a desirable feature for equilibrium models, as pointed out by Le and Singleton (2013). The model endogenously generates time-varying prices of risk through disappointment aversion preferences, based on the work of Gul (1991). Outcomes below the certainty equivalent of future lifetime utility are disappointing. As the certainty equivalent evolves dynamically, the pricing kernel implied by the disappointment aversion preferences exhibits endogenously time-varying market prices of risk. We also show theoretically that the kink in the indifference curve, which arises from the asymmetry in preferences, introduces a volatility of the pricing kernel that is at least as large as that of an investor with symmetrically recursive preferences of Epstein and Zin (1989). These features generate strongly time-varying and countercyclical risk aversion that is able to quantitatively match the predictability patterns in nominal bond returns.

In Section 2, we describe a model for a continuous-state economy with a representative agent who exhibits generalized disappointment aversion preferences, and discuss the solution method. We apply the framework to the term structure of real and nominal interest rates in Section 3. In Section 4, we illustrate implications for the predictability of bond returns. We conclude in Section 5.

2 Model

We describe the generalized disappointment aversion (GDA) preferences of Routledge and Zin (2010) in Subsection 2.1 and characterize a general framework for the endowment dynamics in Subsection 2.2. We discuss the numerical solution method in Subsection 2.3.

2.1 Preferences and Stochastic Discount Factor

The representative agent exhibits aversion for disappointing outcomes. Such preferences are based on the work of Gul (1991) and were generalized by Routledge and Zin (2010). In the spirit of Epstein and Zin (1989) and Weil (1989), the investor derives utility V_t recursively from a weighted average of current and future consumption as follows:

$$
\begin{aligned}
V_t &= \left\{ (1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t \left(V_{t+1}\right)\right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \\
&= C_t^{1-\delta} \left[\mathcal{R}_t \left(V_{t+1}\right)\right]^{\delta} \quad \text{if } \psi = 1,
\end{aligned} \tag{2}
$$

where C_t denotes the level of current consumption and $\mathcal{R}_t \left(V_{t+1}\right)$ is the certainty equivalent of next period lifetime utility, summarizing the utility over all future consumption streams. The time preference parameter is given by $0 < \delta < 1$, and $\psi > 0$ characterizes the elasticity of intertemporal substitution.

As the agent is averse towards disappointing outcomes, she has asymmetric preferences over good and bad outcomes. More specifically, with GDA preferences, the risk-adjustment function $\mathcal{R}(V)$ is implicitly defined by:

$$
U(\mathcal{R}) = E\left[U(\mathcal{R})\right] - \ell E\left[\left(U(\kappa\mathcal{R}) - U(V)\right) I\left(V < \kappa\mathcal{R}\right)\right], \tag{3}
$$

where $I(\cdot)$ is an indicator function defined to be 1 if the condition is met and 0 otherwise, with the utility function $U(\cdot)$ defined as:

$$
U(X) = \frac{X^{1-\gamma}}{1-\gamma} \quad \text{if } \gamma > 0 \quad \text{and } \gamma \neq 1, \tag{4}
$$

where $\gamma > 0$ is the coefficient of relative risk aversion. When $\gamma = 1$, $U(X) = \ln(X)$. The coefficient of generalized disappointment aversion, $0 < \kappa \leq 1$, defines the fraction of the certainty equivalent below which an outcome is considered to be disappointing, while the coefficient of disappointment aversion $\ell \geq 0$ defines by how much utility is reduced in disappointing states. With ℓ equal to zero, the model nests the symmetric Kreps and Porteus (1978) certainty equivalent $\mathcal{R}$, and V_t is restored to be the standard Epstein and Zin (1989) recursive utility. When $\ell > 0$, outcomes below a fraction κ of the certainty equivalent $\mathcal{R}$

lower it by an amount modulated by ℓ.

From (3) it follows that the risk-adjusted future lifetime utility may be rewritten as:

$$\mathcal{R}_t\left(V_{t+1}\right) = \left(E_t\left[\frac{1+\ell I\left(V_{t+1} < \kappa\mathcal{R}_t\left(V_{t+1}\right)\right)}{1+\ell\kappa^{1-\gamma}E_t\left[I\left(V_{t+1} < \kappa\mathcal{R}_t\left(V_{t+1}\right)\right)\right]}V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}, \tag{5}$$

where $E_t\left[\cdot\right]$ denotes the expectation conditional on all information available at time t. Hansen et al. (2007) derive the stochastic discount factor $M_{t,t+1}$ in terms of the continuation value of utility of consumption as:

$$M_{t,t+1} = M_{t,t+1}^{*}\left(\frac{1+\ell I\left(V_{t+1} < \kappa\mathcal{R}_t\left(V_{t+1}\right)\right)}{1+\ell\kappa^{1-\gamma}E_t\left[I\left(V_{t+1} < \kappa\mathcal{R}_t\left(V_{t+1}\right)\right)\right]}\right), \tag{6}$$

where

$$M_{t,t+1}^{*} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\frac{1}{\psi}-\gamma}. \tag{7}$$

2.2 Generalized Endowment Economy

Let $\mathcal{O}\left(\cdot;\cdot\right)$ and $\mathcal{S}\left(\cdot;\cdot\right)$ be two generic functions describing the observation and state equations in a state-space system. We generally define the dynamics of real aggregate consumption growth $\Delta c_{t+1} = \ln\left(C_{t+1}/C_t\right)$, where C_t denotes the level of aggregate consumption, as:

$$\begin{aligned} \Delta c_{t+1} &= \mathcal{O}\left(\mathbf{U}_{t+1};\mathbf{X}_t\right) \\ \mathbf{X}_{t+1} &= \mathcal{S}\left(\mathbf{U}_{t+1};\mathbf{X}_t\right), \end{aligned} \tag{8}$$

where $\mathbf{X}_t$ is an N-dimensional real-valued vector process governing the state of the real economy, and where $\mathbf{U}_{t+1}$ is a K-dimensional vector of independent and identically distributed shocks with density function $h\left(\mathbf{U}\right)$ and support $\mathcal{U} \subseteq \mathbb{R}^K$.

2.3 Model Solution

We need to find a solution to the welfare valuation ratios V_t/C_t and $\mathcal{R}_t\left(V_{t+1}\right)/C_t$, which define the lifetime utility and the certainty equivalent of future lifetime utility to the current consumption level, respectively. These ratios are functions of the N-dimensional state vector $\mathbf{X}_t$ that governs the real economy. In this section, we explain how to explicitly solve for the welfare valuation ratios, which allow for the derivation of the probability of disappointment and the stochastic discount factor. We relegate technical expressions to Appendix A.

Given a specification of the endowment process as defined in equation (8), we solve for the welfare valuation ratios:

$$\frac{V_t}{C_t} = G^V\left(\mathbf{X}_t\right) \quad \text{and} \quad \frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t} = G^{\mathcal{R}}\left(\mathbf{X}_t\right), \tag{9}$$

which are functions of the N-dimensional state vector $\mathbf{X}_t$. From the recursion in Equation (2), it follows that $G^V = F\left(G^{\mathcal{R}}\right)$, where

$$\begin{aligned} F\left(G\right) &= \left\{(1-\delta) + \delta G^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1, \\ &= G^{\delta} \quad \text{if } \psi = 1 \end{aligned} \tag{10}$$

for any positive real number G. Hence the first step is to derive a solution to $G^{\mathcal{R}}$, the ratio of the certainty equivalent of future lifetime utility to the current consumption level. We explicitly show in Appendix A that $G^{\mathcal{R}}\left(\mathbf{X}_t\right)$ can be solved recursively using numerical integration over the real-valued vector process $\mathbf{X}_t$. Our method is conceptually similar to the one used by Campbell and Cochrane (1999) to solve for the price-consumption ratio in the external habit model, and is referred to as the *fixed-point* method by Wachter (2005).[5]

Specifically, we initiate the recursion by conjecturing a solution to $G_0^{\mathcal{R}}\left(\mathbf{X}_t\right)$. $G_1^{\mathcal{R}}\left(\mathbf{X}_t\right)$

[5] Wachter (2005) compares the advantages of the *fixed point* and *series* methods for the speed of convergence of the solution to the price-consumption ratio. Such a comparison is not possible in our framework as the welfare valuation ratios follow a non-linear recursion, which cannot be expressed as a sum of recursive terms.

is then obtained on a grid of values for $\mathbf{X}_t$. We then iterate forward. At each step of the recursion, we evaluate the function obtained in the previous iteration at a set of points $\mathcal{S}(\mathbf{U}_{t+1}; \mathbf{X}_t)$ for each value of $\mathbf{X}_t$, where $\{\mathbf{U}_{t+1}\}$ is determined by the numerical integration routine. Typically, these points lie outside the predefined grid of values for $\mathbf{X}_t$. Thus, we apply an interpolation method to evaluate $G_k^{\mathcal{R}}$ at these points.[6] The solution to the recursion yields a fixed point for $G^{\mathcal{R}}(\mathbf{X}_t)$, which is unique (See Backus et al. (2004) and Marinacci and Montrucchio (2010)).

Given the solutions of $G^{\mathcal{R}}$, we derive the solution of $G^V = F\left(G^{\mathcal{R}}\right)$. It is then straightforward to compute the disappointment probability $\xi_t \equiv \xi(\mathbf{X}_t)$ and the real stochastic discount factor $M_{t,t+1} \equiv M(\mathbf{X}_t, \mathbf{X}_{t+1}, \Delta c_{t+1})$, which are derived using the solutions to the welfare valuation ratios V_t/C_t and $\mathcal{R}_t(V_{t+1})/C_t$. Detailed expressions are reported in equations (A.7) and (A.8), respectively.

3 Application to the Term Structure of Interest Rates

We solve for the real and nominal yield curves in Subsection 3.1, and specify an explicit model for consumption growth and inflation in Subsection 3.2. Calibrations and data are described in Subsection 3.3. Subsection 3.4 is at the heart of this section, as we provide an in depth discussion of the model solutions and how they compare with existing models of the term structure of nominal interest rates.

[6] One example is the log-linear interpolation method, assuming that $\ln G_k^{\mathcal{R}}(\mathbf{X}_t)$ is linear in $\mathbf{X}_t$. This interpolation method is similar to the one used in Campbell and Cochrane (1999) to numerically solve the habit formation model.

3.1 Asset Prices: Real and Nominal Yield Curves

The price of a real zero-coupon bond paying one unit of consumption n-periods ahead from now must satisfy the Euler equation

$$P_{n,t} = E_t \left[M_{t,t+n} \right]. \tag{11}$$

More generally, we can solve for the term structure of real interest rates recursively, given that the price $P_{n,t} \equiv P_n \left(\mathbf{X}_t \right)$ of the real zero-coupon bond that matures in n periods satisfies the recursion

$$P_n \left(\mathbf{X}_t \right) = E_t \left[M_{t,t+1} P_{n-1} \left(\mathbf{X}_{t+1} \right) \right], \tag{12}$$

with the initial condition $P_{0,t} = 1$. Real bond prices are computed recursively using numerical integration, and we report detailed expressions in Equation (B.2) of Appendix B. Given the solution for real bond prices, real yields to maturity n are defined as:

$$y_{n,t} = -\frac{1}{n} p_{n,t}, \tag{13}$$

where $p_{n,t}$ is the logarithm of the real bond price.

Nominal assets are priced by discounting nominal payoffs with the nominal stochastic discount factor, which is simply the difference between the natural logarithm of the real pricing kernel and the inflation rate π_{t+1}. The logarithm of the nominal stochastic discount factor is thus expressed as:

$$\ln M^{\$}_{t,t+1} = \ln M_{t,t+1} - \pi_{t+1}. \tag{14}$$

Similar to the price of the real zero-coupon bond, the price of a nominal zero-coupon bond

$P^{\$}_{n,t}$ paying one currency unit n-periods ahead from now satisfies the recursion

$$P^{\$}_{n,t} = E_t\left[M^{\$}_{t,t+1} P^{\$}_{n-1,t+1}\right], \tag{15}$$

with initial condition given by $P^{\$}_{0,t} = 1$.

For the analysis of nominal prices, we specify a general process for inflation π_{t+1} that embeds a rich class of affine inflation dynamics specified in Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2013), among others. To derive the solution for nominal bond prices, we assume that the dynamics of the inflation rate process are governed by an L-dimensional real-valued vector process $\mathbf{Y}_t$ such that the joint moment generating function, conditional on the real vector of shocks $\mathbf{U}_{t+1}$, is given by:

$$E_t\left[\exp\left(a\pi_{t+1} + b^{\top}\mathbf{Y}_{t+1}\right) \mid \mathbf{U}_{t+1}\right] = \exp\left(A\left(a, b, \mathbf{X}_t, \mathbf{U}_{t+1}\right) + \mathbf{Y}_t^{\top} B\left(a, b\right)\right), \tag{16}$$

which we use to show that:

$$P^{\$}_{n,t} = P^{\$}_{n}\left(\mathbf{X}_t\right) \exp\left(\mathbf{Y}_t^{\top} B^{\$}_{n}\right), \tag{17}$$

where the coefficients $B^{\$}_{n}$ satisfy the recursion

$$B^{\$}_{n} = B\left(-1, B^{\$}_{n-1}\right), \tag{18}$$

with the initial vector-valued condition $B^{\$}_{0} = \mathbf{0}$. We use the law of iterated expectations to ensure that the numerical integration applies only to the vector of real shocks $\mathbf{U}_t$, which allows us to show that $P^{\$}_{n}\left(\mathbf{X}_t\right)$ satisfies the recursion

$$P^{\$}_{n}\left(\mathbf{X}_t\right) = E_t\left[M_{t,t+1} P^{\$}_{n-1}\left(\mathbf{X}_{t+1}\right) \exp\left(A\left(-1, B^{\$}_{n-1}, \mathbf{X}_t, \mathbf{U}_{t+1}\right)\right)\right], \tag{19}$$

with the initial condition $P_0^{\$}(\mathbf{X}_t) = 1$. Detailed expressions for nominal bond prices are reported in Equation (B.6) of Appendix B. Given the solution for nominal bond prices, nominal yields to maturity n are defined as:

$$y_{n,t}^{\$} = -\frac{1}{n} p_{n,t}^{\$}, \tag{20}$$

where $p_{n,t}^{\$}$ is the natural logarithm of the nominal bond price.

3.2 An Explicit Model for Consumption Growth and Inflation

For an empirical application of our framework, we need to specify an explicit process for the endowment economy and inflation, characterized by Equations (8) and (16), respectively. Solving models of generalized disappointment aversion with continuous endowment dynamics requires numerical solution methods. This implies that solutions to asset prices involve integration over the support of each independent source of risk. This computational complexity makes a parsimonious model attractive and desirable. We thus present a model where the state of the real economy is characterized by a single state variable σ_t, the volatility of aggregate consumption growth. The existence of fluctuations in macroeconomic uncertainty is now a well-established fact (Kandel and Stambaugh (1990) and Stock and Watson (2002)) and its importance for asset prices and the real economy has been demonstrated, among many others, by Bansal et al. (2005), Lettau et al. (2008), Bloom (2009), and Jurado et al. (2015). In addition, we show in the application that such a simple framework is powerful enough to explain rich stylized facts for the term structure of interest rates and return predictability in bond markets. In fact, Bonomo et al. (2011) illustrate that a model with generalized disappointment aversion without persistent fluctuations in the mean of aggregate consumption growth improves empirical return predictability patterns in the stock market over a specification with recursive utility and long run risk in expected consumption growth.

We model real aggregate growth Δc_{t+1} to have a constant mean μ_c and affine GARCH dynamics for the variance process, following Heston and Nandi (2000).[7] Assuming a GARCH instead of, for example, stochastic volatility dynamics, avoids multi-dimensional integration when we compute asset prices numerically. In addition, it guarantees the desirable feature of a positive volatility process. Formally, the real economy is defined as:

$$\begin{aligned} \Delta c_{t+1} &= \mathcal{O}\left(u_{t+1};\sigma_t^2\right) = \mu_c + \sigma_t u_{t+1} \\ \sigma_{t+1}^2 &= \mathcal{S}\left(u_{t+1};\sigma_t^2\right) = \left(1-\phi_\sigma\right)\mu_\sigma - \nu_\sigma + \left(\phi_\sigma - \nu_\sigma\beta_\sigma^2\right)\sigma_t^2 + \nu_\sigma\left(u_{t+1} - \beta_\sigma\sigma_t\right)^2, \end{aligned} \tag{21}$$

where μ_σ denotes the unconditional mean volatility, $0 < \phi_\sigma < 1$ modulates the persistence of macroeconomic uncertainty, β_σ determines the correlation between consumption growth and innovations in consumption volatility, and ν_σ defines the sensitivity of growth rates to consumption shocks u_{t+1}. The unconditional variance is given by:

$$\sigma_\sigma^2 = \frac{2\nu_\sigma^2\left(1 + 2\beta_\sigma^2\mu_\sigma\right)}{1-\phi_\sigma^2}, \tag{22}$$

and the volatility process is well behaved if we impose the restrictions $\left(1-\phi_\sigma\right)\mu_\sigma - \nu_\sigma \geq 0$ and $\phi_\sigma - \nu_\sigma\beta_\sigma^2 \geq 0$. Given μ_σ, $0 < \phi_\sigma < 1$ and σ_σ, the two non-negativity constraints imply that the volatility of volatility ν_σ and the leverage coefficient β_σ are given by:

$$\nu_\sigma = \sqrt{\frac{\left(1-\phi_\sigma^2\right)\sigma_\sigma^2}{2\left(1+2\beta_\sigma^2\mu_\sigma\right)}} \quad \text{and} \quad \beta_\sigma^{\min} \leq |\beta_\sigma| \leq \beta_\sigma^{\max},$$

[7] Other consumption growth volatility dynamics, such as an exponential GARCH of Nelson (1991), would be consistent with our general framework. The solution for other dynamics will also remain bound to a single integration as long as the consumption growth dynamics are driven by a single shock. Such dynamics have recently been applied in Tédongap (2015) to study the implications for the cross-section of equity returns in a consumption-based equilibrium framework.

where

$$
\begin{aligned}
\beta_\sigma^{\min} &= \sqrt{\max\left(0, \frac{1}{2\mu_\sigma}\left(\frac{1}{2}\frac{1+\phi_\sigma}{1-\phi_\sigma}\frac{\sigma_\sigma^2}{\mu_\sigma^2} - 1\right)\right)} \\
\beta_\sigma^{\max} &= \sqrt{\frac{2\phi_\sigma^2\mu_\sigma + \sqrt{2\phi_\sigma^2\left(2\phi_\sigma^2\mu_\sigma^2 + \left(1-\phi_\sigma^2\right)\sigma_\sigma^2\right)}}{\left(1-\phi_\sigma^2\right)\sigma_\sigma^2}}.
\end{aligned}
$$

If $\beta_\sigma > 0$, then the two innovations are negatively correlated; they are positively correlated if $\beta_\sigma < 0$, and they are uncorrelated if $\beta_\sigma = 0$. The calibration of the model therefore requires a choice on the value of β_σ.[8]

For the analysis of nominal prices, we also need to specify an explicit process for inflation. More precisely, we assume that growth rates in prices have a time-varying mean and volatility. The dynamics of inflation π_{t+1} depend on the state vector $\mathbf{Y}_{t+1} = \left[z_{t+1}, v_{t+1}\right]^\top$. $\mathbf{Y}_{t+1}$ has two state variables, expected inflation, z_{t+1}, and price growth uncertainty, v_{t+1}, which impacts both expected and realized inflation, as described in the following system of equations:

$$
\begin{aligned}
\pi_{t+1} &= \mu_\pi + z_t + \left(\nu_\pi\sigma_t u_{t+1} + \sqrt{v_t}\varepsilon_{t+1}\right) \\
z_{t+1} &= \phi_z z_t + \nu_z\left(\nu_\pi\sigma_t u_{t+1} + \sqrt{v_t}\varepsilon_{t+1}\right), \\
v_{t+1} &= \left(1-\phi_v\right)\mu_v - \nu_v + \left(\phi_v - \nu_v\beta_v^2\right)v_t + \nu_v\left(\varepsilon_{t+1} - \beta_v\sqrt{v_t}\right)^2,
\end{aligned}
\tag{23}
$$

where ε_{t+1} is an independent and identically distributed standard normal shock, orthogonal to the shocks in consumption growth u_{t+1}.[9] Such a specification implies that innovations in expected and realized inflation are perfectly positively correlated. Shocks to aggregate consumption growth are thus allowed to impact expected and realized inflation. The parameter μ_π denotes the average inflation rate, $0 < \phi_z < 1$ modulates the persistence of expected price growth, ν_z determines the level of the expected inflation shock volatility, and $\nu_\pi < 0$

[8]The leverage coefficient introduces skewness in multi-period consumption growth rates, even though there is no skewness at the monthly horizon.

[9]We specify an exogenous price growth process similar to the model for inflation in Wachter (2005) or Piazzesi and Schneider (2006). The affine dynamics for inflation uncertainty ensure that the numerical solution remains restricted to a one-dimensional integration.

determines how uncertainty about real growth affects both realized and expected inflation. A negative value for ν_π imposes a negative correlation between innovations in consumption growth and realized and future inflation. The volatility process v_t is assumed to be the residual component of inflation volatility orthogonal to consumption volatility.[10] The persistence of inflation uncertainty is parametrized through ϕ_v, μ_v defines the average level of inflation volatility and ν_v governs the residual volatility. The parameter β_v is a leverage coefficient, governing the correlation between inflation volatility innovations and both expected and realized price growth. The leverage parameter introduces skewness in low-frequency price growth, even though inflation has zero skewness at the single-period (monthly) horizon. Thus quarterly and yearly inflation rates are skewed. We note that these dynamics can be mapped into the general framework defined in Equation (16) as follows:

$$
\begin{aligned}
A\left(a, b, \sigma_t^2, u_{t+1}\right) &= a\mu_\pi + b_2\left(\left(1-\phi_v\right)\mu_v - \nu_v\right) - \frac{1}{2}\ln\left(1-2b_2\nu_v\right) + \left(a + b_1\nu_z\right)\nu_\pi\sigma_t u_{t+1} \\
B_1\left(a, b\right) &= a + b_1\nu_z \\
B_2\left(a, b\right) &= b_2\phi_v + \frac{\left(a + b_1\nu_z - 2b_2\nu_v\beta_v\right)^2}{2\left(1-2b_2\nu_v\right)}.
\end{aligned}
\tag{24}
$$

Given the explicit dynamics for aggregate consumption growth, the price of the real zero-coupon bond depends on a single state variable, the volatility of aggregate consumption growth. In contrast to real bonds, the price of a nominal zero-coupon bond $P_{n,t}^{\$}$ paying one currency unit n-periods ahead from now is a function of more than one state variable. In addition to macroeconomic uncertainty, nominal bond prices also depend on expected inflation and inflation uncertainty. Using the law of iterated expectations and conditioning on the realizations of innovations to consumption growth, we keep the numerical solution restricted to a one-dimensional integration. Given the assumption for the inflation dynamics,

[10] The residual inflation volatility dynamics follow an analogous GARCH recursion with similar parameter restrictions to the process for the volatility of aggregate consumption growth.

we can show that:

$$P^{\$}_{n,t} = P^{\$}_{n}\left(\sigma_t^2\right)\exp\left(B^{\$}_{z,n} z_t + B^{\$}_{v,n} v_t\right), \tag{25}$$

where the coefficients $B^{\$}_{z,n}$ and $B^{\$}_{v,n}$ satisfy the recursions

$$\begin{aligned} B^{\$}_{z,n} &= \phi_z B^{\$}_{z,n-1} - 1 \\ B^{\$}_{v,n} &= \phi_v B^{\$}_{v,n-1} + \frac{\left(\nu_z B^{\$}_{z,n-1} - 2\nu_v \beta_v B^{\$}_{v,n-1} - 1\right)^2}{2\left(1 - 2\nu_v B^{\$}_{v,n-1}\right)}, \end{aligned} \tag{26}$$

with the initial conditions $B^{\$}_{z,0} = 0$ and $B^{\$}_{v,0} = 0$. The sequence $\left\{P^{\$}_n\left(\sigma_t^2\right)\right\}$ satisfies the recursion

$$P^{\$}_n\left(\sigma_t^2\right) = E_t\left[M_{t,t+1} P^{\$}_{n-1}\left(\sigma_{t+1}^2\right)\exp\left(A\left(-1, B^{\$}_{n-1}, \sigma_t^2, u_{t+1}\right)\right)\right], \tag{27}$$

with the initial condition $P^{\$}_0\left(\sigma_t^2\right) = 1$, and where $B^{\$}_{n-1}$ denotes the two-dimensional vector with components $B^{\$}_{z,n-1}$ and $B^{\$}_{v,n-1}$. The recursion in Equation (27) has no closed-form solution and is solved by one-dimensional numerical integration over a grid of values for σ_t^2.

3.3 Calibration and Data

The calibration of the model is summarized in Table 1 and is consistent with standard calibrations in long run risk models, such as Bansal et al. (2012) or Bonomo et al. (2011). We calibrate the affine GARCH dynamics in Equation (21) at the monthly decision interval to match the first and second moment of real annual U.S. consumption growth from 1929 to 2011. The mean of consumption growth is calibrated to $\mu_c = 0.0015$. The unconditional volatility of consumption growth, which is equal to $\sqrt{\mu_\sigma}$, is defined to be $\sqrt{\mu_\sigma} = 0.7305 \times 10^{-2}$. We set the persistence and the volatility of consumption volatility to $\phi_\sigma = 0.995$ and $\sigma_\sigma = 0.6263 \times 10^{-4}$. Given μ_σ, ϕ_σ and σ_σ, we choose $\beta_\sigma = \beta_\sigma^{\min}$ and ν_σ is defined in terms of the other parameter values.

The mean level of inflation μ_π is equal to 0.0030 and the inflation leverage on news ν_π

is -0.1294, implying that realized and expected inflation are negatively correlated with innovations in consumption growth. The negative correlation between consumption growth and both contemporaneous and expected inflation is an important feature of the data (Piazzesi and Schneider; 2006), and is important to generate an upward term structure of nominal yields. Recursive utility is helpful in generating greater risk premia for long-maturity bonds, as the negative correlation between innovations in consumption growth and expected inflation imply that that the real payoffs from nominal bonds are low when consumption growth is low. This effectively means that nominal bonds do not provide a good hedge against period of low consumption growth, and, as a result, investors demand greater risk premia for holding long-term bonds.

Expected inflation is highly persistent with a value of $\phi_z = 0.9840$ and the level of expected inflation volatility shock is $\nu_z = 0.3457$. Mean uncertainty in inflation is equal to $\mu_v = 6.3698 \times 10^{-7}$ and the persistence of inflation volatility is $\phi_v = 0.85$. Finally, the level of residual inflation volatility is $\nu_v = 9.5546 \times 10^{-8}$ and the leverage coefficient is given by $\beta_v = -2.9827 \times 10^{+3}$. Thus, inflation volatility is positively correlated with expected and realized price growth, which also implies positive low-frequency inflation skewness.

Regarding preferences, we calibrate the intertemporal elasticity of substitution ψ at 1.5 and the constant relative risk aversion parameter γ at 2. This parameter configuration implies a preference for early resolution of uncertainty, as is suggested empirically by the estimations in Bansal and Shaliastovich (2013) and Augustin and Tédongap (2016), among others. The disappointment aversion parameter ℓ is fixed at 1 and we set the threshold of the certainty equivalent below which outcomes become disappointing, κ, equal to 0.95. Thus, in our benchmark calibration, any outcome that is more than 5% below the certainty equivalent will be considered disappointing and will lead to a reshuffling of the state price probabilities. The subjective discount factor δ is equal to 0.9985.

We define a benchmark grid of 501 points for σ_t^2, and we later evaluate the numerical precision of the solutions for alternative grid scenarios.[11] The grid is defined in terms of the natural logarithm of σ_t^2, which has approximately a mean of $\mu_h = \ln \mu_\sigma - \sigma_h^2/2$ and a standard deviation of $\sigma_h = \ln(1 + \sigma_\sigma^2/\mu_\sigma^2)$.[12] We use 334 logarithmically spaced points between $\mu_h - 7\sigma_h$ and μ_h, and 167 logarithmically spaced points between μ_h and $\mu_h + 5\sigma_h$. The lower segment is finer and includes values for volatility that are much closer to zero. This allows to better capture the non-linear behavior of the welfare valuation ratio as volatility approaches zero. Given the assumed dynamics, more than 99.9% of the population distribution of $\ln \sigma_t^2$ lies between seven standard deviations below and five standard deviations above the mean.

To evaluate the implications of the model, we simulate a time series of 300,000 months of data and compare the population moments to the sample data.[13] We use real data sampled at an annual frequency over the period 1929 to 2011. Data for consumption and price growth are taken from the Bureau of Economic Analysis National Income and Product Accounts Tables. To compare the model's solutions to the term structure of nominal interest rates, we use monthly Fama-Bliss discount bond prices from the CRSP U.S. Treasury Database from January 1964 through December 2011. Fama-Bliss Discount Bonds Files contain artificial discount bonds with 1 to 5 years to maturity, constructed after first extracting the term structure from a filtered subset of the available bonds. This database is a refinement of the one used in Fama and Bliss (1987).

3.4 Model Solutions and Analysis

We first discuss the model implications for real and nominal growth. Next, we discuss the numerical solution to the welfare valuation ratios. We then report the results for the term

[11] To solve the integrals, we choose the adaptive Simpson quadrature method as the numerical integration routine and force the integral to be bounded by -8 and +8 standard deviations.

[12] More precisely, μ_h and σ_h are approximate solutions to the mean and standard deviation of the logarithm of σ_t^2 when we apply a log-normal approximation to the volatility dynamics and if we ignore skewness. We emphasize that we obtain a grid for σ_t^2 by taking the exponent of the grid on $\ln \sigma_t^2$ element-by-element.

[13] A simulation of 300,000 observations is equivalent to the simulation of 100,000 quarters in Wachter (2005). In all simulations, we use a burn-in period of equal size, i.e. 300,000 months of data.

structure of nominal interest rates. We end with a discussion of the model's mechanism and compare it to existing macrofinance models of the term structure of interest rates.

3.4.1 Real and Nominal Growth

In Table 2, we compare the time-averaged annualized moments, computed from a simulated time series of 300,000 monthly observations, to the sample moments in the data, estimated using annual data over the period 1929 to 2011. Focusing first on the dynamics of consumption growth, the left panel shows a close fit between the (statistically significant) estimated moments and the model-implied unconditional moments. The mean growth rate is 1.97% in the data, while it is 1.79% in the model. Similarly, the comparison of the volatilities of consumption growth indicates 2.02% in the data versus a model-implied value of 2.08%. We obtain an annualized first-order auto-correlation of 0.24, which is a bit lower than the estimated value of 0.48. This is expected, however, as we specify monthly consumption growth to be unpredictable.[14] Finally, we also obtain a reasonable fit for the skewness and kurtosis of the aggregate consumption growth dynamics.[15] Overall, the dynamics we have chosen for our empirical application closely reflect the distribution of aggregate consumption growth, as all values are within two standard deviations of the sample estimates.

In the right-hand panel in Table 2, we compare the model-implied population values of the inflation process to the data estimates. Expected inflation is 3.17% (3.57%) in the sample (model), the annualized volatility is 3.29% (2.89%), the first-order auto-correlation coefficient is 0.83 (0.86), and the kurtosis is 8.64 (6.15). The skewness of inflation is estimated negatively at -0.80, although the estimate is not statistically different from zero. The model-implied value for inflation skewness is 1.34. This result arises because of the negative leverage parameter β_v in the inflation volatility dynamics, and is consistent with those authors who

[14]We could model the conditional mean of aggregate consumption growth as a function of consumption volatility. This would be one way to increase the auto-correlation coefficient implied by the model.

[15]The time aggregation introduces skewness at the annual horizon because of the leverage effect, even though the non-predictable consumption growth dynamics have zero skewness at the monthly horizon by construction.

argue for positive skewness in inflation. See, for example, Aizenman and Hausmann (1994), Chaudhuri et al. (2013), and references therein.

3.4.2 Welfare Valuation Ratios

We next discuss the numerical solution to the utility-consumption ratio. A solution to this welfare valuation ratio is the primary input to solutions for the stochastic discount factor, and therefore for asset prices. In Figure 1a, we plot the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility σ_t^2 for our benchmark scenario with 501 grid points. The negative slope suggests that the ratio of utility to the level of consumption is decreasing for higher levels of consumption volatility. This is consistent with the view that agents dislike macroeconomic uncertainty. To shed some light on the robustness of the numerical solution, we evaluate the solution to the welfare valuation ratio for different grids. We specify different densities ranging from the coarsest grid with 24 points to the finest grid with 750 points. We plot in Figure 1b the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility for each of these grids. There is a significant difference in the results between the solution derived from the coarsest grid with 24 points and the one with 75 points. The difference in solutions based on the grids with 75 and 123 points is substantially smaller. There is hardly any improvement for the solution using 750 grid points over the solution using a grid of 498 points. This suggests that the solution is accurate and that increasing the number of grid points beyond 501 points is unnecessary.

Another statistic of interest in models with (generalized) disappointment aversion is the disappointment probability. In Figure 1c, we report the probability of disappointment $\xi(\sigma_t^2)$ as a function of consumption volatility σ_t^2. Without macroeconomic uncertainty, it is unlikely to be disappointed (i.e., probability close to zero percent) as there is little probability of falling below the certainty equivalent threshold. As consumption volatility increases, the disappointment probability increases as well. It equals approximately 8% when the level of the monthly consumption growth volatility equals $\sigma_t^2 = 0.01$.

We further evaluate the precision of the numerical solution by focusing on Epstein and Zin (1989) recursive utility without disappointment version. For that model, we can derive closed-form solutions and compare them to numerical solutions obtained using the same method described above. The Epstein and Zin (1989) model is nested in the model with disappointment and easily obtained by setting the disappointment intensity to zero, i.e., $\ell = 0$, and by relying on the Campbell and Shiller (1988a) log-linearization of returns. Details of the analytical solutions are reported in Appendix C. In Figure 2, we plot the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility σ_t^2 for both the analytical (dotted line) and the numerical (solid) solution for our benchmark scenario with 501 grid points. The consumption growth parameters are identical to those reported in Table 1. The preference parameters for this example are $\delta = 0.9989$, $\psi = 1$, and $\gamma = 4$. The graph visually illustrates that the numerical solution is accurate. There is a slight discrepancy between the two lines. This is expected, as the analytical solution relies on a log-linear approximation to the wealth-consumption ratio, while the numerical solution solves the model without any approximation. Quantitatively, the relative root mean squared error is equal to 1.56%, with a maximum of 1.59% when the volatility is close to zero.

3.4.3 The Term Structure of Real and Nominal Interest Rates

We plot in Figure 3a the real yields $y_t^{(n)}$ as a function of consumption volatility for maturities one ($n = 1$) to five ($n = 5$) years. The model implies a downward sloping term structure of real interest rates. This is consistent with the intuition that inflation-indexed bonds represent a valuable hedge for long-term investors, which are willing to pay a premium to hold such assets (Campbell et al.; 2009). Payoffs of real bonds are fixed in consumption units, which are more highly valued when macroeconomic uncertainty is high. As real bond returns are negatively correlated with the level of consumption and stock prices, they command a negative risk premium that increases with the asset horizon. This channel is particularly true if shocks to macroeconomic uncertainty are persistent. In that case, positive innovations in economic uncertainty can lead to extended periods of slow growth, which increases real bond prices in recessions. A negative slope of the real yield curve is also consistent with a negative term structure of real interest rates found in the long-term U.K. data (Evans; 1998). For very high levels of macroeconomic uncertainty, the slope of the term structure of real interest rates becomes slightly more negative, as can be seen through the wider dispersion in the lines as we move closer to the right of the figure. At very high levels of consumption volatility (and for high asset horizons), real yields become negative. This is a common feature of recursive utility models with long run risks when the parameter calibration implies a preference for early resolution of uncertainty (Bansal and Shaliastovich; 2013).

We next turn to the term structure of nominal interest rates. In Table 3, we report the model-implied term structure of nominal yields and the corresponding volatilities from the simulation with 300,000 months of data, corresponding to 25,000 years. The one-year and the five-year nominal yields is 5.26% and 5.95%, compared to the values of 5.20% and 5.83% in the data. Thus, we match both the level and the slope of the term structure well. The volatility of nominal bond yields is a bit lower than in the data, but we match the downward sloping pattern well.

3.4.4 A Discussion of the Model Mechanism

Equation (25) highlights that nominal bond prices are non-linear functions of three sources of risk: macroeconomic uncertainty, expected inflation, and inflation uncertainty. To sharpen the intuition about the model's mechanism, we first plot in Figures 3b and 3c the sensitivities of nominal bond yields $y_t^{(n)}$ to expected inflation $(-B_{z,n}^{\$}/n)$ and inflation volatility $(-B_{v,n}^{\$}/n)$, as these maturity-dependent coefficients are known in closed form.[16] The loading of nominal bond yields to expected inflation is positive, implying that high expected inflation raises risk premia and increases nominal yield spreads. Nominal bond payoffs are fixed in terms of price levels, and therefore they pay off when expectations about future price growth are

[16]Since $y_{n,t}^{\$} = -\frac{1}{n}p_{n,t}^{\$}$, we have that $y_{n,t}^{\$} = -\frac{1}{n}p_n^{\$}\left(\sigma_t^2\right) - \frac{1}{n}B_{z,n}^{\$}z_t - \frac{1}{n}B_{v,n}^{\$}v_t$.

high. Innovations in consumption growth are negatively correlated with both realized and expected inflation. In other words, high expected inflation reflects a negative innovation to consumption growth. Thus, investors will issue bonds to borrow from future consumption. This depresses nominal bond prices and raises nominal yields. Since the loadings $-B^{\$}_{z,n}/n$ are negative functions of the asset horizon, short-run yields are comparatively more sensitive to expected inflation shocks than longer-term yields.

The loadings of nominal bond yields to inflation uncertainty are negative at all maturities. Thus, inflation volatility lowers nominal bond risk premia, which reflects a flight-to-quality effect across all asset horizons. In times of high nominal uncertainty, investors develop a precautionary savings motive. This leads to them to buy nominal bonds, which raises their prices and lowers nominal yields. Given that $-B^{\$}_{v,n}/n$ is negative and has a negative slope too, the reduction in risk premia is comparatively greater at longer horizons than at short-term maturities. Expected inflation and inflation uncertainty thus have opposing effects on the term structure of nominal interest rates.

The third source of risk that impacts nominal bond yields is real uncertainty. Since all three risk factors enter Equation (25) in a non-linear way, we cannot study the sensitivity of nominal bond yields to real uncertainty independently from expected price growth and inflation uncertainty. To investigate the sensitivity of nominal bond yields to consumption volatility, we thus fix the values of expected inflation and inflation uncertainty at their long run average values. Conditional on these values, we plot in Figure 4 nominal bond yields for maturities of one to five years as a function of consumption volatility. All nominal yields are lower for higher levels of real uncertainty. This reflects a flight-to-quality effect, whereby bond prices (yields) respond positively (negatively) to macroeconomic uncertainty. Figure 4 also illustrates that long-term nominal yields are more sensitive to economic uncertainty than short-term nominal yields. This is characterized by the differences in the steepness of each maturity line plot.

To provide some perspective, we compare our results to those in Bansal and Shaliastovich

(2013). In our framework, nominal yields respond negatively to inflation and real uncertainty at all maturities. Thus, uncertainty induces a flight-to-quality effect, which dominates the intertemporal substitution motive. This effect is opposite to the one described in Bansal and Shaliastovich (2013). Our implications are, however, not directly comparable to theirs for several reasons. First, they specify a model in which real and nominal uncertainties affect expected real and nominal growth, respectively. The conditional volatilities of realized consumption growth and inflation are constant, while they are time-varying in our set-up. Second, the authors allow for non-neutrality of inflation. Hence, expected inflation negatively affects future growth. This specification is supported by a negatively estimated relationship in the data, although the coefficient is statistically insignificant. Our model features inflation-neutrality as in Wachter (2005) and Piazzesi and Schneider (2006), among many others, but we allow innovations in consumption growth to affect both realized and expected inflation.[17] Understanding how these different models differentially affect the intertemporal substitution and precautionary savings motives through the nominal and real uncertainty channels is an interesting question to pursue in future research.

In Table 4, we illustrate the similarities and differences between the main preference-based models for the term structure of nominal interest rates and our framework. Piazzesi and Schneider (2006) examine the nominal term structure of interest rates using EZ recursive preferences, homoscedastic consumption growth and inflation. The authors focus on a special case with $\psi = 1$, which allows to generate closed-form solutions, patient investors, $\delta > 1$, and large coefficients of relative risk aversion γ above 40 (59 in the benchmark case). While the authors match the moments of an upward (downward) sloping nominal (real) yield curve, they do not investigate any implications for return predictability and do not verify whether their model reconciles the failure of the expectations hypothesis. Their focus is on the relation between surprise inflation and future consumption growth. The authors emphasize

[17] The working paper version of Bansal and Shaliastovich (2013) featured inflation neutrality and sensitivity of expected and realized inflation to innovations in consumption growth.

the importance of this relation being negative, which we enforce in our calibration. In addition, the term structure of real yields is flat (constant), which is not supported by the data.

Gabaix (2012) develops a variable rare disaster model with time separable expected utility and both a real and a nominal disaster shock, each of which has both time-varying intensity and probability of occurrence. That model generates an upward sloping nominal term structure of interest rates and reconciles the failure of the expectations hypothesis using both the CP and CS regressions. However, the model produces counterfactual implications for the behavior of risk premia across assets (stocks and bonds) and for cross-asset predictability (prediction of bond risk premia by the PD ratio, and prediction of long-horizon equity excess returns by the CP factor). In particular, nominal assets (bonds) are driven by the inflation disaster shock, while equity is driven by a real disaster shock. Thus, they are independent, something that is difficult to reconcile with the evidence on time variation in stock and bond risk premia. Tsai (2016) extends the work of Gabaix (2012) and builds a recursive utility model with variable rare disasters, focusing on the special case with unitary EIS ($\psi = 1$) in order to yield closed-form solutions. Nominal bond prices are driven by three state variables that drive risk premia: expected inflation, real disaster shocks and inflation disaster shocks. The author also stresses the importance of a negative relation between consumption growth and expected inflation for matching an upward sloping nominal yield curve, similar to Piazzesi and Schneider (2006). The time variation in the disaster probabilities introduces time variation in risk premia, and the model generates opposite effects on the long end and the short end of the yield curve. At the short end, the risk-free rate effect dominates, while at the long-end, nominal prices and risk premium effects dominate. This is conceptually similar to the flight to quality effect from consumption volatility on the yield curve that we document in the framework with generalized disappointment aversion. Long-term and short-term nominal yields have different sensitivities to consumption volatility. As a result, the nominal yield curve may be both upward and downward sloping (see Figure 2).

There are other models that study the term structure of nominal interest rates or predictability puzzles in bond markets. However, these models are not directly comparable, as they consider (i) the effect of heterogeneous agents, (ii) a production economy in which the endowments arise endogenously, and (iii) they postulate a monetary economy or a full-fletched DSGE model with capital and labor (Buraschi and Jiltsov; 2005, 2007; Ehling et al.; 2018). What all of these models have in common is that they rely on symmetry in preferences, which is inconsistent with the experimental evidence on decision theory. We find it important to understand whether micro-founded macroeconomic models calibrated to be consistent with asymmetric preferences can match asset prices in the data. For instance, Delikouras (2014) refers to Choi et al. (2007) in the context of portfolio choice problems, who find disappointment aversion coefficients that range from 0 to 1.876 with a mean of 0.390, and second-order risk aversion parameters that range from -0.952 to 2.87 with a mean of 1.448. Similarly, using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients ranging from 1.260 to 2.070. See also Artstein-Avidan and Dillenberger (2010) for experimental evidence on disappointment aversion.

In addition, disappointment aversion endogenously produces a time-varying market price of risk, which helps to increase the volatility of the pricing kernel (and therefore helps to explain the failure of the expectations hypothesis), without introducing too much predictability of future consumption growth through long run risk in expected growth. There are other ways of *exogenously* imposing a time-varying market price of risk (like learning, or preference shocks), while this feature arises endogenously through preferences with disappointment aversion. Having both a time-varying market price and quantities of risk is an important feature in order to be consistent with the behavior and shape of bond risk premia implied by Gaussian affine term structure models (ATSM) (Creal and Wu; 2016).

To summarize, in a simple economy with three sources of risk – expected inflation and both real and inflation uncertainty – and preferences that include disappointment aversion, we can generate an upward sloping term structure of nominal bond yields. Risk premia rise in response to shocks to expected inflation and decrease in response to shocks to inflation and economic uncertainty. Thus, the flight-to-quality effect dominates the intertemporal substitution effect for the response of nominal yields to a rise in real and nominal uncertainty. We generate an upward sloping term structure of nominal bond yields without shocks to expected growth, nor do we rely on inflation non-neutrality as in Bansal and Shaliastovich (2013). At the same time, we generate countercyclical real interest rates, in contrast to an upward sloping term structure of real interest rates as in Wachter (2005). The success of the model is partly due to the ability of the GDA preferences to generate variation in the pricing kernel that is much larger than what is obtained in standard recursive utility with symmetric preferences. To see this, we use Equation (6) to express the conditional variance of the log stochastic discount factor, $m_{t,t+1} = \ln M_{t,t+1}$, as:

$$\begin{aligned} Var_t[m_{t,t+1}] &= Var_t[m^*_{t,t+1}] + (\ln(1+\ell))^2 \, Var_t[I(V_{t+1} < \kappa \mathcal{R}_t(V_{t+1}))] \\ &\quad + 2\ln(1+\ell)\, Cov_t[m^*_{t,t+1}, I(V_{t+1} < \kappa \mathcal{R}_t(V_{t+1}))], \end{aligned} \tag{28}$$

where $Var_t[m^*_{t,t+1}]$ represents the conditional variance of the pricing kernel with Epstein and Zin (1989) recursive utility. The two additional terms are strictly non-negative. As a consequence, the conditional variance of the log pricing kernel with GDA preferences is always at least as large as that of an equivalent pricing kernel without GDA. This feature is a useful ingredient to solve asset pricing puzzles such as the equity premium and the risk-free rate puzzles of Mehra and Prescott (1985) and Weil (1989), respectively.[18] Another merit of the framework with asymmetric preferences is that state probabilities are reshuffled if an outcome is below a fraction κ of the certainty equivalent. Through this mechanism, the model endogenously generates effective countercyclical risk aversion. This property is useful to explain stylized predictability patterns. In that regard, we exploit in the next section the simulated time series of nominal bond yields to show that the model also accounts for the

[18]See Bonomo et al. (2011) for an application of GDA preferences to the equity and risk-free rate puzzles.

failure of the expectations hypothesis.

4 Predictability

We next show how the model rationalizes the failure of the expectations hypothesis. The empirical failure of the expectations hypothesis of interest rates, documented by Fama and Bliss (1987) and Campbell and Shiller (1991), has motivated the development of economic models seeking to explain the economic mechanism at work. Backus et al. (1989) show that the standard Consumption Capital Asset Pricing Model (CCAPM) with power utility cannot account for the anomaly. Recent theoretical explanations that have been suggested use the long run risk framework of Bansal and Yaron (2004) or the external habit set up of Campbell and Cochrane (1999). Wachter (2006) uses external habits with countercyclical interest rates to generate an upward sloping term structure of nominal bonds and predictability in bond returns. An alternative set up with habit preferences has been suggested by Le et al. (2010). In contrast, Bansal and Shaliastovich (2013) suggest that time-varying expected growth rates, expected inflation, and real and inflation uncertainty, together with Epstein and Zin (1989) recursive preferences and preference for early resolution of uncertainty, yield time-varying bond risk premia and an upward sloping term structure of interest rates.[19] Our model, which generates an upward sloping term structure of nominal bonds, is also able to reproduce predictability patterns in bond returns that are quantitatively close to the standard tests of the expectations hypothesis. We achieve these results without predictability in consumption growth.

[19] While these are the most recent articles on the topic, other relevant references are Bekaert et al. (1997), Longstaff (2000), Bekaert and Hodrick (2001), Bansal and Zhou (2002), Dai and Singleton (2002) and Buraschi and Jiltsov (2005).

4.1 The Expectations Hypothesis

The expectations hypothesis predicts that excess returns on bonds are unpredictable and that risk premia are constant. Various versions of the theory have been tested empirically. All yield the same conclusion that there is significant evidence of predictability in bond returns, and that risk premia are time-varying. We evaluate the success of our model by its ability to reproduce the multiplicity of empirical regression results suggested over the last decade. To be more specific, Fama and Bliss (1987) project holding period returns on the corresponding forward-spot spread:[20]

$$rx_{t+12}^{(n)} = \alpha^{(n)} + \beta^{(n)} \left(f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+12}^{(n)}, \tag{29}$$

where $rx_{t+12}^{(n)}$ indicates the annual excess log return of a n-year bond over the one-year yield $y_t^{(1)}$ defined as $rx_{t+12}^{(n)} = r_{t+12}^{(n)} - y_t^{(1)}$, with the return given by the difference in log prices, that is $r_{t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}$. The forward spread $f_t^{(n)}$ for a loan between time $t + 12(n-1)$ and time $t + 12n$ is defined as $f_t^{(n)} = p_t^{(n-1)} - p_t^{(n)}$. Alternatively, Campbell and Shiller (1991) regress changes in long yields on the yield spread as follows:

$$y_{t+12}^{(n-1)} - y_t^{(n)} = \alpha^{(n)} + \beta^{(n)} \frac{1}{n-1} \left(y_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+12}^{(n)}. \tag{30}$$

All these regressions predict a slope coefficient of one, an outcome that is strongly rejected in the data. Dai and Singleton (2002) suggest that adding the bond risk premium to the left-hand side of the regression can restore the unity regression coefficient. Thus, a model that is able to bring the slope coefficient closer to its predicted value should help resolve the expectations hypothesis puzzle. On that account, we also test our model on their adjusted

[20] All t subscripts refer to a monthly sampling frequency, while all n superscripts refer to the bond maturity in years.

regression defined as:

$$y_{t+12}^{(n-1)} - y_t^{(n)} + \frac{1}{n-1}\hat{E}_t\left[r_{t+12}^{(n)} - y_t^{(1)}\right] = \alpha^{(n)} + \beta^{(n)}\frac{1}{n-1}\left(y_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+12}^{(n)}, \tag{31}$$

where $\hat{E}_t\left[r_{t+12}^{(n)} - y_t^{(1)}\right]$ defines the risk premium component of nominal bond yields. Our last test is based on the results of Cochrane and Piazzesi (2005), who show that a single factor projection based on one-year to five-year forward rates captures a significant variation in bond returns. The single tent-shaped factor is obtained in a two-step estimation procedure, whereby first the average of one-year excess bond returns of two to five years to maturity are regressed on one to five year forward rates, $\frac{1}{4}\sum_{n=2}^{5} rx_{t+12}^{(n)} = \gamma_0^{(n)} + \gamma_1^{(n)} y_t^{(1)} + \gamma_2^{(n)} f_t^{(2)} + ... + \gamma_5^{(n)} f_t^{(5)} + \varepsilon_{t+12}^{(n)}$. The predicted single bond factor $CP_t = \hat{rx}_{t+12}$ is then used in a second step to forecast excess bond returns at each maturity from two to five years:

$$rx_{t+12}^{(n)} = \beta^{(n)} CP_t + \varepsilon_{t+12}^{(n)}. \tag{32}$$

4.2 Results

We report in Table 5 the results of the model-implied predictability regressions and compare them to the data. We start with the description of the Cochrane-Piazzesi factor restricted single factor regressions in Panel A of Table 5. There is a close match between both the regression coefficients and the R^2 of the regressions. The model-implied slope coefficients range from 0.42 to 1.56 for the two- and five-year maturity, compared with a range of 0.46 to 1.44 in the data. The model-implied standard errors are significantly smaller, which is due to the regressions with 300,000 observations; the observed sample has 564 data points. In addition, over the sample period 1964 to 2011, the single factor forecasts excess bond returns with an R^2 statistic of 21%. Predictability increases up to 4 years with an R^2 of 26% and flattens out a bit with a value of 24% for 5-year bonds. The model statistics range from 19% at the 2-year horizon to 20% at the 5-year maturity.

In Panels B, C, and D of Table 5, we report the model-implied results for the Fama and Bliss (1987) regressions of changes in short rates on forward-spot spreads, the Campbell and Shiller (1991) and the Dai and Singleton (2002) regressions, respectively. We compare all model-implied results to the observed counterparts, estimated in the data using annual prices from 1964 to 2011. Overall, we confirm the previous evidence of predictability of excess bond returns by bond yields. The implied statistics are quantitatively close to the data counterparts, both in terms of regression coefficients and explanatory power. In the Campbell and Shiller (1991) regressions, the beta coefficients for longer maturities are less negative than in the data. In unreported results, we have verified that this is easily adjustable by increasing the persistence of expected inflation. For the adjusted Dai-Singleton regressions, the beta coefficients are indistinguishable from one. This suggests that the model generates a sizable time-varying risk premium in bond returns that helps explain the failure of the expectations hypothesis for nominal bonds.

5 Conclusion

This paper provides a general framework for preference-based models with generalized disappointment aversion when the economy is modeled to have continuous states. We apply the framework to the term structure of nominal interest rates using a parsimonious model with a single state variable for the real economy, and two state variables for inflation. We model consumption growth as non-predictable with time-varying uncertainty, specified as affine GARCH volatility dynamics. Thus, macroeconomic uncertainty inhibits the same shocks as realized aggregate consumption growth, which has the advantage of restricting the numerical solutions to a single integration. Likewise, we define affine GARCH dynamics for inflation uncertainty, which impacts both realized and expected inflation.

The key ingredients to the model are disappointment aversion and preference for early resolution of uncertainty. Real bonds depend only on real uncertainty, while nominal bonds also depend on expected inflation and nominal uncertainty. The ability of the model to generate strong countercyclical risk aversion and a high volatility of the pricing kernel enables us to generate an upward (downward) sloping term structure of nominal bond yields (volatilities), consistent with the data. The model also accounts for the failure of the expectations hypothesis. We generate predictability in excess bond returns with substantial time variation in risk premia. Our model-implied regression coefficients are quantitatively close to the sample estimates obtained by most standard tests of the expectations hypothesis.

Investors are disappointed in bad states of the world. This begs the question of whether disappointment aversion improves over other preference frameworks for contingent claims securities that better capture tail events, such as interest rate options. We leave this question for future research.

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A Solutions to Welfare Valuation Ratios

The welfare valuation ratios are defined as:

$$\frac{V_t}{C_t} = G^V(\mathbf{X}_t) \quad \text{and} \quad \frac{\mathcal{R}_t(V_{t+1})}{C_t} = G^{\mathcal{R}}(\mathbf{X}_t), \tag{A.1}$$

where $\mathbf{X}_t$ characterizes the N-dimensional state vector that governs the real economy. Define the function $Z^{\mathcal{R}}(\cdot;\cdot)$ as:

$$Z^{\mathcal{R}}(\cdot;\cdot) = F\left(G^{\mathcal{R}}(\mathcal{S}(\cdot;\cdot))\right)\exp(\mathcal{O}(\cdot;\cdot)), \tag{A.2}$$

where $\mathcal{O}(\cdot;\cdot)$ and $\mathcal{S}(\cdot;\cdot)$ characterize two generic functions describing the observation and state equations in a state-space system, which we have specified as:

$$\begin{aligned} \Delta c_{t+1} &= \mathcal{O}(\mathbf{U}_{t+1};\mathbf{X}_t) \\ \mathbf{X}_{t+1} &= \mathcal{S}(\mathbf{U}_{t+1};\mathbf{X}_t), \end{aligned} \tag{A.3}$$

with $\mathbf{U}_{t+1}$ being a K-dimensional vector of independent and identically distributed shocks with density function $h(\mathbf{U})$ and support $\mathcal{U} \subseteq \mathbb{R}^K$. It follows from the recursion in Equation (2) and the certainty equivalent (5) that:

$$\begin{aligned} G^{\mathcal{R}}(\mathbf{X}_t) &= \left(\frac{E_t\left[\left(1+\ell I\left(Z^{\mathcal{R}}(\mathbf{U}_{t+1};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)\right)\left(Z^{\mathcal{R}}(\mathbf{U}_{t+1};\mathbf{X}_t)\right)^{1-\gamma}\right]}{1+\ell\kappa^{1-\gamma}E_t\left[I\left(Z^{\mathcal{R}}(\mathbf{U}_{t+1};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)\right]}\right)^{\frac{1}{1-\gamma}} \\ &= \left(\frac{\int_{\mathcal{U}}\left(1+\ell I\left(Z^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)\right)\left(Z^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t)\right)^{1-\gamma}h(\mathbf{U})\,d\mathbf{U}}{1+\ell\kappa^{1-\gamma}\int_{\mathcal{U}} I\left(Z^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)h(\mathbf{U})\,d\mathbf{U}}\right)^{\frac{1}{1-\gamma}}, \end{aligned} \tag{A.4}$$

where $\int_{\mathcal{U}}$ defines the integral over the domain $\mathcal{U}$. We solve equation (A.4) recursively using numerical integration. We initiate the recursion by conjecturing a solution $G_0^{\mathcal{R}}(\mathbf{X}_t)$. $G_1^{\mathcal{R}}(\mathbf{X}_t)$ is then obtained on a grid of values for $\mathbf{X}_t$, as:

$$G_1^{\mathcal{R}}(\mathbf{X}_t) = \left(\frac{\int_{\mathcal{U}}\left(1+\ell I\left(Z_0^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G_0^{\mathcal{R}}(\mathbf{X}_t)\right)\right)\left(Z_0^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t)\right)^{1-\gamma}h(\mathbf{U})\,d\mathbf{U}}{1+\ell\kappa^{1-\gamma}\int_{\mathcal{U}} I\left(Z_0^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G_0^{\mathcal{R}}(\mathbf{X}_t)\right)h(\mathbf{U})\,d\mathbf{U}}\right)^{\frac{1}{1-\gamma}}, \tag{A.5}$$

where $Z_0^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) = F\left(G_0^{\mathcal{R}}(\mathcal{S}(\mathbf{U};\mathbf{X}_t))\right)\exp(\mathcal{O}(\mathbf{U};\mathbf{X}_t))$. More generally, for any k, given a value for $G_k^{\mathcal{R}}(\mathbf{X}_t)$, we obtain the value of $G_{k+1}^{\mathcal{R}}(\mathbf{X}_t)$ on a grid of values for $\mathbf{X}_t$, as:

$$G_{k+1}^{\mathcal{R}}(\mathbf{X}_t) = \left(\frac{\int_{\mathcal{U}}\left(1+\ell I\left(Z_k^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G_k^{\mathcal{R}}(\mathbf{X}_t)\right)\right)\left(Z_k^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t)\right)^{1-\gamma}h(\mathbf{U})\,d\mathbf{U}}{1+\ell\kappa^{1-\gamma}\int_{\mathcal{U}} I\left(Z_k^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G_k^{\mathcal{R}}(\mathbf{X}_t)\right)h(\mathbf{U})\,d\mathbf{U}}\right)^{\frac{1}{1-\gamma}}, \tag{A.6}$$

where $Z_k^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) = F\left(G_k^{\mathcal{R}}(\mathcal{S}(\mathbf{U};\mathbf{X}_t))\right)\exp(\mathcal{O}(\mathbf{U};\mathbf{X}_t))$. The iterations are repeated until convergence to the fixed point, which is the solution $G^{\mathcal{R}}(\mathbf{X}_t)$ to Equation (A.4). This solution is unique, as is discussed in Backus et al. (2004) and also shown in Marinacci and Montrucchio (2010). Given the solutions of $G^{\mathcal{R}}$, we derive the solution of $G^V = F\left(G^{\mathcal{R}}\right)$. It is then straightforward to compute the disappointment probability ξ_t as:

$$\begin{aligned} \xi_t \equiv \xi(\mathbf{X}_t) &= E_t\left[I\left(Z^{\mathcal{R}}(\mathbf{U}_{t+1};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)\right] \\ &= \int_{\mathcal{U}} I\left(Z^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right) h(\mathbf{U})\, d\mathbf{U}. \end{aligned} \tag{A.7}$$

It follows from Equation (A.1) that the real stochastic discount factor is given by:

$$\begin{aligned} M_{t,t+1} = {} & \delta \exp(-\gamma \Delta c_{t+1}) \left(\frac{G^V(\mathbf{X}_{t+1})}{G^{\mathcal{R}}(\mathbf{X}_t)}\right)^{\frac{1}{\psi}-\gamma} \\ & \times \left(\frac{1 + \ell I\left(G^V(\mathbf{X}_{t+1})(C_{t+1}/C_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)}{1 + \ell \kappa^{1-\gamma} \xi(\mathbf{X}_t)}\right), \end{aligned} \tag{A.8}$$

which is derived given the solutions to the welfare valuation ratios V_t/C_t and $\mathcal{R}_t(V_{t+1})/C_t$ based on Equation (A.4), and the disappointment probability in Equation (A.7) .

B Solutions to Real and Nominal Bond Prices

The term structure of real interest rates is solved recursively, given that the price $P_{n,t} \equiv P_n(\mathbf{X}_t)$ of the n-period zero-coupon real bond satisfies the recursion:

$$P_n(\mathbf{X}_t) = E_t\left[M_{t,t+1} P_{n-1}(\mathbf{X}_{t+1})\right], \tag{B.1}$$

with the initial condition $P_{0,t} = 1$. The real bond price is computed recursively using numerical integration as follows:

$$\begin{aligned} P_n(\mathbf{X}_t) = {} & \delta \left(\frac{1}{G^{\mathcal{R}}(\mathbf{X}_t)}\right)^{\frac{1}{\psi}-\gamma} \left(\frac{1}{1 + \ell\kappa^{1-\gamma}\xi(\mathbf{X}_t)}\right) \\ & \times \int_{\mathcal{U}} \Big\{ \exp(-\gamma \mathcal{O}(\mathbf{U};\mathbf{X}_t)) \left(G^V(\mathcal{S}(\mathbf{U};\mathbf{X}_t))\right)^{\frac{1}{\psi}-\gamma} \\ & \qquad \times \left(1 + \ell I\left(Z^{\mathcal{R}}(\mathbf{U};\mathbf{X}_t) < \kappa G^{\mathcal{R}}(\mathbf{X}_t)\right)\right) P_{n-1}(\mathcal{S}(\mathbf{U};\mathbf{X}_t)) \Big\} h(\mathbf{U})\, d\mathbf{U}. \end{aligned} \tag{B.2}$$

To derive the solution for nominal bond prices, assume that the dynamics of the inflation rate process are governed by an L-dimensional real-valued vector process $\mathbf{Y}_t$ such that the joint moment generating function, conditional on the real vector of shocks $\mathbf{U}_{t+1}$, is given by:

$$E_t\left[\exp\left(a\pi_{t+1} + b^{\top}\mathbf{Y}_{t+1}\right) \mid \mathbf{U}_{t+1}\right] = \exp\left(A(a,b,\mathbf{X}_t,\mathbf{U}_{t+1}) + \mathbf{Y}_t^{\top} B(a,b)\right). \tag{B.3}$$

Given the assumption for the inflation dynamics, we conjecture and verify that:

$$P^{\$}_{n,t} = P^{\$}_{n}\left(\mathbf{X}_t\right)\exp\left(\mathbf{Y}_t^{\top} B^{\$}_{n}\right), \tag{B.4}$$

where the coefficients $B^{\$}_n$ satisfy the recursion $B^{\$}_n = B\left(-1, B^{\$}_{n-1}\right)$, with the initial vector-valued condition $B^{\$}_0 = \mathbf{0}$. We use the law of iterated expectations to ensure that the numerical integration applies only to the vector of real shocks $\mathbf{U}_t$. This allows us to derive the recursion for the sequence $\left\{P^{\$}_n\left(\mathbf{X}_t\right)\right\}$ as follows:

$$P^{\$}_{n}\left(\mathbf{X}_t\right) = E_t\left[M_{t,t+1}P^{\$}_{n-1}\left(\mathbf{X}_{t+1}\right)\exp\left(A\left(-1, B^{\$}_{n-1}, \mathbf{X}_t, \mathbf{U}_{t+1}\right)\right)\right], \tag{B.5}$$

with the initial condition $P^{\$}_0\left(\mathbf{X}_t\right) = 1$. The recursion (B.5) has no closed-form solution and is solved by numerical integration over a grid of values for $\mathbf{X}_t$. It follows that:

$$\begin{aligned}
P^{\$}_{n}\left(\mathbf{X}_t\right) = \delta\left(\frac{1}{G^{\mathcal{R}}\left(\mathbf{X}_t\right)}\right)^{\frac{1}{\psi}-\gamma}&\left(\frac{1}{1+\ell\kappa^{1-\gamma}\xi\left(\mathbf{X}_t\right)}\right)\\
\times\int_{\mathcal{U}}&\Big\{\exp\left(-\gamma\mathcal{O}\left(\mathbf{U};\mathbf{X}_t\right)+A\left(-1, B^{\$}_{n-1}, \mathbf{X}_t, \mathbf{U}_{t+1}\right)\right)\left(G^{V}\left(\mathcal{S}\left(\mathbf{U};\mathbf{X}_t\right)\right)\right)^{\frac{1}{\psi}-\gamma}\\
&\quad\times\left(1+\ell I\left(Z^{\mathcal{R}}\left(\mathbf{U};\mathbf{X}_t\right)<\kappa G^{\mathcal{R}}\left(\mathbf{X}_t\right)\right)\right)P^{\$}_{n-1}\left(\mathcal{S}\left(\mathbf{U};\mathbf{X}_t\right)\right)\Big\}\,h\left(\mathbf{U}\right)d\mathbf{U}.
\end{aligned} \tag{B.6}$$

C Welfare Valuation Ratios With Recursive Utility

We derive solutions to the welfare valuation ratios when the representative investor has Epstein and Zin (1989) preferences, without disappointment version ($\ell = 0$), and the Kreps and Porteus (1978) certainty equivalent. Define the log welfare valuation ratios $z_{V,t} = \ln\left(C_t \,/\, V_t\right)$ and $z_{\mathcal{R},t} = \ln\left(C_t \,/\, \mathcal{R}_t\left(V_{t+1}\right)\right)$, which are given by the two recursions:

$$\begin{aligned}
z_{V,t} &= -\frac{1}{1-\frac{1}{\psi}}\ln\left(\left(1-\delta\right)+\delta\exp\left(-\left(1-\frac{1}{\psi}\right)z_{\mathcal{R},t}\right)\right) \quad \text{if} \quad \psi\neq 1\\
&= \delta z_{\mathcal{R},t} \quad \text{if} \quad \psi = 1,
\end{aligned} \tag{C.1}$$

and

$$\begin{aligned}
z_{\mathcal{R},t} &= -\frac{1}{1-\gamma}\ln\left(E_t\left[\exp\left(\left(1-\gamma\right)\left(\Delta c_{t+1}-z_{V,t+1}\right)\right)\right]\right) \quad \text{if } \gamma\neq 1,\\
&= E_t\left[z_{V,t+1}-\Delta c_{t+1}\right] \quad \text{if} \quad \gamma = 1.
\end{aligned} \tag{C.2}$$

Solving for these ratios is standard in the literature and necessitates the use of the affine property of the dynamics of consumption growth, in conjunction with the log-linear approximation of the first recursion in Equation (C.1) around the average log welfare valuation ratio: $\bar{z}_{\mathcal{R}} = E\left[z_{\mathcal{R},t}\right]$,

$$z_{V,t} = q_0 + q_1 z_{\mathcal{R},t}, \tag{C.3}$$

where

$$q_1 = \frac{\delta \exp\left(-\left(1-\frac{1}{\psi}\right)\bar{z}_{\mathcal{R}}\right)}{(1-\delta)+\delta\exp\left(-\left(1-\frac{1}{\psi}\right)\bar{z}_{\mathcal{R}}\right)} \text{ and } q_0 = \frac{1}{1-\frac{1}{\psi}}\left[q_1 \ln\frac{(1-\delta)\,q_1}{\delta\,(1-q_1)} - \ln\frac{1-\delta}{1-q_1}\right]. \tag{C.4}$$

These coefficients are equivalent to the coefficients of the log-linear approximation of Campbell and Shiller (1988a) of the unobserved return on the claim over future consumption stream, around the average consumption-wealth ratio.

Given the consumption growth dynamics defined in equation (21), and the conjecture

$$z_{V,t} = \beta_{V0} + \beta_{V\sigma}\sigma_t^2 \text{ and } z_{\mathcal{R},t} = \beta_{\mathcal{R}0} + \beta_{\mathcal{R}\sigma}\sigma_t^2, \tag{C.5}$$

we use the method of undetermined coefficients and the property that

$$E\left[\exp\left(a\epsilon^2 + b\epsilon\right)\right] = \exp\left(-\frac{1}{2}\ln(1-2a) + \frac{b^2}{2\,(1-2a)}\right) \tag{C.6}$$

for any real numbers a and b and any standard normal random variable ϵ, to show that:

$$\begin{aligned}\beta_{\mathcal{R}0} &= \beta_{V0} - \mu_c + \beta_{V\sigma}\left((1-\phi_\sigma)\,\mu_\sigma - \nu_\sigma\right) + \frac{\ln\left(1+2\,(1-\gamma)\,\nu_\sigma\beta_{V\sigma}\right)}{2\,(1-\gamma)} \\ \beta_{\mathcal{R}\sigma} &= \phi_\sigma\beta_{V\sigma} - \frac{(1-\gamma)\left(1+2\nu_\sigma\beta_{V\sigma}\beta_\sigma\right)^2}{2\left(1+2\,(1-\gamma)\,\nu_\sigma\beta_{V\sigma}\right)}.\end{aligned} \tag{C.7}$$

The parameter $\bar{z}_{\mathcal{R}}$ is endogenous to the recursive utility model, and can be found as the solution to the non-linear fixed-point equation $\bar{z}_{\mathcal{R}} = \beta_{\mathcal{R}0} + \beta_{\mathcal{R}\sigma}\mu_\sigma$, since $\beta_{\mathcal{R}0}$ and $\beta_{\mathcal{R}\sigma}$ depend on q_0 and q_1, which in turn depend on $\bar{z}_{\mathcal{R}}$. The loglinear approximation $z_{V,t} = q_0 + q_1 z_{\mathcal{R},t}$ of the lifetime utility recursion implies that $\beta_{V0} = q_0 + q_1\beta_{\mathcal{R}0}$ and $\beta_{V\sigma} = q_1\beta_{\mathcal{R}\sigma}$, implying from Equation (C.7) that:

$$\begin{aligned}\beta_{V0} &= \frac{q_0}{1-q_1} + \frac{q_1}{1-q_1}\left[-\mu_c + \beta_{V\sigma}\left((1-\phi_\sigma)\,\mu_\sigma - \nu_\sigma\right) + \frac{\ln\left(1+2\,(1-\gamma)\,\nu_\sigma\beta_{V\sigma}\right)}{2\,(1-\gamma)}\right] \\ \beta_{\mathcal{R}0} &= \frac{\beta_{V0}-q_0}{q_1} \quad\text{and}\quad \beta_{\mathcal{R}\sigma} = \frac{\beta_{V\sigma}}{q_1},\end{aligned} \tag{C.8}$$

where $\beta_{V\sigma}$ is solution to the quadratic equation

$$\beta_{V\sigma}^2 - S\beta_{V\sigma} + P = 0 \tag{C.9}$$

with

$$
\begin{aligned}
S &= -\frac{(1 - q_1\phi_\sigma) + 2\,(1-\gamma)\, q_1 \nu_\sigma \beta_\sigma}{2\,(1-\gamma)\,\nu_\sigma\left(1 - q_1\left(\phi_\sigma - \nu_\sigma \beta_\sigma^2\right)\right)} \\
P &= \frac{q_1}{4\nu_\sigma\left(1 - q_1\left(\phi_\sigma - \nu_\sigma \beta_\sigma^2\right)\right)}.
\end{aligned}
\tag{C.10}
$$

Given the calibrated volatility parameters implying $\beta_\sigma > 0$, the quantities S and P are real and the quantity $S^2 - 4P$ is nonnegative as long as γ is smaller than γ^{max}, where γ^{max} is the largest value beyond which the model does not allow for a converging solution. In this case, the two solutions $\beta_{V\sigma}^-$ and $\beta_{V\sigma}^+$ to this equation, with $\beta_{V\sigma}^- \leq \beta_{V\sigma}^+$, are given by:

$$
\beta_{V\sigma}^- = \frac{S - \sqrt{S^2 - 4P}}{2} \quad \text{and} \quad \beta_{V\sigma}^+ = \frac{S + \sqrt{S^2 - 4P}}{2}. \tag{C.11}
$$

We follow Bollerslev et al. (2012) and choose $\beta_{V\sigma} = \beta_{V\sigma}^-$, such that $\underbrace{\lim}_{\alpha_\sigma \to 0} \alpha_\sigma \beta_{V\sigma}^+ = 0$.

Table 1: Model Parameter Calibration

In this table, we report model and preference parameter values, which are calibrated at a monthly decision interval.

Parameter		Value
Consumption Growth Dynamics		
Mean consumption growth	μ_c	0.0015
Persistence of volatility	ϕ_σ	0.9950
Volatility level	$\sqrt{\mu_\sigma}$	0.7305e-02
Volatility of volatility	σ_σ	0.6263e-04
Inflation Dynamics		
Mean inflation rate	μ_π	0.0030
Persistence of expected inflation	ϕ_z	0.9840
Inflation leverage on news	ν_π	-0.1294
Level of expected inflation shock volatility	ν_z	0.3457
Inflation Volatility level	μ_v	6.3698e-07
Persistence of inflation volatility	ϕ_v	0.8500
Level of residual inflation volatility	ν_v	9.5546e-08
Inflation leverage coefficient	β_v	-2.9827e+03
Preference Parameter Values		
Subjective discount factor	δ	0.9985
Intertemporal elasticity of substitution	ψ	1.5
Coefficient of relative risk aversion	γ	2.0
Coefficient of disappointment aversion	ℓ	1.0
Coefficient of generalized disappointment aversion	κ	0.95

Table 2: Cash-flows

In this table, we present moments of consumption and inflation dynamics from the data and the model. The data are real, sampled at an annual frequency, and cover the period 1929 to 2011. Standard errors are Newey-West with one lag. For the model, we report population statistics based on a simulation of 300,000 months. Consumption and price growth rates in the model are time-averaged. Data for consumption and price growth are taken from the Bureau of Economic Analysis National Income and Product Accounts Tables.

Consumption				Inflation			
Moment	Estimate	SE	Model	Moment	Estimate	SE	Model
$E[\Delta_c]$ (%)	1.97	0.28	1.79	$E[\pi]$ (%)	3.17	0.52	3.57
$\sigma[\Delta_c]$ (%)	2.02	0.38	2.08	$\sigma[\pi]$ (%)	3.29	0.76	2.89
$AC1[\Delta_c]$	0.48	0.12	0.24	$AC1[\pi]$	0.83	0.13	0.86
$Skew[\Delta_c]$	-1.58	0.67	-0.70	$Skew[\pi]$	-0.80	1.23	1.34
$Kurt[\Delta_c]$	9.63	2.29	7.88	$Kurt[\pi]$	8.64	2.13	6.15

Table 3: Asset Pricing Implications in Population

In this table, we report the term structure of nominal interest rates and the corresponding volatilities. All asset pricing implications in population are based on simulations of 300,000 months of data. Data statistics are based on the Fama-Bliss zero-coupon database from CRSP over the sample period 1964 until 2011.

Nominal Term structure of Interest Rates - Model					
	1y	2y	3y	4y	5y
Mean Yield (%)	5.26	5.44	5.71	5.78	5.95
Std (%)	2.08	1.93	1.82	1.75	1.71
Nominal Term structure of Interest Rates - Data					
	1y	2y	3y	4y	5y
Mean Yield (%)	5.20	5.40	5.58	5.73	5.83
Std (%)	3.03	2.97	2.90	2.85	2.78

Table 4: Prerence-based models for the Term Structure of Interest Rates

In this table, we summarize some of the key preference-based models that examine the term structure of nominal interest rates (time-separable CRRA preferences, Habit, Epstein-Zin recursive preferences, and Generalized Disappointment Aversion). We report the preference model, the key state variables for the real and nominal endowments, and the main preference parameters. The real endowment contains expected growth (x_t), time-varying volatility in expected growth (σ_t^x), time-varying volatility in realized growth (σ_t^c), jumps in consumption growth (J_t^c), surplus consumption ratio (s_t), expected inflation (z_t), volatility in expected inflation (σ_t^z), volatility in realized inflation (σ_t^π), jumps in inflation (J_t^z), and joint jumps in inflation and consumption growth ($J_t^{c,\pi}$). The reported preference parameters are the subjective discount factor δ, the coefficient of relative risk aversion γ, the coefficient of intertemporal elasticity of substitution ψ, the disappointment threshold κ, and the disappointment intensity ℓ. Wachter (2006) calibrates the subjective discount factor at a quarterly frequency with $\delta = 0.98$. Le et al. (2010) calibrate the subjective discount factor at a quarterly frequency with $\delta = 0.9904$. We relate to their calibration scheme CS in Table 1. Gabaix (2012) uses a time preference parameter ρ equal to 6.57%, which we map into a subjective discount factor at a monthly frequency of $\delta = \exp(-\rho)^{(1/12)}$ for comparability. Tsai (2016) uses a time preference parameter of 0.010 in continuous time, which we have mapped into a comparable monthly value of $\delta = \exp(-\rho \times (1/12)) = 0.9992$. Preference parameters for Creal and Wu (2016) are based on their model 2 benchmark estimation (Table 2, global maximum).

	Preferences				Endowment										Benchmark Parameters				
Study	CRRA	Habit	EZ	GDA	x_t	σ_t^x	σ_t^c	J_t^c	s_t	z_t	σ_t^z	J_t^z	σ_t^π	$J_t^{c,\pi}$	δ	γ	ψ	κ	ℓ
Piazzesi and Schneider (2006)			✓		✓					✓					1.0050	59	1	–	–
Wachter (2006)		✓							✓	✓					0.9933	2	–	–	–
Le et al. (2010)		✓			✓				✓	✓					0.9968	2.20	–	–	–
Bansal and Shaliastovich (2013)			✓		✓	✓				✓	✓				0.9940	20.9	1.81	–	–
Gabaix (2012)	✓							✓		✓		✓			0.9945	4	–	–	–
Tsai (2016)			✓					✓		✓		✓		✓	0.9992	3	1	–	–
Creal and Wu (2016)		✓	✓		✓	✓	✓		✓	✓	✓		✓		0.9998	6.75	1.02	–	–
This study				✓			✓			✓	✓				0.9985	2	1.5	0.95	1

Table 5: Benchmark Regressions

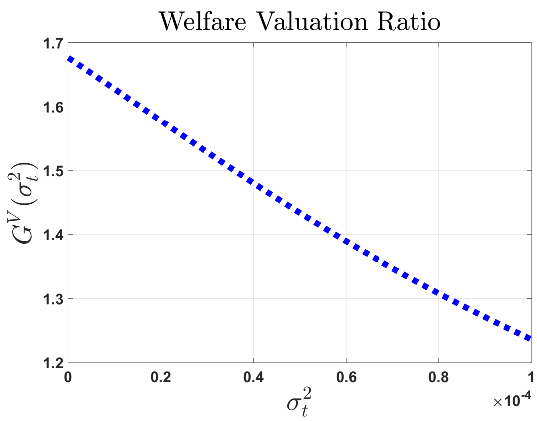
Panel A reports the restricted Cochrane-Piazzesi regressions from the projection of holding period returns on the single CP factor; Panel B reports the Fama-Bliss regression results from the projection of holding period returns on forward-spot spreads; Panel C reports the Campbell-Shiller regressions from the projection of changes in long rate spreads on yield-spot spreads; Panel D reports the Dai-Singleton regressions from the projection of adjusted changes in long rate spreads on yield-spot spread.

	Model					**Data**			
Panel A: Cochrane-Piazzesi: regression of holding period returns on single CP factor $rx^{(n)}_{t+12} = b_n\left(\gamma^\top f_t\right) + \varepsilon^{(n)}_{t+12}$									
	n=2	n=3	n=4	n=5		n=2	n=3	n=4	n=5
b_n	0.42	0.82	1.20	1.56	b_n	0.46	0.86	1.25	1.44
HH,12 lags	0.01	0.02	0.02	0.03	HH,12 lags	0.08	0.16	0.22	0.27
R^2	0.19	0.19	0.20	0.20	R^2	0.21	0.23	0.26	0.24
Panel B: Fama-Bliss: regression of holding period returns on forward-spot spread $rx^{(n)}_{t+12} = \alpha^{(n)} + \beta^{(n)}\left(f^{(n)}_t - y^{(1)}_t\right) + \varepsilon^{(n)}_{t+12}$									
	n=2	n=3	n=4	n=5		n=2	n=3	n=4	n=5
β_n	0.77	0.82	0.86	0.90	β_n	0.83	1.14	1.38	1.10
HH,12 lags	0.02	0.02	0.02	0.02	HH,12 lags	0.26	0.35	0.41	0.46
NW,18 lags	0.02	0.02	0.02	0.02	NW,18 lags	0.23	0.31	0.36	0.42
R^2	0.09	0.10	0.10	0.11	R^2	0.12	0.13	0.15	0.08
Panel C: Campbell-Shiller: regression of changes in long rate spreads on yield-spot spread $y^{(n-1)}_{t+12} - y^{(n)}_t = \alpha^{(n)} + \beta^{(n)}\frac{1}{n-1}\left(y^{(n)}_t - y^{(1)}_t\right) + \varepsilon^{(n)}_{t+12}$									
	n=2	n=3	n=4	n=5		n=2	n=3	n=4	n=5
β_n	-0.55	-0.59	-0.62	-0.66	β_n	-0.67	-1.07	-1.47	-1.48
HH,12 lags	0.04	0.04	0.04	0.04	HH,12 lags	0.53	0.64	0.70	0.74
NW,18 lags	0.04	0.04	0.04	0.04	NW,18 lags	0.47	0.56	0.62	0.66
R^2	0.01	0.01	0.02	0.02	R^2	0.02	0.04	0.06	0.06
Panel D: Dai-Singleton: regression of adjusted changes in long rate spreads on yield-spot spread $y^{(n-1)}_{t+12} - y^{(n)}_t + \frac{1}{n-1}\hat{E}_t\left[r^{(n)}_{t+12} - y^{(1)}_t\right] = \alpha^{(n)} + \beta^{(n)}\frac{1}{n-1}\left(y^{(n)}_t - y^{(1)}_t\right) + \varepsilon^{(n)}_{t+12}$									
	n=2	n=3	n=4	n=5		n=2	n=3	n=4	n=5
β_n	0.99	0.99	1.00	1.00	β_n	1.10	1.13	1.02	0.74
HH,12 lags	0.04	0.04	0.04	0.04	HH,12 lags	0.46	0.57	0.63	0.66
NW,18 lags	0.04	0.04	0.04	0.04	NW,18 lags	0.42	0.51	0.56	0.59
R^2	0.04	0.04	0.05	0.05	R^2	0.06	0.05	0.04	0.02

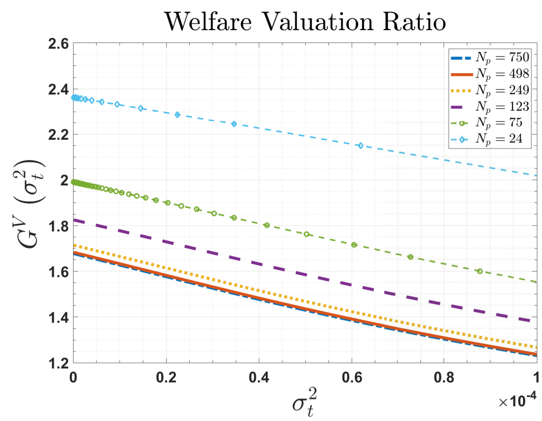
Figure 1: Model Solutions

In Figure 1a, we plot the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility σ_t^2 for our benchmark scenario with 501 grid points. In Figure 1b, we plot the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility σ_t^2 for different grids, ranging from 24 to 750 points. In Figure 1c, we report the probability of disappointment $\xi(\sigma_t^2)$ as a function of consumption volatility σ_t^2.

(a)



(b)



(c)

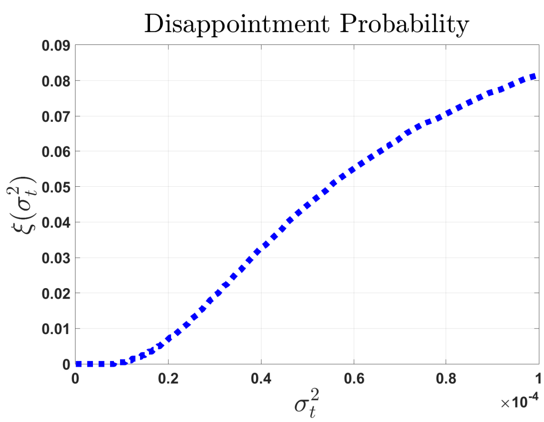


Figure 2: Accuracy of Numerical Solution Method

In this figure, we plot the welfare valuation ratio $V_t/C_t = G^V(\sigma_t^2)$ as a function of consumption volatility, σ_t^2, for an investor with Epstein-Zin recursive utility without disappointment aversion. For this case, we report both the analytical solution (dotted line) and the numerical (solid) solution for our benchmark scenario with 501 grid points. The consumption growth parameters are identical to those reported in Table 1. The preference parameters for this example are $\delta = 0.9989$, $\psi = 1$, and $\gamma = 4$.

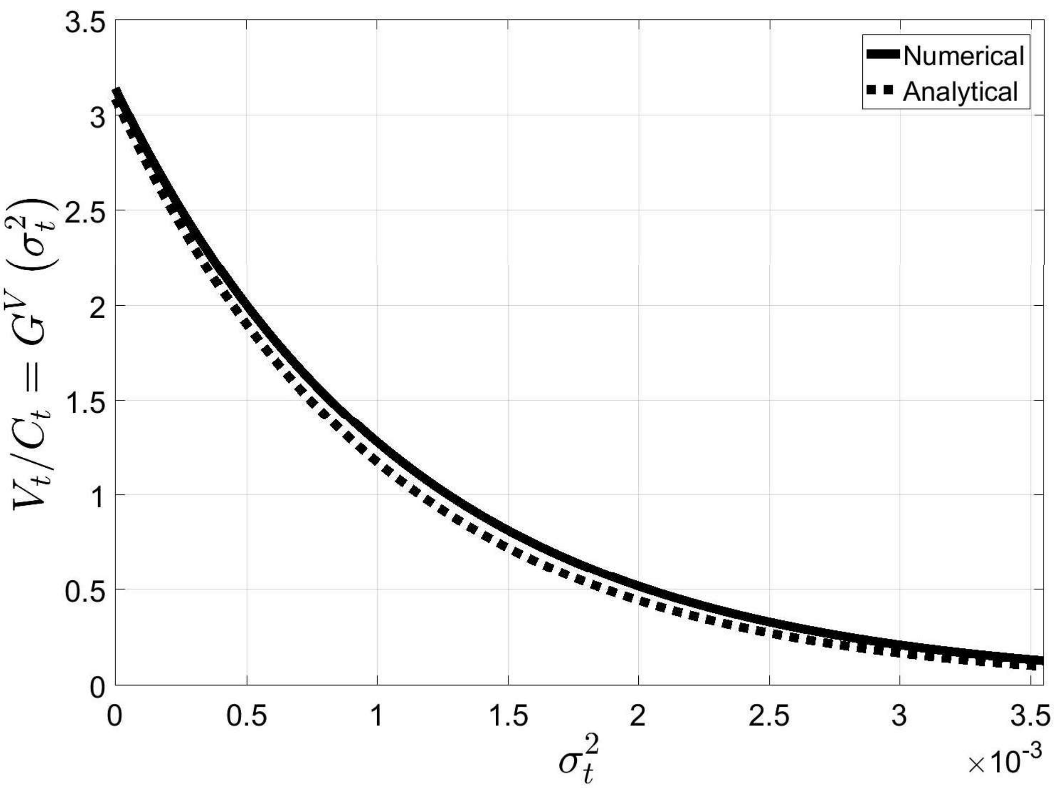
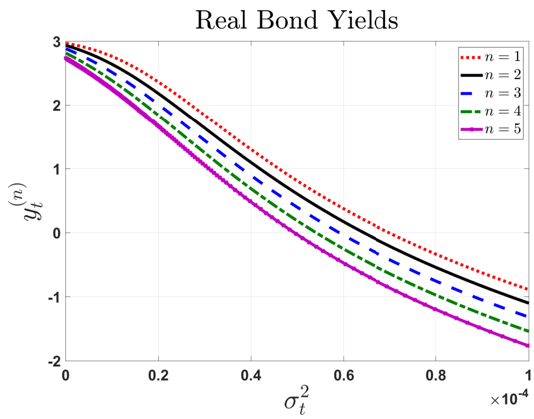


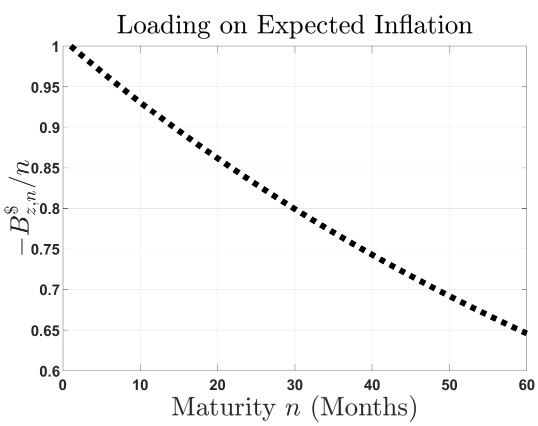
Figure 3: Asset Pricing Solutions

In this figure, we plot asset pricing solutions for our benchmark scenario with 501 grid points. In Figure 3a, we plot the real yields, $y_t^{(n)}$, as a function of consumption volatility for maturities $n = 1$ year to $n = 5$ years. In Figures 3b and 3c, we plot the sensitivities of nominal bond yields, $y_t^{(n)}$, to expected inflation $(-B^{\$}_{z,n}/n)$ and inflation volatility $(-B^{\$}_{v,n}/n)$, respectively.

(a)



(b)



(c)

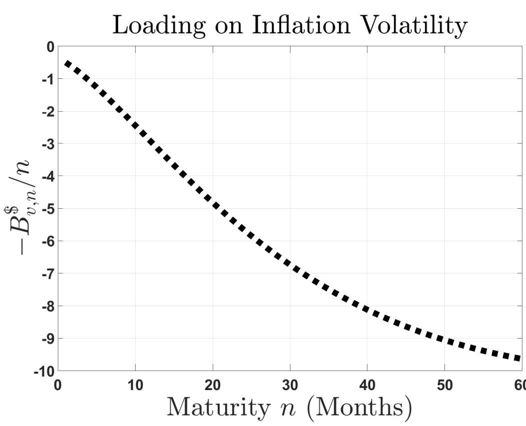


Figure 4: Nominal Bond Yields

In this figure, we plot nominal bond yields, $y_t^{\$,(n)}$, as a function of consumption volatility for maturities $n = 1$ year to $n = 5$ years when expected inflation and inflation uncertainty are fixed at their long-run values.